Ambiguity and the brain: a probabilistic perspective

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Neuroeconomics
A „rational“ decision

second order probability: \( p_{x.2}(x \text{ white balls in urn B}) \sim B(n, 1/2) \)
conditional first order probability: \( p_{x.1}(\text{white}|x \text{ white balls}) = x/n \)
resulting first order probability:
\[
p(\text{black}|\text{urn B}) = \sum p_{i.2} \cdot p_{i.1} = 0.5
\]

Ellsberg (1961)
The rational and irrational agents

information

The rational agent

decision

irrationality
How does information reach the agent?
The rational and irrational agents

- Information
  - The rational agent
  - Decision
Perception: Integrating noisy information

More precision - more weight
Less precision - less weight

\[ f(\theta) \propto L(\text{visual}) \times L(\text{auditory}) \times \text{prior} \]
The „Bayesian“ Brain

Classical statistics:
parameters are unknown, but fixed
point estimation of parameters
uncertainty in the estimation: confidence interval, based on sample distribution of the parameter estimate

Bayesian statistics:
parameters are treated as random variables
estimation of parameters depends on loss function
uncertainty of the parameter is explicitly described as a probability distribution
„Bayesian probability“: a degree of belief
... a few consequences

(1) for each parameter, we need to represent a degree of belief about their possible values: we can then summarise the ensuing distribution by describing its „uncertainty“ or „imprecision“

(2) we need to specify, and take into account, a prior belief on the expected states of the environment

(3) after making our inference, we can specify a predictive distribution of what to expect next, taking the uncertainty on the estimation into account (... predictive coding)
Neuroeconomics

Assuming a rational agent, can we deduce the structure of the available information, and the prior beliefs?
The rational and irrational agents

information

The rational agent

decision
Prior beliefs: ... is there certainty out there?

and does it depend on the state?

*rule of law, supermarket*
Neuroeconomics

Can economic paradigms enable us understand how the brain handles uncertainty?
What we can be uncertain about

- Sensory uncertainty
  - Environmental uncertainty
  - Internal noise

- State uncertainty
  - Current state
  - Inference of current state

- Rule uncertainty
  - Probability of transition to state A
  - Probability of transition to state B

- Outcome uncertainty
  - Possible future state A
  - Possible future state B

Ambiguity and Risk

Rule uncertainty in reinforcement learning

posterior uncertainty of rule predictions can arise from large prediction errors ("unexpected uncertainty")

induces more updating of associations (Pearce-Hall model)

not under experimental control

Yu & Dayan (2005); Daw, Niv, & Dayan (2005)
How do the two urns differ?

(1) Rule uncertainty: „missing information about probabilities“
(2) other factors: saliency, social factors, need for information search

Ambiguity aversion is reduced when decisions are made privately
Ambiguity aversion is reduced when missing information is not potentially knowable
(i. e. when uncertainty is indeed about future states)

Camerer 1995; Curley et al. (1986); Trautmann et al. (2008); Chow & Sarin (2002)
„Brain responses“ to ambiguity

\[(AC + AR) > (RC + RR)\]

Huettel et al. (2006)
Experiment 1. Knowable or not?

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
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<td>set 1</td>
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<tr>
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<td>CS+</td>
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F

<table>
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<th>frequency</th>
<th>averaged shock probability</th>
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<td>0.75</td>
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<td>8</td>
<td>0.38</td>
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Frequency: averaged shock probability

A

<table>
<thead>
<tr>
<th>outcome probability</th>
<th>preceding Pavlovian conditioning 80 trials</th>
<th>scanning study 150 trials (2 sessions)</th>
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<tbody>
<tr>
<td>.75</td>
<td>20 trials</td>
<td>risk, ambiguity, ignorance</td>
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<tr>
<td>.50</td>
<td>45 trials</td>
<td>45 trials, 45 trials</td>
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<tr>
<td>.25</td>
<td>20 trials</td>
<td>45 trials</td>
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<td>.00</td>
<td>20 trials</td>
<td>15 trials</td>
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</table>

B

In ambiguous trials, the original cue is shown

Jittering ITI: 6.3 s ± 1.5 s

Experiment 1. Knowable or not?

Second order probabilities

Experiment 1. Knowable or not?

Conclusions

Brain responses to ambiguity in pIFG and pPAR are
(a) independent of choice, independent of gain/loss
(b) not due to rule uncertainty, but to some other factor

What other factor(s) could that be?
Experiment 2. Can we quantify rule uncertainty?

\[ p_{i.1} \in \{0.2, 0.5, 0.8\} \]

\[ p_{i.2} = p(\text{color } i | \text{position}) \]

\[ p_1 = \sum p_{i.1} \times p_{i.2} \]

Shannon entropy:
\[ H = - \sum p_{i.2} \times \log(p_{i.2}) \]
Experiment 2. Can we quantify rule uncertainty?

144 learning trials (operant conditioning), results from final 24

Bach, Hulme, Penny, & Dolan (2011) J Neurosci
Experiment 2. Can we quantify rule uncertainty?

<table>
<thead>
<tr>
<th>Non-ambiguous („risky“)</th>
<th>84</th>
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<tr>
<td>Ambiguous</td>
<td>276</td>
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<tr>
<td></td>
<td>360</td>
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<tr>
<td>Ambiguous: Effect of entropy</td>
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<td></td>
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Experiment 2. Can we quantify rule uncertainty?

Table 1. Summary of models analyzed to account for the influence of ambiguity and second-order uncertainty on choice (see also Fig. 3)

| Models accounting for ambiguity | Models accounting for ambiguity and second-order uncertainty
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Baseline</td>
<td>Baseline SOP</td>
</tr>
<tr>
<td>Utility weighting</td>
<td>Utility weighting model, using different utility functions for ambiguous and nonambiguous decisions</td>
</tr>
<tr>
<td>EU weighting additive</td>
<td>EU weighting additive model, adding a constant to the expected utility of ambiguous choices</td>
</tr>
<tr>
<td>EU weighting multiplicative</td>
<td>EU weighting multiplicative model, multiplying the expected utility of ambiguous choices with a constant</td>
</tr>
<tr>
<td>Ep weighting</td>
<td>Ep weighting model, nonlinearly weighting the expected probabilities in ambiguous choices by exponentiating them with a constant (Hsu et al., 2005)</td>
</tr>
<tr>
<td>Pessimistic weighting</td>
<td>Pessimistic weighting model, biasing the second-order probabilities toward the worse scenario in ambiguous choices (Ghirardato et al., 2004; Huettel et al., 2006)</td>
</tr>
<tr>
<td>Minimax</td>
<td>Minimax model, where choice in ambiguous trials is based on the worse scenario only</td>
</tr>
<tr>
<td>SOP</td>
<td>SOP model, in which the conditional expected outcomes are nonlinearly weighted before being combined with the unbiased second-order probabilities into an expected outcome (Segal, 1987; Kilbanoff et al., 2005)</td>
</tr>
<tr>
<td>Combined SOP</td>
<td>Combined SOP model, in which the second-order probabilities are additionally weighted by entropy</td>
</tr>
</tbody>
</table>

*Based on the second-order probability (SOP) model, additional models were formed that accounted for entropy within ambiguous choices by having an additional free parameter that is modulated by mean-centered entropy. EU, Expected utility; Ep, expected probability.*
Experiment 2. Can we quantify rule uncertainty?

**Diagram 1:** Ambiguity vs. non-ambiguity

- Baseline
- Utility weighting
- EU weighting additive
- EU weighting multiplicative
- Ep weighting
- Pessimistic weighting
- Minimax
- SOP

**Diagram 2:** Entropy within ambiguous gambles

- Baseline SOP
- Utility weighting
- EU weighting additive
- EU weighting multiplicative
- Ep weighting
- Pessimistic weighting
- Combined SOP

Bach, Hulme, Penny, & Dolan (2011) J Neurosci
Experiment 2. Can we quantify rule uncertainty?

A-D: Second order certainty. E: Ambiguity vs. non-ambiguity. F: Relation of ambiguity vs. non-ambiguity with ambiguity aversion on a between-subject level

Bach, Hulme, Penny, & Dolan (2011) J Neurosci
Experiment 2. Can we quantify rule uncertainty?

Conclusions I

Ambiguity vs. non-ambiguity on the one hand, and second-order uncertainty with ambiguous gambles on the other

(a) covary with different brain activations

(b) both increase aversion to those gambles, but via different computational algorithms

Bach, Hulme, Penny, & Dolan (2011) J Neurosci
Experiment 2. Can we quantify rule uncertainty?

Conclusions II

Ambiguity vs. non-ambiguity is not a good model for rule uncertainty

Rule uncertainty can be manipulated and investigated as entropy over second-order probabilities (rule probabilities)

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What we can be uncertain about

- Sensory uncertainty
  - Environmental uncertainty
  - Internal noise

- State uncertainty
  - Current state
  - Probability of transition to state A
  - Probability of transition to state B

- Rule uncertainty
  - Transition rules
  - Possible future state A
  - Possible future state B

- Outcome uncertainty
  - Future states
  -entropy in ambiguous gambles
  -risk

Neuroeconomics