Using Prospect Theory to Analyze New Risks (Ambiguity) in a Large Representative Sample and to Explain Real Investment Decisions

by Peter P. Wakker, Erasmus Sch. Econ. Rotterdam Univ, the Netherlands; joint with:
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(b) Steve Dimmock & Roy Kouwenberg

Amb. in the Ec. & Ecs, Nijmegen, 05Oct12

Purposes: 1. Present a descriptive model of ambiguity. Convince you that it is (a) well founded & (b) easy to use; You can do it at your home!
2 (if time) Example of application (to finance): ambiguity → nonparticipation puzzle.
Outline:

1. Motivation, history (Ellsberg) & preparation for our contribution;
2. Our contribution: Setup of experiment, analysis, & predictions;
3. Underlying theory (source method);
4. Experiment & results;
5. Discussion/Conclusion.
Ellsberg paradox

Known urn K

<table>
<thead>
<tr>
<th></th>
<th>R</th>
<th>B</th>
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<tr>
<td>50</td>
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Ambiguous urn A

<table>
<thead>
<tr>
<th></th>
<th>R &amp; B in unknown proportion</th>
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<tr>
<td>?</td>
<td>100–?</td>
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\[
\begin{align*}
P(R_K) & > P(R_A) \\
P(B_K) & > P(B_A)
\end{align*}
\]

Common, classical interpretation:
Violates subjective probabilities:

Violates subjective probabilities!
Subjective probabilities surely do not work in any classical way.

Understood since 1921/1961: In the economy and in economics we sorely need something fundamentally new.

Lasted till late 1980s until someone was clever enough to find something fundamentally new:

Gilboa & Schmeidler (‘87, ’89). This explains
- why something as important as ambiguity, only took off in late 1980s;
- why we have to catch up much;
- why ambiguity is so popular today;
- why this conference today.
But:
Gilboa & Schmeidler’s models are normative. Our aim today is descriptive.
Currently popular models of ambiguity:

- **Smooth model** (Klibanoff, Marinacci, Mukerji 2005);
- **Choquet expected utility** (CEU/RDU; Gilboa 1987; Schmeidler 1989);
- **Multiple priors** (maxmin, $\alpha$-maxmin, ...; Chateauneuf 1991; Gajdos, Hayashi, Tallon, & Vergnaud 2008; Ghirardato, Maccheroni, Marinacci 2004; Gilboa & Schmeidler 1989; Jaffray 1994; Luce & Raiffa 1957 Ch. 13; Wald 1950);
- **Robust** (Hansen & Sargent 2001);
- **Variational** (Maccheroni, Marinacci, Rustichini 2006; cf. Chateauneuf & Faro 2009);
- **Vector** (Siniscalchi 2009);
- **2-stage maxmin** (Jaffray 1989; Olszewski 2007);
- **Multiplier**: Strzalecki (2011).

- Franz van Winden will better organize this plethora!? 

- **But**: **All normative!** (& Most are too general for empirical work.)

Abdellaoui, Baillon, Placido, & Wakker’11 (AER) made prospect theory tractable for the lab (source method). Could get ambiguity premiums and the like.
Hey, Lotito, & Maffioletti’2011, in (only) comparative study of ambiguity models today, claims: prospect theory < multiple priors. But did not use source method and used wrong formulas.

Vitalie Spinu et al’2012 show, after correction + source method: prospect theory > all others!

To explain the source method, first an idea by Chew & Sagi (2008): Revival of subjective probabilities for ambiguity!
Ellsberg paradox

**Known urn K**

- 50 R
- 50 B

**Ambiguous urn A**

- 100 R&B in unknown proportion

- (R^K: €15)
- (B^K: €15)

- P(R^K) > P(R^A)
- P(B^K) > P(B^A)

Common, classical interpretation:
Violates subjective probabilities:

More precisely:
(they use a hidden assumption)

Violates subjective probabilities + same weighting of probs for two urns (probabilistic sophistication).
Expained more on next slide.
Ellsberg paradox

**Known urn K**

- 50 R
- 50 B

**Ambiguous urn A**

- 100 R&B in unknown proportion

Chew & Sagi (2008):

\[
P(R_K) = \frac{1}{2} \quad P(B_K) = \frac{1}{2}
\]

Weighted more pessimistically! Gives up probabilistic sophistication.
Thus: subjective probabilities are not completely excluded by Ellsberg. More than that: Chew & Sagi gave symmetry-type preference conditions justifying subj. probs. Abdellaoui et al. (2011) found the conditions empirically satisfied.

Revival of subjective probabilities for ambiguity!

For our Ellsberg urn A, $P(R_A) = P(B_A) = \frac{1}{2}$ must be, because of symmetry.
Abdellaoui et al. had to do complex measurements.

We (Dimmock, Kouwenberg, & Wakker) make the source method tractable for the field. So, now to our contribution:
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5. Discussion/Conclusion.
We did experiment with LISS panel: representative sample from the Netherlands. N = 1935 subjects. Paid €7,650 for real incentives.

Abdellaoui et al. (2011) needed 30 minutes per academic subject to measure ambiguity. Measured U, P, w, source functions wₜ, etc. We had 5 minutes per non-academic subject.

So we needed a powerful tool for analyzing ambiguity:

Matching probabilities.

Remember: You can do this at your home!
Matching probability of $R_A$:

We find $X$ s.t. indifference here.

We write

$X/100 = \text{matching probability of } R_A = M(0.5)$.

Kahn & Sarin (1988) heuristically used

$0.5 - M(0.5)$ as index of ambiguity aversion;

$0.5 - M(0.5) > 0$: ambiguity aversion;

$0.5 - M(0.5) = 0$: ambiguity neutrality;

$0.5 - M(0.5) < 0$: ambiguity seeking.

Plausible indexes! We will use more sophisticated indexes, explained later. We first discuss various general ambiguity attitudes.
Matching probabilities $M(p)$ can also be determined for ambiguity-neutral probabilities $p \neq 0.5$. E.g., $p = 1/10$ or $p = 9/10$ by taking unknown urn with 10 colors.
Examples of matching probability functions & ambiguity attitudes

- Ambiguity neutrality: homoeconomicus(!?))
- Ambiguity aversion
- Convex: prevailing finding
- Inverse-S: Insensitivity; ambiguity-generated (a-insensitivity)

This is Ellsberg’s ambiguity-seeking prediction for unlikely. Is descriptively important. This is a-insensitivity.

Maybe Dominik Bach will give neurofoundation for two components this afternoon.

Empirical findings deviate from theoretical wisdom. A-insensitivity is descriptively important.
For econometric regressions we want quantitative indexes of:
- ambiguity aversion,
&
- a-insensitivity.

As analogs of traditional risk aversion indexes.

Abdellaoui et al. (2011 AER) introduced them (idea of Craig Fox):
1. Take best-fitting line on (0,1);
2. c: intercept; d: anti-intercept;
3. **ambiguity aversion (anti-elevation):**
   \[ b = d - c. \]
4. **a-insensitivity (anti-slope):**
   \[ a = 1 - \text{slope} = c + d; \]

Indexes are heuristically plausible at first sight (Kahn-Sarin). There is more to them: a decision-theoretical model (the source method).
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3. Decision theory: foundation for matching probabilities to capture ambiguity attitudes

We use source method of Abdellaoui et al. (2011). Is descriptive!
Summarizing evaluation of prospect $x_E y$:

$$w(m_{S_0}(P(E)))U(x) + (1-w(m_{S_0}(P(E))))U(y).$$

Utility (Bernoulli 1738)
subj./amb.neutral prob. (Savage’54; Chew & Sagi’08)
Probability weighting for nonexpected utility
under risk (Allais’53)

ambiguity function (Ellsberg’61).

How measure ambiguity function $m_{S_0}$??

Abdellaoui et al. (2011) measured whole model.
Took half an hour per student.
LISS: only 5 minutes per nonacademic subject.
Must simplify!

We resorted to the following very convenient algebraic result:
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4. Experiment

LISS panel:
representative sample from the Netherlands.
N = 1935 subjects; €7,650 real incentives.
We used urns with 10 balls and measured
M(0.1), M(0.5), M(0.9) (for gains ...).

We first tested with N = 85 students: OK!

I only report n = 675
(real incentives & complete financial data).
Results:

Literature-lab findings are confirmed:
(1) Much ambiguity aversion.
(2) Also much a-insensitivity.

Ad (2):
For unlikely (1:10) event ambiguity seeking!
As predicted by Ellsberg;
agreeing with all empirical studies;
going against all common theoretical ideas.
Our results on portfolio choices:

1. Ambiguity aversion: unrelated to market participation. ????
2. Insensitivity ↓ market participation. ?
   (Also ↓ private business ownership.)
   (Also: non-trust (Guiso, Sapienza, & Zingales J. of Finance’08) ↓ nonparticipation if strong ambiguity perception.)

Result 1 is surely very surprising.

**Trautmann** already showed:
Classical views on ambiguity need readjustement for empirical applications.

Both results we found can be explained by: reference dependence.
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Reference dependence:

Stocks involve possible losses. ambiguity aversion: for losses $\neq$ for gains. Cohen, Jaffray, & Said (1987): no empirical relation between gains & losses at the individual level. (Trautmann!?)

Our Ellsberg-choice measurements were for gains (traditional!). Do not speak to losses.
Explanation for insensitivity using reference dependence:

- Insensitivity is cognitive. Will be the same for gains & losses. So our gain-measurements do give predictions for losses here.
- + loss aversion: overweighting losses > overweighting gains.

Hence negative impact.
Conclusion

Descriptive source method is well implementable (matching probabilities!)
You can do it at your home!

Is theoretically well founded.

For general population, besides ambiguity aversion, also a-insensitivity.

Ambiguity can explain part of nonparticipation puzzle. Mostly a-insensitivity does so.

We should study losses!
The end