Dynamic segmentation in a static environment
Radboud University Nijmegen

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Abstract

This thesis has two major research purposes: (1) to investigate the implications of transparency concerning credit risk management practices by banks, guided by Basel Accord I, II and III, and (2) to examine the effects of dynamic segmentation for mortgage portfolio hedges. For the first purpose, (1), a literature study was conducted using publications from the Basel Committee on Banking Supervision, and published analyses thereupon. The progress of risk regulations and banks’ responses to these regulations has been studied. It is concluded that part of the regulations in the Accords have effectuated opposite of what they aimed. For the second part (2), interest rate paths were provided by ING Bank’s model validation department, mortgage portfolios used were simplified versions of ING Bank’s 2014 mortgage portfolio. Net Present Value and Basis Point Value were used as measures to compare static and dynamic segmentation models. It can be concluded that a dynamic segmentation model is more robust against bumps in the interest rate path. Furthermore, the dynamic segmentation model yields higher margins.
Preface

This thesis concludes my Master’s programme in Mathematics. During my years of study at the Radboud University I have studied different fields within Mathematics, and applied that knowledge through internships outside the University. During my studies, I have gotten to appreciate the beauty of Mathematics. The existence of a single truth within its own laws is what I find most beautiful about Mathematics. I have especially enjoyed specializing in Stochastics, my favourite field within Mathematics, because this field bravely attempts to apply the flawless logic of Mathematics onto a flawed real world.

The nature of this thesis - a combination of Mathematics and Management Sciences - has challenged me to step outside my comfort zone, and helped me acquire new ways of thinking. I am grateful that the Faculty of Sciences offers this programme to its students.

I would like to thank Heleen de Coninck for her ongoing support, patience and trust during the writing of this thesis. Also, I want to thank Arjan van Rooij for helping me reflect on my experiences during my internship.

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Chapter 1

Introduction

The focus of this thesis is on financial risk management practices by banks. The study covers the development of regulations on risk management practices, the impact of these supervisory regulations, and practical examples of risk management. Furthermore, we go into detail on a market risk model that can be used to hedge the risk on mortgage portfolios. We cover the mathematical theory, model details and an analysis of the model’s performance.

The contents of this thesis find their theoretical underpinning in two fields of study. The first being economics and business, the second being stochastics and financial mathematics.

1.1 Host organization

From February until June 2014, I was based at the Market Risk Management (MRM) department of ING Domestic Bank NL. This department’s projects and tasks are organized into four teams: Mortgages, Asset & Liability management, IT and Savings, Payments & Loans. These four teams cooperate to manage the market risk on the mortgage portfolio, the savings and payment accounts, and loans.

The research conducted in this thesis is mostly related to the work of the Mortgages team. At ING I have studied a dynamic model for mortgage portfolio segmentation and the general route of risk model design and implementation within the bank.
1.2 Problem description

Financial risk management is one of the major disciplines within any bank. It is a complex topic and remains a large field of study in the financial world as well as in the academic world. The types of financial risks that banks incur range from client risk and credit risk, to liquidity risk and market risk. All types of risk require their own management strategy. Risk management - in any form - is subject to international and national regulations.

In 1998, Altman and Saunders published an article describing the development of risk measurement practices in the last 20 years. One year later, the General Manager of the Bank for International Settlements was appointed Chair of the Financial Stability Forum (FSF), a forum created by the G7 Finance Ministers and Central Bank Governors in 1999. The goal of the forum was to rethink the global financial architecture after the Asian crisis of 1997 and the Russian crisis of 1998. To prevent a global financial crisis, there was a call for increased regulations on financial institutions, which included regulations for risk management. The Basel committee did respond to the request with regulations and minimum capital requirements for financial institutions, but a financial crisis could not be averted. Although the regulations on credit risk models and capital requirements were public, the implementations of the models by banks were not. We will dive into the technical aspects of the regulations, and study whether they were logical choices, and we will have a brief look on why the crisis could still find its way, despite the large amount of regulatory precautions.

1.3 Research questions

To guide risk management, the Bank for International Settlements (BIS) has published three major accords on banking practices that are supported by financial institutions and banks worldwide. Good risk management practices and good models are a requirement for financial stability. Although a lot of academic research on credit risk modeling has been done, information about implementation of these models by banks is hardly available. The BIS, aiming to serve central banks in their pursuit of monetary and financial stability, states that "Monetary and financial stability is a precondition for sustained economic growth and prosperity". [Bank for International Settlements, 2014] Over the years the Basel Committee of the BIS has agreed on a series of accords that provide regulations, guidelines and principles for financial risk management. These publications have been established in cooperation with the G10 countries, and have been adopted by central banks and supervising authorities in the G10 countries. Hence, the main question to be answered in this thesis is

“How can banks improve their financial risk management in a transparent and effective manner?”

In chapter 3 we study the interaction between the BIS and the banks and its influence on risk management practices by banks. We will do so by answering the following questions:

A.1 How transparent are the risk modeling practices of European banks?

A.2 What are the societal impacts of a lack of transparency on risk models?

A.3 What are the benefits of transparency on risk management by banks for the public and what would be the drawbacks?

After having studied the creation and development of the regulations and the impact on the market, we will focus on a particular risk management model for mortgage portfolios. The research on this model is an in depth comparison of the new dynamic model with the current static model. Since market risk management is about reducing the risk on the mortgage portfolio from the perspective of market risk, we will answer the following questions:

B.1 How can we measure the performance of a segmentation model?

B.2 How can two segmentation models be compared?

B.3 Which model performs best under chosen performance measures?

[2More details about the role of the G-10 countries are given in chapter 3]
1.4 Scope

Within the scope of this research is a literature study of the regulations in the years that before the financial crisis of 2007, and a technical study of the workings of the dynamic segmentation model for mortgages. A thorough evaluation of the dynamic model’s performance is out of scope due to limitations regarding available computing power and the impossibility to make technical adaptations to the model’s implementation.
Chapter 2
Methodology

2.1 Approach

This study consists of two parts. The first part covers regulations on risk management practices by banks and financial institutions, and their impact on the actual implementation of risk models by banks. A documentary analysis is performed to answer research questions A.1, A.2 and A.3.

In the second part of this study we address research questions B.1, B.2 and B.3. To answer questions B.1 and B.2, which cover evaluation metrics on the risk models, we used academic literature as well as internal documents from ING. The measures used are widely accepted economic measures, such as BPV (Definition 2.1) and NPV (Definition 2.2), and hence they are described in academic documents. The interpretation of their values is explained by internal documents from ING. The last research question, B.3 — requires an analytic approach. We use accepted statistical methods and measures, such as the Sharpe ratio [Sharpe, 1994], to assess the stability of the dynamic segmentation model in comparison to the static model.

2.2 Data collection and analysis methods

The documents used in the documentary analysis are retrieved via internet search\(^1\) references from within academic papers, publications from the Basel Committee for Banking Supervision, and internal documents from ING DB NL\(^2\). In chapter 3 the documents are analysed and discussed chronologically to give a clear overview of the development of the regulations, and the dynamics between regulatory organizations and the financial institutions that are subject to these very regulations.

The sole variable dependency in the prepayment models is the interest rate path that functions as input. We have attempted to simulate interest rates paths using Hull-White
trees and the Nelson-Siegel-Svensson method. The theory behind these methods is presented in [chapter 5](#). We did, however, decide to use 200 interest rate paths and variation thereupon, provided by the Model Validation department, to test the stability of the prepayment models, since the simulated interest rate paths could not match the required quality. The stability of the prepayment models is tested using the basis point value and the net present value.

**Definition 2.1 (Basis Point Value).** The basis point value tells how the position of a bond changes under a 0.01% parallel move in the underlying yield curve. One basis point is equal to 0.01%. This measure is often used to measure interest rate risk. [Martellini et al., 2003, pag. 169](#)

**Definition 2.2 (Net Present Value).** The net present value of an investment is its current discounted value. It can be computed by

\[
\text{NPV} = \sum_{t=1}^{T} \frac{C_t}{(1 + r)^t}
\]

where \(C_t\) is the cash flow at time \(t\) and \(r\) is the discount rate. [Marget Risk Management Bank, 2012](#)

### 2.3 Validity, reliability and limitations

Implementation of the model was already done by the Market Risk Management department, since it had to be integrated in their hedging application. Using the description of the algorithm in this study, the implementation can be reproduced.

The 200 interest rate paths used to test the models were provided by the Model Validation Department. The validity of these interest rate paths has been approved by De Nederlandsche Bank for testing purposes. Furthermore, the variety within these interest paths is sufficient to provide significant results and draw valid and sound conclusions about the performances of the models. This is shown in [chapter 5](#) and elaborated on in [chapter 6](#) and [chapter 7](#). The major limitation of using these 200 interest rate paths is that the research is hard to reproduce exactly.

### 2.4 Summary and outline

This study combines documentary analysis with quantitative methods to answer the research questions.

In [chapter 3](#) the regulations on credit risk modeling and the effect on implementations by financial institutions are discussed. [chapter 4](#) provides a mathematical introduction into mortgages and loans, the prepayment model and the Hull-White interest rate tree. In [chapter 5](#) we discuss the theory behind interest rate simulations and yield curve estimation. [chapter 6](#) shows the result of applying the static segmentation model and the dynamic
segmentation model to a replication of the largest mortgage portfolio of ING in June 2014. Thereafter, chapter 7 compares the behaviour of the dynamic and static segmentation model under the variation of several parameters. The conclusion of the study is presented in chapter 9 followed by the discussion in chapter 8.
Chapter 3

Regulations

There is hardly any public documentation on the implementation of financial risk models by banks and financial institutions. Academic research on such models is available in scientific papers, but the details of the actual implementation by banks and financial institutions are not published. Banks have a clear reason to do so; their competitive advantage regarding financial risk management practices depends on confidentiality.

In this chapter we will focus on the development of credit risk models, how they were influenced by regulations and vice versa. And we will try to find an answer to the following questions, that have risen after the most recent financial crisis.

- How transparent are the risk modeling practices of European banks?
- What are the societal impacts of a lack of transparency on risk models?
- What are the benefits of transparency on risk management by banks for the public and what would be the drawbacks?

The answers to these questions are related to the topic of risk regulations for banks and financial institutions. The development of regulations and agreements goes back to 1962, when a group of ten countries called the Group of Ten (G-10) was established. The ten original members of the G-10 are Belgium, Canada, France, Germany, Italy, Japan, Netherlands, Sweden, United Kingdom and the United States. Switzerland joined the G-10 in 1964, making it a group of eleven countries, but the name remained. All members of the G-10 agreed to participate in General Arrangements to Borrow (GAB). By participating in the GAB countries agree to support the International Monetary Fund (IMF) by providing it with financial resources to increase the lending ability of the IMF. The GAB is activated when the normal funds of the IMF cannot match the needs of its borrowers. [International Monetary Fund, 2016]

In 1974 the governors of the G-10 countries established the Basel Committee, a committee of the Bank for International Settlements (BIS). This committee has since then published

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1 This point was also made by [Treacy and Carey, 2000], who studied few banks and their respective models.
three important accords on regulations regarding credit risk management for banks and financial institutions.

After a brief introduction on the BIS and the Basel Committee, this chapter will discuss the three accords, their implications for, and their impact on risk management practices. Section 3.2 covers the first Accord, followed by an analysis of its implications in section 3.3. We then continue chronologically with Basel II in section 3.5 and a study of its implications in 3.6. The final section covers the activities of the committee and activities of the banks in section 3.8.
3.1 Bank for International Settlements

On May 17, 1930 the Bank for International Settlements (BIS) was established. The organisation is the world’s oldest international financial organisation. Members of the BIS are 60 central banks representing countries that together comprise about 95% of the world GDP.\(^2\)

The mission of the BIS is to “serve central banks in their pursuit of monetary and financial stability, to foster international cooperation in those areas and to act as a bank for central banks”.

The BIS hosts six committees to support member central banks and supervisory authorities with policy recommendations and background analysis of the financial market. We will focus on only one of the six committees, The Basel Committee on Banking Supervision\(^3\).

The objective of the Basel Committee on Banking Supervision is to “enhance understanding of key supervisory issues and improve the quality of banking supervision worldwide”. It does so by providing a forum for cooperation on banking supervisory matters. We will further refer to this committee as the Basel Committee. \[Bank for International Settlements, 2014\]

\(^2\)This percentage is provided by the BIS. \[Bank for International Settlements, 2014\]. For an extensive list of all member central banks, visit https://www.bis.org/about/member_cb.htm?m=1%7C2%7C601.

\(^3\)Other committees are Committee on the Global Financial System, Committee on Payments and Market Infrastructures, Markets Committee, Central Bank Governance Forum, Irving Fisher Committee on Central Bank Statistics \[Bank for International Settlements, 2016\].
3.2 Basel I Accord

The Basel I Accord, published in 1988 and enforced by law in the G-10 countries in 1992, consists of a set of minimum capital requirements for banks. Mainly focused on credit risk, the goal of this accord is to converge standards on capital adequacy of banks, which means:

(i) increase the stability and soundness of the international banking system, and

(ii) implement a fair, consistent framework for banks in different countries that decreases the competitive inequality among international banks.

The Basel Committee divides all capital and assets held by banks in risk categories, each category having its own risk weight. The risk weight varies between 0, 10%, 20%, 50% and 100%. A risk weight of 0% means that the asset has no effect on the risk based capital of the bank. Examples of assets with 0% risk weight include cash, claims on central governments and central banks, and other types of claims on central governments.

The weighted sum of these assets is called the risk weighted assets (RWA). The minimal capital requirements are defined in terms of a ratio compared to the RWA.

3.2.1 Capital ratio

The Basel Committee suggested that capital be divided into two categories; tier 1 and tier 2 capital. Tier 1 capital consists of equity capital and published reserves from post-tax retained earnings. All other types of capital are tier 2 capital. Tier 2 capital holds undisclosed reserves, revaluation reserves, general provisions, general loan-loss reserves, hybrid debt capital instruments, subordinated term debt, etc. National supervisory authorities were free to include or exclude some types of the tier 2 capital - the supplementary capital - while evaluating the capital requirements.

The framework described in Basel I states that banks that act internationally should hold a capital corresponding to 8% of their risk weighted assets, and the sum of their tier 1 and tier 2 capital should be at least 8% of the risk weighted assets, with tier 1 capital making up at least 4% of that.

3.2.2 Credit risk

Credit risk is defined by the Basel Committee as ”the potential that a bank borrower or counterparty will fail to meet its obligations in accordance with agreed terms”.

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4See [Basel Committee on Banking Supervision, 1988, p. 21] for a full list of assets per risk weight.
Since credit risk is inherent to loans, the goal of credit risk management departments within banks is to practise banking activities that yield a maximum return while still keeping the credit risk within acceptable boundaries. Credit risk is included in entire portfolios and in individual loans and transactions. Managing this risk well is necessary for a financially healthy bank.

After publishing the first accord in 1988 and monitoring the developments on capital standards during the years since 1988, the Committee proceeded in 1998 to set up a framework that aims to establish minimum levels of capital for internationally active banks. The framework is a revision of the 1988 Accord that is ought to further strengthen the soundness and stability of the international banking system whilst maintaining sufficient consistency. According to the Basel Committee, the new framework will promote the development and implementation of stronger risk management practices by banks.

One of the major innovations in the revised framework is the use of assessment of risk by the banks’ internal risk assessment systems as an input to capital calculations. [Basel Committee on Banking Supervision, 1998c] Since this framework is designed for banks that are internationally active, supervisory authorities are free to require stricter limits on minimum capital requirements on a national level. Though directed mainly at assessing capital levels related to credit risk, other risks should be taken into account by supervisory authorities as well.

This attempt by the Basel Committee on convergence of capital standards was widely supported by the banks from the G10 countries.

### 3.2.3 Framework for internal control systems

In 1998, the Basel Committee published a paper that proposes a framework for internal control systems in banks. We shall refer to this framework as the FICS, no to be confused with the framework discussed in the previous subsection 3.2.2. This paper is in line with the ongoing effort of the Basel Committee to improve supervision that encourages correct and safe risk management. The FICS provides a guideline for banks to set up an internal control system.

The goals of the system are to help banks achieve long term profitability targets, maintain reliable reporting and help comply with internal and external regulations, rules, policies, etc. [Basel Committee on Banking Supervision, 1998b]

The first three principles of the FICS cover management oversight and control culture. They address the responsibilities of directors, senior management and supervisors regarding the quality of the internal control system. The fourth principle in this paper covers risk recognition and assessment. The Basel Committee stresses that a sound and effective internal control system should require that all risks faced by the bank are assessed correctly and timely. This implies restrictions on the bank’s activities, since banks should map out all risks faced in their banking activities. The risks could be caused by internal factors, such as operational risk, as well as external factors, such as market risk and interest rate risk.
Evaluating risk factors should be done while taking into account their environment. Supervisory authorities ought to keep the competitive environment of a specific bank in mind.

One of the implicit goals of this framework was to prevent the banks from participating in shady or reckless business activities. Keeping credit risk in mind, banks should have a safe distribution of risk over their regulatory capital. The solution to ensuring banks to have this safe distribution of risk lies with the supervisory authorities. The final principle of the paper describes the duty of the supervisory authorities to require banks to “an effective system of internal controls that is consistent with the nature, complexity and inherent risk in their [...] activities and that responds to changes in the bank’s environment and conditions”. [Basel Committee on Banking Supervision, 1998b, p. 22]

The FICS is mostly theoretical. Practical implementations by banks can be derived from the extensive guideline that accompanies the principles. Supervisory authorities are supposed to interpret the framework in a way that fits the national financial market, accounting principles, and keep in mind the financial activities in which banks are involved.

### 3.2.4 Principles of Credit Risk Management

In 2000 the Risk Management Group of the Basel Committee on Banking Supervision issued a paper in which they propose international standards for the management of credit risk by banks and financial institutions. The observations that gave rise to this paper were the serious banking problems and major financial difficulties experienced in both G-10 and non-G-10 countries in the years before [][5] The problems encountered can be of one of the following types:

- **Credit concentration problems.** Credit concentration is any risk exposure where the potential loss is large relative to the bank’s capital, its total assets or the bank’s overall risk level.

- **Credit process problems.** A thorough and correct credit risk assessment is a challenge to many banks. This causes weaknesses in the credit granting and monitoring process. Some problems in the credit process arise because senior management makes subjective decisions, lead by intuition, on credit assessment of risk management. Other banks experience problems because they do not have an effective credit review process.

A full list of issues can be found in [Basel Committee on Banking Supervision, 2000, p. 22-26].

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5The paper does not go into detail about the reason for the major problems, other than “lax credit standards by borrowers and counterparties, poor portfolio risk management, or a lack of attention to changes in economic or other circumstances that can lead to a deterioration in the credit standing of a bank’s counterparties.” [Basel Committee on Banking Supervision, 2000 p. 1].
The Committee claimed that these problems can all be attributed to the lack of credit standards for borrowers and counterparties, poor portfolio management and absence of thorough monitoring of changes in circumstances that could lead to a destabilisation of the credit status of the bank’s borrowers or counterparties. Counterparties refer to the party on the other end of a financial transaction. A counterparty can be an individual, a corporation, a government or any other legal entity. [Basel Committee on Banking Supervision, 2000]

Loans are not the sole origin of credit risk; it can also follow from the bank’s activities and - with increasing frequency - banks incur credit risk from various financial instruments, such as mortgages or bonds. Loans are, however, the largest source of credit risk for a bank. This research therefore mainly focusses on managing credit risk on outstanding loans and estimating credit risk on new loans, and the regulations thereupon.

I refer to the principles proposed by the Basel Committee, stated in [Basel Committee on Banking Supervision, 2000], for an overview of the principles that ought to be taken into account for successful credit risk management according to the Basel Committee. All these principles are covered by either of the following topics

- Establishing an appropriate credit risk environment
- Operating under a sound credit granting process
- Maintaining an appropriate credit administration, measurement and monitoring process
- Ensuring adequate controls over credit risk
- The role of supervisors

Note that no explicit description of the implementation of these principles is provided by the Basel Committee.
3.3 Impact of Basel I and capital frameworks

The deadline for implementation of the Basel Accord was 1992. In the following years, the effects of the minimal requirements have been monitored. Both the Basel Committee and independent parties have analysed the data.

3.3.1 Analysis by Basel Committee

Research constituted by the Basel committee was focused on the effects of the Basel I accord with regard to the capital ratios maintained by banks.

Recall the main two objectives presented in section 3.2. The review of the impact of the first Accord was done ten years after the Accord was signed. [Basel Committee on Banking Supervision, 1998a]

Collected data showed that indeed the capital ratio of banks had risen in 10 years, by 2 percent points, but the Committee emphasised that it is not clear if this increase is caused by the capital requirements set in the Accord, or by increased market discipline.

The latter is a logical explanation since the agreements on the Accord show that banks are willing to be, and agree to the necessity of being, more transparent. However, research by [Lopez and Saidenberg, 2000] and [Jones, 2000] show different conclusions. According to Jones (2000), banks perceive the cost of equity to be greater than the cost of debt. That makes RCA more appealing. He further claims that RCA activites “erode the regulatory capital standards and could impair regulatory discipline that is needed to limit systemic risk”. Lopez and Saidenberg (2000) evaluate different credit risk models that would yield different valus for the capital ratio, and hence, show that the requirement is not an objective value of the risk present in a bank’s portfolio.

3.3.2 Rising problems with the Basel Accord

Minimum requirements on a ratio rather than a value allow for some freedom. The capital ratio, which consists of tier 1 capital in the denominator and RWA in the numerator, can be increased by either increasing the tier 1 capital, or decreasing the RWA.

Most banks in the short run have adjusted their assets and reacted to the requirements in the way the Basel Committee envisioned. [Jones, 2000]

The main problem that has risen is the activity of regulatory capital arbitrage, RCA. This is the practice of reducing the effective amount of regulatory capital to meet the requirements in the accord whilst not reducing the risks incurred on the portfolio.

Jones (2000) mentions some techniques that are commonly used in the practice of RCA. Most of these methods involve decreasing the amount of risk based capital, RBC.

The Market Risk Amendment to the Basel Accord, issued in 1997, opened up new possibilities for further regulatory capital arbitrage. Accounting rules in some countries may allow banks to shift credit risk from the banking books to the trading books, which lowers the risk based capital, according to the VaR-models. VaR stands for Value-at-Risk. It is a measure for market risk on a set of investments or assets. VaR estimates the loss on a set of
investments or assets for a fixed time window. Gordy (2003) emphasises that RCA lowers - or even undermines - the effectiveness of the regulatory capital requirements set in the Basel Accord. He points out that it is necessary to discern the levels of risk in underlying assets, to achieve the full potential of the regulatory rules. [Gordy, 2003, p. 2-4]

### 3.3.3 Implications of regulatory capital arbitrage

First of all, it is clear that regulatory capital arbitrage allows banks to shift risks to other types of capital, at which moment the risk based capital does not truly represent the bank’s financial position.

The direct implications of capital arbitrage are possibly false figures about the health of the financial institutions, since capital levels can be lowered while the risk incurred on loan portfolios remains the same. The indirect implications may be even more destructive. While the Accord attempts to reduce international competitive inequalities and create a stable and sound international banking system, capital arbitrage challenges the morality of the international banking systems, because it is possible to satisfy the requirements set by the Accord, while still adopting increasing risk levels on the underlying assets. Supporting a fair and stable banking system would require the banks to contribute equally to the rules that were agreed upon, and not have requirements that - as they turn out - do not represent the true risk present in the portfolios. Various regulatory capital arbitrage strategies might yield different capital ratios, which is the limiting factor in risky investments.

The moral hazard that arises when banks seek the boundaries of the framework using capital arbitrage is an unwelcome, yet expectable side effect of the accord. Since the aim of the Basel Committee is to encourage safe and correct risk management, they should be expected to take measures. One could argue that no matter the boundaries, they will always be pushed. Setting up strict guidelines could prevent unsafe risk management.

Capital arbitrage furthermore could negatively impact other parties on the financial market, since it could mask weaknesses in a bank’s financial position, and this could lead to more positive risk assessments by counterparties.

On the other hand, capital arbitrage may benefit the market because banks have more room to initiate in low risk activities that would otherwise yield insufficient profits. If most of the risk can be transferred so that it does not influence the risk based capital, while still allowing banks to take part in high risk investment activities, there is more room for banks to invest in low risk markets or assets. This follows from the risk based capital requirement; if risk is transferred without changing the value of the risk based capital, banks can invest in low risk markets or assets that add only little to the risk based capital, without it becoming a burden. Last, the rough division in credit risk sources discourages banks to hedge entire portfolios, since that could imply requirements on regulatory capital.

On a final note, [Calem and LaCour-Little, 2004](#) suggest that the rules and requirements from Basel I and following frameworks are not sufficient for mortgage loans. They suggest
that a geographical parameter be taken into account when assigning credit risk. Their paper proposes a method to measure credit risk on a geographically diversified mortgage portfolio. The suggestions made by Calem and LaCour-Little (2004) are, however, more suitable for supervisory authorities who rate the credit risk model, than as rules to be incorporated in the framework for internal control systems, since they go into detail about the implementation.
3.4 Modelling credit risk (1980 - 2004)

Now that we have discussed the general implications of the frameworks, we will delve into the credit risk models that are implemented by the banks.

Since the requirements on regulatory capital depend on correct credit risk assessment, the need for good credit risk models is high. Credit risk models are tools to both assess the risk to banking activities and to estimate the amount of economic capital that is required to meet international minimum requirements. These models may be developed and implemented by banks themselves, as following for the framework for internal control systems. However, a consequence of the Market Risk Amendment to the Basel I Accord is that models need to be validated and satisfy standards before they may be used for regulatory capital purposes.

3.4.1 The early years

[Altman and Saunders, 1998] cover the development of credit risk models from 1980 to 2000. In the beginning of the 1980’s - before any minimum requirements were set - banks and financial institutions relied on experts to analyse the risk of lending to a certain borrower. The so-called four C’s (character, capital, capacity and collateral) were used to characterise estimate the risk of non-repayment. The judgement of the experts based on the risk would determine whether a loan was granted. Financial institutions then shifted to using analytical models. The most simple form of models searched for a function that could separate repayments and non-repayments by means of a transformation of the multivariate input space. The variables are mostly accounting and market variables. These types of models are called discriminant analysis credit rate models. More advanced models, like Black-Scholes-Merton, KMV, Altman’s model, incorporate stock data. These models were popular in the early 1990’s. After the framework for internal control systems had been introduced, various articles providing an overview of credit risk models have been published. [Crouhy et al., 2000], [Gordy, 2000] and [Lopez and Saidenberg, 2000] compare current models. They propose methods to evaluate these models that allow for comparison between them. [Altman and Saunders, 1998]

3.4.2 Credit risk models after 2000

So far we have seen that the interpretation of the framework and choices for implementation may benefit banks, depending on the requirements set on a national level. One example of a benefit is regulatory capital arbitrage, as explained in subsection 3.3.2 and subsection 3.3.3. Note that the following papers mentioned are rather old; transparency nowadays about the
internal credit risk models is even less likely; details about implementations by banks are unavailable to the public.

[1] Treacy and Carey, 2000 discuss credit risk rating methods in use at the fifty largest banks in the United States in 2000. They compare the methods, as far as they know of them, to the methods used by credit rating agencies.

The big difference between banks and credit rating agencies is that the methods used by agencies should be understandable for outsiders, whereas credit rating models used by banks are mainly used and reported on internally.

The paper does not provide much detail about how credit rating is done at the selected banks, other than discussing general risk frameworks that are also used by the credit rating agencies. Since the paper does not provide any information about the number of risk levels that banks use in their frameworks, we assume the authors did not have this information.

The number of risk levels could influence the risk weight assessment on loans, and is thus an important parameter in risk assessment.

Treacy and Carey suspect that senior employees still have a large role in interpreting the risk ratings and tuning the parameters of the models. [Treacy and Carey, 2000]

Banks may use the credit risk models differently, depending on the type of customers they assign loans to, and depending on the type of competitive market they operate in.

A bank’s credit risk reports mainly serve two goals: analysis/reporting and administration. The former is used by other departments inside the bank, the latter is used by supervisory authorities.

Thomas (2000) has looked into the way credit score models are used by financial institutions. He warns that banks that have confidence in the quality of their models could start cherry-picking their customers to reduce the risk on loans they approve. Although discriminating on certain parameters (e.g. gender, age) is forbidden, it cannot be ruled out that this is done.

Thomas concludes that, due to the strong connection between scoring risk for provisional capital and scoring risk for future sales, banks will likely use the models to forecast future sales and profits. [Thomas, 2000]

Crouhy et al. (2000) review some of the publicly available credit scoring models that incorporate the market risk requirements from the 1998 Market Risk Amendment to the Basel I Accord. The challenge is to build a model that combines market and credit risk in a correct way. Since the two types of risk are not independent, this turns out to be hard. No model is yet capable of doing so. Crouhy et al. conclude that the models do not outperform an naive method of estimating both risks. [Crouhy et al., 2000]

Finally, Lopez (2000) points out that the evaluation of credit risk models is very hard in practice. There is not enough data available on credit losses to test the quality of the model. It takes many years to capture enough data that spans more than one credit cycle. Simulations might give an indication for the performance of credit risk models, but finding the right parameters for a simulation is based more on experience than backed by evidence,
and hence, the results cannot be fully trusted. This makes it hard for banks and for supervisory authorities to validate models. [Lopez and Saidenberg, 2000, p. 7]

Lopez suggests to use cross-section evaluation on credit risk models by sampling data from existing portfolios. Since gathering enough data is very time-consuming when evaluating credit risk models, there is a practical problem caused by the demand of standards on credit risk models.

The standards are necessary consequences of the regulatory capital requirements, since these are based on the outcome of the credit risk models.

Summary on credit risk models

We conclude that credit risk models are widely in use since the Accord of 1992. However, information on the exact implementation of these models by banks is hard to get by. Credit rating agencies provide information about their models, since they are required to, but these models seem less sophisticated and still very much rely on experts rather than research.

Academic contributions to credit risk modeling are plentiful, but it is not clear whether and how these contributions are used by banks. The academic literature suggests that banks use models that are somewhat similar to those used by credit rating agencies. The models used by the latter are based on academic contributions to the field. The paper of [Treacy and Carey, 2000] suggests that models used by banks are also based on academic contributions, but this could not be verified.

We can conclude that credit risk models are useful to banks, not only for reporting or credit scoring of customers, but also as a measure of forecasting future sales. Supervisory authorities should focus on the capital requirements that must be met by the banks, as well as on the details of the models.

Advanced models may rule out certain groups of customers to benefit the profits of the banks, but this is in contradiction with the second goal of the Accord 3.2; if certain banks are able to filter out risky customers; banks with less sophisticated models could - unknowingly - have a larger base of risky customers.
3.5 Basel II accord

In response to the problems that arose with the Basel I Accord the Basel II Accord was published in 2004. This Accord supersedes the Basel I Accord and attempts to cover the shortcomings of the requirements in the first Accord. We discussed these shortcomings, such as regulatory capital arbitrage, in section 3.3.

The new Accord aims to promote “the adoption of stronger risk management practices by the banking industry” [Basel Committee on Banking Supervision, 2004]. The new framework is based on three pillars:

1. Minimum capital requirements,
2. Supervisory review process,
3. Market discipline,

which should improve capital regulation on changing risk management practices, whilst remaining a generic framework that is specific up to the national level. As previously concluded by Jones (2000) and Gordy (2003), this framework should specify more risk-sensitive requirements than the previous version.

This framework can only be implemented in practice if internal risk calculations can be used as input to the capital calculations. The Committee recognised this need and has hence set up minimum requirements for these internal risk assessment models. Note that only banks that have received permission from their supervisory authority are allowed to use this internal ratings based (IRB) approach to assess credit risk.

3.5.1 Supervisory review process

Since the Basel II gives more freedom and responsibility to the bank in regard of credit risk models, the role of the supervisory authorities changed. The authorities should not only validate the capital requirements but also the risk assessment practices of the banks.

The revisited framework constitutes four new principles regarding the review process of the supervisory authorities: [Basel Committee on Banking Supervision, 2004]

1. **Banks should have a process for assessing their overall capital adequacy in relation to their risk profile and a strategy for maintaining their capital levels.**
   This principle requires the banks to set up risk models to qualify and quantify the risks on all their banking activities. Since results of the models may be used in the capital calculations, this increases the competition on good risk models.

2. **Supervisors should review and evaluate banks’ internal capital adequacy assessments and strategies, as well as their ability to monitor and ensure their compliance with regulatory capital ratios.** Supervisors should take
appropriate supervisory action if they are not satisfied with the result of this process.

3. **Supervisors should expect banks to operate above the minimum regulatory capital ratios and should have the ability to require banks to hold capital in excess of the minimum.**
   Supervisory authorities gain increasing regulatory power; on suspicion of an unhealthy bank or unstable financial market, the supervisory authority might require one or more banks to hold more regulatory capital than the 8% stated in the Accord.

4. **Supervisors should seek to intervene at an early stage to prevent capital from falling below the minimum levels required to support the risk characteristics of a particular bank and should require rapid remedial action if capital is not maintained or restored.**
   In addition to the third principle, this principle implies that supervisory authorities should not only check the current state of affairs at banks, regarding the capital and risk management strategies, but they should also be able to predict the financial situation in the (near) future and intervene if necessary.

### 3.5.2 Market Discipline

Following the first Accord, RCA became common practice amongst banks. By further specifying risk weights and recognising more types of risk, the Basel II Accord took away some of the competitive inequalities. To increase transparency and equality between banks that participate in international banking activities, the Committee set up the third pillar of the Accord. The Committee believes that for the framework to perform as well as it envisions, they should introduce disclosure requirements for banks that want to use the framework.

The third pillar - Market Discipline - should complement the first two pillars. The Committee “aims to encourage market discipline by developing a set of disclosure requirements which will allow market participants to assess key pieces of information on the scope of application, capital, risk exposures, risk assessment processes, and hence the capital adequacy of the institution.” [Basel Committee on Banking Supervision, 2004](#)

A higher degree of transparency concerning internal (credit) risk assessment methods is required now that banks can rely more on their own methods when assessing capital requirements.

The Committee leaves it to the supervisory authorities to set the level of transparency regarding the disclosure on their risk assessment methods. The framework does provide guidelines for the disclosure requirements.
3.6 Implications of Basel II Accord

[Gordy, 2003, p. 23] expected that validating the internal grading systems of banks will be the biggest challenge for supervisory authorities. The consequences of the new framework would only become visible after 2006, when it was implemented.

Under the new framework the capital requirements for banks would be based on the bank’s current risk assessment. The new problem that arises is that these capital requirements could vary significantly at different points in the economic cycle.

One big consequence could be that banks hold back on lending during periods of recession. This would all depend on how much the model used by the bank is forward-looking. If a model uses only a single point in time, capital requirements in time of recession could be higher, allowing the bank to issue a smaller amount of loans. At the same point in time, banks that use a more forward-looking model could have lower capital requirements. This problem is called procyclicality.

[Catarineu-Rabell et al., 2005] warn that these differences could cause severe effects on the economy at macro level. Banks could be responsible for a credit crunch in time of a recession, or practise excessive lending in peak periods. This idea is confirmed by [Rösch, 2005, p. 13-14].

[Catarineu-Rabell et al., 2005] think that banks would typically choose for a countercyclical ratings based approach, meaning that credit ratings would be lower in times of recession. Supervisory authorities will probably require the opposite, based on the new Accord, which would mean a procyclical approach, which has the effects stated above.
3.7 Credit risk modelling after Basel II

Now that risk assessment has become more dynamic under the new framework, [Thomas et al., 2005] have conducted a survey to determine what part of research concerning credit risk modeling has to be done in the coming years. [Crook et al., 2007] looked into the current state of research to conclude that credit risk models can yield large competitive advantages under the Basel II Accord capital requirements, given the freedom of using internal risk rating systems.

As mentioned before, and as noted by [Thomas et al., 2005, p. 1008], risk assessment is not only used to estimate the default probabilities of individual customers, but also to forecast the sales and profits on loans. The authors also notice a shift in interest where the possibilities of estimating the risk on portfolios, instead of individual loans, is researched. [Gordy, 2003] already discussed the advantages of being able to calculate the risk on portfolios, since it saves in computational time and effort. The existing risk models on loan portfolios are suitable for corporate loans, and not for consumer loans.

3.7.1 Internal Risk Rating Systems

Risk assessments made by banks are increasingly important for banks. The requirements on risk assessment methods still allow for some freedom. We have seen two types of models - pro-cyclical and point-in-time - which yield different capital requirements.

Little is known about the exact working of internal rating systems used by banks. Nor do we know a lot about how stable they are during a credit cycle. [Jacobson et al., 2006] have investigated the different internal risk rating systems that assess the credit risk on loan portfolios.

The authors use data from two major commercial Swedish banks and show that the two banks have not implemented internal rating systems that result in the consistent credit loss (e.g. credit risk) estimates on a loan portfolio. The differences of credit loss estimates on the same loan portfolio by the two different rating systems are significant. A different assessment of credit risk on a portfolio of loans would result in different regulatory capital requirements for the respective banks.

This shows the shortcomings of the internal risk ratings based risk assessment.

3.7.2 Procyclicality of Basel II

We have briefly addressed the problem with procyclical risk models in [section 3.6]. Gordy and Howells propose three methods to solve this problem and study these methods from the perspective of the third pillar of the Accord; market discipline. Most importantly in terms of market discipline is that forward-looking models show a distorted image about regulatory capital, since the risk estimates on a portfolio are flattened.
by the credit cycle that is implicit in the model. [Gordy and Howells, 2006, p. 408-409]

Heid, 2007 studies the macroeconomic effects of the cyclical property of the new Accord and concludes that this cyclicality has influence on the loan demand and loan supply. In the three different scenarios studied, loan supply decreases.
3.8 Basel II’s heritage

In spite of committed attempts by the Basel Committee and cooperating financial situations to set up rules for a supervised and regulated financial system, flaws and loops remain in the framework. Where the second Basel Accord was supposed to clear up the shortcomings from the first Accord, it opens up a new range of problems:

- Implicit procyclicality,
- Decreased transparency in risk models of banking institutions,
- Unseen diversification and inconsistency in credit risk models.

We can say that the regulatory power has shifted from the supervising authorities - where it is supposed to be - to the banks which control the regulatory capital with their internal risk rating frameworks.

After the financial crisis in 2008, Gerding (2009) suggests that prohibiting banks to regulate their capital based on their own internal models and requiring banks to publish their source code could prevent the mistakes that are inherent to the Basel II framework. Not only could open source code of the models used by banks improve the quality of the models - which would benefit the entire financial system, it could also be a safeguard for the incentives of the people who use these models. Under the Basel II framework, it is practically infeasible for supervisory authorities to inspect and validate the models implemented by banks. Firstly because they differ per bank, secondly because they are almost always part of a larger, complex system. [Gerding, 2009]

Basel III

After the financial crisis, the Basel Committee designed a new framework that would help banks withstand a bank run. Rather than replacing or improving the Basel II Accord, the Basel III Accord is supposed to be implemented alongside Basel II. As a consequence, the one issue that we have addressed multiple times so far - the internal ratings based approach to risk assessment - still remains. Attempts to find literature on current credit risk models used by banks yields no results. This shows that the transparency, insisted on by Gerding, is still far from common practice.

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Chapter 4

Mortgages and prepayments

In the previous chapters we have discussed regulations on risk modeling practices. The following chapters will cover a model that estimates prepayments on mortgage portfolios to manage market risk.

4.1 Repayment structure

When a client is granted a mortgage by a bank, he is then obliged to make regular - often monthly - payments to the bank to pay back that mortgage. The monthly payments on a mortgage always contain some interest payment, and may contain a cash payment. With different kind of mortgages come different monthly payment schemes. There are several types of mortgages, the most common of which we will discuss in section 4.2.

A cash payment is a payoff on the total outstanding amount of the mortgage, also called the outstanding notional. Cash payments can be contractual, and lower the interest on the remaining outstanding principal of the mortgage.

Clients usually have some flexibility in repaying their mortgage. A client might make a repayment higher than the contractual repayment and hence lower the outstanding principal on his mortgage. These repayments are called prepayments. Prepayments are beneficial to the holder of the mortgage, since his interest payments will go down, but prepayments are often disadvantageous to the issuer of the mortgage, the bank. Banks will receive less interest on the outstanding notional of the mortgage than expected based on the mortgage contract.

Upon an unexpected repayment by a client, the mortgage funding has to be adjusted to match the new expected repayment scheme. The bank still has to pay interest on the deposits that were used for funding the old repayment profile. If the market rate$^1$ has decreased, this is particularly harmful to the banks, and it will usually yield an economic loss.

In most cases of an early repayment, the client has to pay a penalty to the bank, but these penalties are often not sufficient to cover the loss for the bank.

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$^1$Market rate refers to the usual price in the market. In this case we refer to the interest rate that is used in the market.
4.2 Mortgages

To understand the differences in prepayment behaviour amongst clients with either a fixed rate or floating rate coupon mortgage, consider the following. When interest rates go up, the holder of a mortgage with a fixed coupon rate can profit. Rates on savings accounts go up, as they are corresponding to the rising market rates, whereas the costs on the mortgage stays fixed, by the fixed coupon rate. In the other case, when mortgage rates go down, the holder of the mortgage is at a disadvantage. He can then decide to do prepayments on his mortgage. This will benefit him, since the rates on savings accounts will decrease, while the coupon rates remain at the same level.

One can see that it is thus favorable to make prepayments on a mortgage with fixed coupon rate when market rates are declining. This reasoning does not apply to a mortgage with floating rate coupons, because the floating rates, by definition, follow interest rate movements. This reasoning suggests that prepayment behaviour on a mortgage portfolio dominated by floating rate mortgages is best described by a constant prepayment function, and a mortgage portfolio with mostly fixed rate mortgages is best modeled by a interest rate dependent prepayment function. This suggestion is acknowledged by the prepayment model analysis of countries that have either a portfolio dominated by floating rate mortgages or a portfolio dominated by fixed rate mortgages.

Among the mortgages with fixed coupon rates, we can distinguish four types: bullet mortgages, linear mortgages, annuity mortgages and savings mortgages. We will briefly discuss the cash flows of these mortgages.

4.2.1 Bullet mortgage

A bullet mortgage is a mortgage in which the entire amount of the mortgage is repaid at the end of the term. Regular repayments on the mortgage consist only of interest payments. Assuming that no prepayments are made on the outstanding notional, the amount to be paid at every contractual payment date is constant. At the end of the term, the outstanding notional should be repaid at once.

The cash flow profile a bullet mortgage is shown in figure 4.1.
4.2.2 Linear mortgage

Payments on a linear mortgage consist of a cash repayment and a coupon payment. Characteristic for the linear mortgage is that the amount of cash repaid at every payment date remains constant. Due to the regular cash repayments, the outstanding notional decreases, and hence, the interest payments decrease. As a consequence, the amount of the regular contractual payments decreases over time. The payments are constructed such that, at the end of the term, the mortgage is fully repaid.

A cash flow scheme of a linear mortgage is shown in figure 4.2. Notice that the cash part of the payment remains constant, whereas the interest part decreases.

4.2.3 Annuity mortgage

An annuity mortgage has the benefit of stable, constant payments for the full term of the mortgage. Just like the linear mortgage, payments for an annuity mortgage consist of a cash
repayment and a coupon payment. The contractual payments are also set up so that the mortgage is fully repaid at the end of the term. As the payments have a fixed amount for the full term of the mortgage, the ratio of the amount of cash and interest paid at every contractual payment date changes over time. The outstanding notional amount decreases due to the cash repayments, and hence, the interest payments decrease too. But, as the contractual payments have a fixed amount, the amount of cash repaid per coupon payment increases during the term. By this construction, the payments at the beginning of the term consist mainly of interest payments and for a small part of cash repayments. Whereas, near the end of the term, this is exactly the other way around; a large share of the amount on the regular contractual payments is taken up by the cash repayments and only a little share is taken up by the interest payment.

Figure 4.3 illustrates the cash flow scheme of an annuity mortgage.

![Annuity mortgage](image)

Figure 4.3: Cash flow scheme of an annuity mortgage.

### 4.2.4 Savings mortgage

A savings mortgage consists of a loan part and a savings part. Payments on a savings mortgage are deposited into a savings account. The interest received on the savings account is equal to the client rate on the loan part of the mortgage contract. At the end of the term, the amount on the savings account will have accumulated to the amount of the outstanding notional of the mortgage. As the interest rate received on the savings account is equal to the interest to be paid on the loans part, it is not considered to be beneficial for the client to prepay on a savings mortgage. The combined profile of the savings part and the loan part of a savings mortgage is equal to the profile of an annuity mortgage. The loan part of a savings mortgage follows the profile of a bullet mortgage. The loan part is captured by the scope of this research, and hence, we will not discern a savings mortgage as an extra type of mortgage from here on.
4.3 Modelling prepayments

A client’s prepayment behaviour must be analysed in order to minimise the risk and losses caused by prepayments on his mortgage. The reduction of risk and minimisation of losses can be achieved by trying to account for prepayments in advance. The entire process of estimating prepayments and accounting for future prepayments is done with the following steps:

- historical analysis of economic loss incurred by the bank due to prepayments,
- segmentation of mortgage portfolio based on differences in prepayment behaviour,
- calculation of prepayment rates in every segment of the segmentation,
- estimation of parameters of prepayment function based on historical prepayment rates,
- application of prepayment functions to outstanding contracts during hedge to predict future prepayments,
- adjusting the funding portfolio.

The prepayment function, as mentioned in the fourth bullet point, will be further explained in section 4.3.3. In short, the prepayment function takes as input the interest rate, and outputs a percentage; the prepayment rate. This rate represents the percentage of the outstanding notional a client is expected to prepay, given the interest rate. Prepayment behaviour is thus dependent of the interest rate, according to the prepayment function. The cause that drives a client to prepay on his mortgage is called a driver. The portfolio may be segmented based on the extent to which a client responds to the prepayment driver. The following section elaborates on this.

4.3.1 Segmentation and drivers

In order to analyse the prepayment data on the entire prepayment portfolio, and to make better estimates of future prepayment behaviour, the mortgage portfolio is clustered in groups with similar characteristics. The clustering may be based on the remaining fixed interest rate period (FIRP), remaining contract term, client rate, mortgage type or specific client’s characteristics. This entire set of characteristics is assumed to determine the client’s prepayment behaviour, and hence, the characteristics are chosen so that the segmentation is also a first step in modelling the prepayments.

Currently, the remaining FIRP and mortgage type are the characteristics used for clustering. The difference between the client rate and the actual market mortgage rate is the only incentive currently used in the prepayment model. Together with the characteristics for segmentation, analysis has shown that this financial incentive is an important driver for prepayment behaviour.
4.3.2 Prepayment rate

Prepayment rates are calculated in two steps. First, the gross prepayment rate is calculated. This rate is defined as

\[ GMP_{t,C} = \frac{\sum_{i \in C} P_{t,i}}{\sum_{i \in C} N_{t-1,i}}, \]  

(4.1)

for a certain mortgage cluster \( C \) at time \( t \). Here, \( P_{t,i} \) denotes the amount prepaid at time \( t \) on mortgage contract \( i \), and \( N_{t-1,i} \) denotes the outstanding notional on contract \( i \) at time \( t - 1 \). The sums are taken over all mortgages \( i \) in a cluster \( C \). The monthly gross prepayment rate is then converted to an annual gross prepayment rate in the following way:

\[ GP_{t,C} = 1 - (1 - GMP_{t,C})^{12}. \]  

(4.2)

As we are only interested in the prepayments that actually incur a loss for the bank, we need to correct the gross prepayment percentage. Let \( I_{t,i} = P_{t,i} - PV_{t,i} \) be the economic impact for the bank, where \( P_{t,i} \) is the prepaid amount and \( PV_{t,i} \) the value of the future contractual cash flow until the end of the FIRP. We then define \( R_{t,i} \) as the percentage of the economic impact that is covered by a penalty paid by the client, i.e.: 

\[ R_{t,i} = \begin{cases} 1 & I \leq 0, \\ \min \left( \frac{Q_{t,i}}{I}, 1 \right) & I > 0 \end{cases}, \]  

(4.3)

where \( Q_{t,i} \) denotes the penalty amount paid by the client at time \( t \) on mortgage \( i \). Note that \( 0 \leq R_{t,i} \leq 1 \). We can now use the penalty ratio to calculate the net prepayment amount; the amount of the prepayment

\[ P_{t,i}^{\text{net}} = P_{t,i} \cdot (1 - R_{t,i}). \]  

(4.4)

Analogously to the gross prepayment rate, the monthly net prepayment rate and the annual net prepayment rate can be calculated as follows:

\[ NMP_{t,C} = \frac{\sum_{i \in C} P_{t,i}^{\text{net}}}{\sum_{i \in C} N_{t-1,i}}, \]  

(4.5)

and

\[ NP_{t,C} = 1 - (1 - NMP_{t,C})^{12}. \]  

(4.6)

The next step in prepayment modelling is to find a prepayment function that fits the plot of the coordinates \( (r(t,C), NP_{t,C}) \), where \( r(t,C) \) denotes the interest rate spread between the market rate and the client’s contract rate for a given cluster \( C \) at time \( t \).

4.3.3 Prepayment function

The prepayment function that is used to predict future prepayments is a function with one variable, \( r \), and four model parameters \( \Omega = (\alpha_0, \alpha_1, \alpha_2, \alpha_3) \). The variable \( r \), representing the incentive, is defined as

\[ r = r_{\text{client}} - r_{\text{market}}, \]  

(4.7)
the difference between the client’s contractual interest rate and the market observable mortgage rate. The prepayment function $P(r, \bar{A})$ is then given by the following expression:

$$P(r, \Omega) = \alpha_0 + \frac{\alpha_1}{1 + \exp(\alpha_2 + \alpha_3 \cdot r)}.$$  \hfill (4.8)

Note that the derivative

$$\frac{dP(r, \Omega)}{dr} = \frac{-\alpha_1 \cdot \alpha_3 \cdot e^{\alpha_2 + \alpha_3 \cdot r}}{(1 + \exp(\alpha_2 + \alpha_3 \cdot r))^2}$$  \hfill (4.9)

is positive whenever $-\alpha_1 \cdot \alpha_3 > 0$, meaning that under this condition, the prepayment function is increasing. This is a reasonable assumption; when $r$ increases, it is beneficial for the client to refinance his mortgage with a lower contractual interest rate. The larger the value of $r$, or the incentive, the more clients will take the opportunity to refinance their mortgage, resulting in more prepayments, and hence, a higher prepayment rate.

Following this reasoning we assume that $\alpha_1 \geq 0$, $\alpha_3 \leq 0$. This prepayment function has two asymptotes; one at $\alpha_0$ as $r \to -\infty$ and one at $\alpha_0 + \alpha_1$ as $r \to \infty$. The lower asymptote can be interpreted as prepayments that will always happen, independent of the market rate, due to, for example, death, divorces, heritages, etc. The higher asymptote represents the prepayments that will always happen plus the prepayments made by all the clients that show interest dependent prepayment behaviour. Clients that did not prepay when the incentive has risen to a certain level, will very likely neither prepay when the incentive rises even further.

It can also happen that for certain clusters, no interest driven prepayment behaviour is observed, en for those clusters, the prepayment function is constant; $P(r, (\alpha_0, 0, 0, 0))$.

By modelling future interest rate and using an interest dependent prepayment function, we can estimate future prepayments on the mortgage portfolios. One way of modelling future interest rates is by using a Hull White interest rate tree.
4.4 Hull-White interest rate tree

The Hull-White interest rate tree can be used to model interest rates using a trinomial tree. ING uses the Hull-White extended Vasicek short model as a basis for the tree building process. The dynamics of this short rate model are given by

\[ dr(t) = (\theta(t) - a \cdot r(t)) \, dt + \sigma dW(t). \]  

(4.10)

Here \( r(t) \) denotes the short rate at time \( t \), \( a \) is the mean reversion rate, \( \sigma \) is the volatility constant and \( \theta(t) \) is a deterministic function that fits the model to the initial yield curve.\(^2\)

As the HW tree is a discrete representation of an interest rate path with discrete time steps \( \Delta t \), we assume that there is a different discrete time short rate process, \( R(t) \), with step \( \Delta t \), which follows the same process as \( r(t) \):

\[ dR(t) = (\theta(t) - a \cdot R(t)) \, \Delta t + \sigma \Delta W(t). \]

(4.11)

One can see that this assumption is reasonable if we let \( \Delta t \rightarrow 0 \). We then define a new process \( R^*(t) \) with the same dynamics as \( R(t) \), but starting at zero:

\[ dR^*(t) = -a \cdot R^*(t) \cdot \Delta t + \sigma \Delta W(t). \]

(4.12)

This process forms the basis for our tree building process. Each central node in the tree will have value \( R^*(t) = 0 \). At the other nodes \((i, j)\) we set \( t = i \cdot \Delta t \) and \( R^*(t) = j \cdot \Delta R^* \). Following the recommendation of Hull and White, we take \( \Delta R^* = \sigma \sqrt{3 \Delta t} \). This value of \( \Delta R^* \) ensures that the volatility of \( R^* \) is represented well. To determine the transition probabilities between the nodes, we first observe the following:

\[ \mathbb{E}[R^*(t + \Delta t) - R^*(t)|R^*(t)] = -a \cdot R^*(t) \cdot \Delta t \]

(4.13)

\[ \text{Var}[R^*(t + \Delta t) - R^*(t)|R^*(t)] = \sigma^2 \cdot \Delta t \]

(4.14)

To incorporate mean reversion for very low or very high interest rates, we use three types of branching:

\(^2\)See appendix for further elaboration on \( \theta(t) \) and the fitting technique.
The transition probabilities for each of the three types of branching must then satisfy the following set of equations:

\[ p_u \Delta R_u^* - p_d \Delta R_d^* = \mathbb{E}[R^*] = -a \cdot R^*(t) \cdot \Delta t, \quad (4.15) \]

\[ p_u (\Delta R_u^*)^2 + p_d (\Delta R_d^*)^2 = \sigma^2 \Delta t + a^2 j^2 \cdot R^*(t)^2 \cdot \Delta t^2, \quad (4.16) \]

\[ p_u + p_m + p_d = 1. \quad (4.17) \]

It is suggested in [Hull, 2006] that we set \( \Delta R^* = \sigma \sqrt{3\Delta t} \). We can solve the equations for each type of branching:

**Regular branching:**

\[ p_u = \frac{1}{6} + \frac{1}{2} (a^2 j^2 \Delta t^2 - aj \Delta t), \quad (4.18) \]

\[ p_m = \frac{2}{3} - a^2 j^2 \Delta t^2, \]

\[ p_d = \frac{1}{6} + \frac{1}{2} (a^2 j^2 \Delta t^2 - aj \Delta t). \]

**Branching downwards:**

\[ p_u = \frac{7}{6} + \frac{1}{2} (a^2 j^2 \Delta t^2 - 3aj \Delta t), \quad (4.19) \]

\[ p_m = -\frac{1}{3} - a^2 j^2 \Delta t^2 + 2aj \Delta t, \]

\[ p_d = \frac{1}{6} + \frac{1}{2} (a^2 j^2 \Delta t^2 - aj \Delta t). \]
Branching upwards:

\[ p_u = \frac{1}{6} + \frac{1}{2} (a^2 j^2 \Delta t^2 + aj \Delta t) , \]

\[ p_m = -\frac{1}{3} - a^2 j^2 \Delta t^2 - 2aj \Delta t , \]

\[ p_d = \frac{7}{6} + \frac{1}{2} (a^2 j^2 \Delta t^2 + 3aj \Delta t) . \]

Note that, as \( a \) and \( \Delta t \) are constants, these transition probabilities only depend on \( j \). One can see that the probabilities attain values between 0 and 1, when \( 1 - \sqrt{2/3} < j \cdot a \cdot \Delta t < \sqrt{2/3} \). Therefore, we choose \( j_{\text{max}} \) to be

\[ j_{\text{max}} = \left\lceil \frac{1 - \sqrt{2/3}}{a\Delta t} \right\rceil , \quad j_{\text{min}} := -j_{\text{max}} , \]

the smallest integer greater than the lower bound for \( j \). We now have gathered all the necessary building blocks to build the tree for the process \( R^* \):

![Hull-White interest rate tree for discretized process \( R^* \).](image)

Note that the process is symmetrical around \( R(0) \) and that the tree has a bounded height. Albeit the process represented by this tree has the right mean reversion coefficient and the
right volatility, we still need it to fit the initial term structure. That is, we have to find \( \alpha(t) \), such that \( R(t) = \alpha(t) + R^*(t) \). We are going to calculate the values of \( \alpha(i \cdot \Delta t) \) iteratively. We define a new variable \( Q(i, j) \); the value of the security which pays \( \mathbf{E}1 \) if node \( (i, j) \) is reached and zero otherwise.

The interest rate tree needs to be able to correctly price a zero coupon bond for all \( t \). Let \( n_m \) be the number of nodes at time \( m \). We have to find \( \alpha_m \) such that the following relation holds:

\[
P_{m+1} = \sum_{j=-n_m}^{n_m} Q(m, j) \cdot \exp(-\alpha_m + j\Delta R)\Delta t).
\]

(4.22)

We can solve equation (4.22) for \( \alpha_m \):

\[
P_{m+1} = \sum_{j=-n_m}^{n_m} Q(m, j) \cdot \exp(-\alpha_m) \Delta t)
\]

\[
\ln P_{m+1} = \ln \left( \sum_{j=-n_m}^{n_m} Q(m, j) \cdot \exp(-\alpha_m) \Delta t) \right) - \alpha_m \Delta t
\]

(4.23)

We have now gathered all expressions required to correctly model the interest rates under the H-W short rate model with a fit on the initial term structure. Another method to find an expression for \( \alpha(t) \) is derived in chapter 5.

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4.5 Dynamic and static segmentation

The difference between dynamic and static segmentation lies in the way in which the expected cash flows are calculated. Expected cash flows are calculated using a Hull-White interest rate tree, which models interest rates on future time steps. The tool that we will use to compare the performances of dynamic and static segmentation performs multiple hedges at different time steps. Therefore, we are dealing with two dimensions in this research.

The first dimension, denoted by $T$, represents the ‘real world’ time; the time instances at which the hedges are executed. The second dimension, $t$, denotes the time instances in the Hull-White tree, which is generated at every time step $T$. Based on empirical research, we know that client prepayment behaviour changes over time. Suppose a client’s prepayment behaviour is currently modelled by prepayment function $A$. We know that the prepayment behaviour will change over time, and based on our clustering method, this means that from a future time $T^*$ on, the client’s prepayment behaviour should be modelled by prepayment function $B$.

The static segmentation method accounts for this prepayment behaviour by applying prepayment function $A$ for all $t$ in the Hull-White tree when $T < T^*$ and applying prepayment function $B$ for all $t$ in the Hull-White tree when $T \geq T^*$.

The dynamic segmentation method, however, accounts for the change in prepayment behaviour from $T = 0$. For all $T$, prepayment function $A$ is applied in the Hull-White tree if $t < T^* - T$ and prepayment function $B$ is applied if $t \geq T^* - T$. The sudden change in the modelled prepayment behaviour, which occurs with the static segmentation method, does no longer come about under the dynamic segmentation. This effect is visualised in chapter 7 where we will test both models.
Chapter 5

Interest rate simulation

5.1 Hull-White tree

The input for the prepayment function is the observable five year swapFTP. In order to determine which internal contracts have to be arranged, an expected cash flow has to be calculated. The expected cash flow depends on both the future contractual payments as well as the future prepayments. To find a good estimate for the future prepayments, a Hull-White trinomial interest rate tree is used to model the future five year swapFTP. We use the Hull-White interest rate model to model the five year swap rate:

\[ dr(t) = (\theta(t) - \alpha \cdot r(t)) \, dt + \sigma dW(t) \, . \] (5.1)

To model the interest rates in an interest rate tree, we need a discrete version of the process. We take the time step in the tree to be constant \( \Delta t \). Next, we assume that the \( \Delta t \) rate \( R \) follows the process

\[ dR(t) = (\theta(t) - \alpha R(t)) \, \Delta t + \sigma dW(t) \, . \] (5.2)

This assumption is reasonable as \( \Delta t \) tends to zero. In this model, \( \alpha \) and \( \sigma \) are the so-called Hull-White parameters. \( \alpha \) Represents the mean-reversion coefficient; the rate at which the process will return to the mean. \( \sigma \) Represents the drift of the process. The time dependent function \( \theta(t) \) is a function that fits the model to the initial forward curve:

\[ \theta(t) := \frac{\partial}{\partial t} f(0, t) + \alpha \cdot f(0, t) + \alpha \frac{\sigma^2}{2} \left(1 - e^{-2\alpha t}\right) \, . \] (5.3)

The Hull-White model is an affine term structure, which means that the price at time \( t \) of a zero coupon bond maturing at time \( T \) can be written as:

\[ P(t, T) = A(t, T) e^{B(t, T)} \, . \] (5.4)

Where \( A(t, T) \) and \( B(t, T) \) satisfy the following:

\[ B(t, T) = \frac{1}{-\alpha} \left(e^{\alpha(t-T)} - 1\right) \, , \]
\[ A(t, T) = -\frac{\sigma^2}{2} \int_t^T B^2(s, T) ds + \int_t^T \theta(s) B(s, T) ds \, . \] (5.5)
The goal of the prepayment model is to value the prepayment option on the contract. Thus we value a zero coupon bond at the end of the tree, as to value the option which is countered by an internal contract.

**Yield curve fitting**

For a sound comparison between dynamic segmentation and static segmentation, regarding market risk, one needs significantly many interest rate scenarios before one can draw any conclusions on the difference of both methods. The challenge is thus to simulate ‘realistic’ yield curves that can be used to test both methods. We will discuss a few methods that can be used for this simulation:

- Using affine term structures and the short rate model,
- Nelson Siegel (Svensson) yield curve fitting,
5.2 Yield curve construction from short rate modeling

A future yield curve is essentially a forward curve. In this section we will develop an expression for the forward curve by using the theory of affine term structures. In the end, this expression can be used to simulate a yield curve at a certain time \( t \), given the initial yield curve and the short rate at time \( t \).

We assume the short rate follows the process as given in (5.6). Other short rate models can be found in [Filipovic, 2009]. We choose this version of the short-rate model since it is used by model we study later in chapter 7.

\[
dr(t) = (\theta(t) - ar(t)) dt + \sigma dW(t). 
\]

(5.6)

This short rate model, the Hull-White extended Vasicek model, is an affine term-structure (ATS).

**Definition 5.1 (Affine Term Structure model).** An affine term structure model is a financial model that relates zero-coupon bond prices to a spot rate model. If the short rate model is an ATS, then the price of a zero coupon bond \( P(t,T) \) can be written as

\[
P(t,T) = \exp(-A(t,T) - B(t,T)r(t)),
\]

(5.7)

for smooth functions \( A(t,T) \) and \( B(t,T) \).

**Proposition 5.1 (Conditions for ATS).** The short rate model

\[
dr(t) = b(t_0 + t, r(t)) dt + \sigma(t_0 + t, r(t))dW^*(t), \quad r(0) = r_0
\]

provides an ATS if and only if its diffusion and drift terms are of the form

\[
\sigma^2(t, r(t)) = a(t) + \alpha(t)r(t) \quad \text{and} \quad b(t, r(t)) = b(t) + \beta(t)r(t),
\]

for some continuous functions \( a, \alpha, b, \beta \) and the functions \( A \) and \( B \) satisfy the system of ordinary differential equations, for all \( t \leq T \),

\[
\partial_t A(t, T) = \frac{1}{2} a(t)B^2(t, T) - b(t)B(t, T), \quad A(T, T) = 0, \quad (5.8)
\]

\[
\partial_t B(t, T) = \frac{1}{2} \alpha(t)B^2(t, T) - \beta(t)B(t, T) - 1, \quad B(T, T) = 0. \quad (5.9)
\]

**Proof.** The proof of this proposition can be found in [Filipovic, 2009][p. 84].

\[\square\]
Given that the zero coupon bond price and the forward rate are related through
\[ P(t, T) = e^{-\int_t^T f(t, u) \, du}, \]  
we can find an explicit expression for the forward curve using (5.7). \[Filipovic, 2009\]
After we find expressions for \( A(t, T) \) and \( B(t, T) \) we can conclude that
\[ f(t, T) = \partial_T A(t, T) + \partial_T B(t, T) r(t). \]  
(5.11)
From the HW short rate model we derive that our functions for the ATS, \( a, \alpha, b, \beta \), are
\[ a(t) = \sigma^2 \]
\[ b(t) = \theta(t) \]
\[ \alpha(t) = 0 \]
\[ \beta(t) = -a \]  
(5.12)
We start by solving (5.9):
\[ \partial_t B(t, T) = aB(t, T) - 1 \]
\[ B(t, T) = \frac{1}{a} (1 - e^{a(t-T)}). \]  
(5.13)
Next, solving (5.8) gives:
\[ \partial_t A(t, T) = \frac{1}{2} \sigma^2 B^2(t, T) - \theta(t) \cdot B(t, T) \]
\[ A(t, T) = -\frac{\sigma^2}{2} \int_t^T B^2(s, T) ds + \int_t^T \theta(s) B(s, T) ds. \]  
(5.14)
The next step to finding an expression for \( f(t, T) \) is finding an expression that fits the initial yield curve \( f(0, T) \). We use the equality
\[ f_0(T) = \partial_T A(0, T) + \partial_T B(0, T) \cdot r(0) \]
and substitute the known expressions:
\[ f_0(T) = \partial_T A(0, T) + \partial_T B(0, T) \cdot r(0) \]
\[ = \partial_T \left[ -\sigma^2 \int_0^T B^2(s, T) ds + \int_0^T \theta(s) B(s, T) ds \right] + \partial_T B(0, T) r(0) \]
\[ = \sigma^2 \int_0^T \partial_t B^2(s, T) ds + \int_0^T \partial_T \theta(s) B(s, T) ds + \partial_T B(0, T) r(0) \]  
(5.15)
\[ = \sigma^2 \int_0^T \left[ \frac{1}{a^2} \left( 1 - e^{a(t-T)} \right)^2 \right] ds + \int_0^T \partial_T \theta(s) B(s, T) ds + \partial_T B(0, T) r(0) \]
\[ = -\frac{\sigma^2}{2a^2} \left( 1 - e^{-aT} \right)^2 + \int_0^T \theta(s) e^{a(s-T)} ds + e^{-aT} r(0). \]  
(5.16)
\[ = g(T) \]
\[ = \phi(T) \]
In (5.15) we have used that \( \partial_T B(t,T) = -\partial_t B(t,T) \). Taking the derivative to \( T \) of \( \phi(T) \) yields

\[
\partial_T \phi(T) = \theta(T) - a \cdot \int_0^T \theta(s)e^{a(s-T)} ds - ae^{-aT} \cdot r(0)
\]

(5.17)

Note that \( f_0(T) + g(T) = \phi(T) \) by (5.16). We can use this to rearrange the result from (5.17) and find an expression for \( \theta(T) \):

\[
\theta(T) = \partial_T \phi(T) + a \phi(T)
\]

\[
\theta(T) = \partial_T (f_0(T) + g(T)) + a \cdot (f_0(T) + g(T)).
\]

(5.18)

We have completed all preliminary work to complete the expression for \( f(t,T) \). The last step consists of substituting (5.18) into (5.11). After some simple calculations we will find the expression we are looking for:

\[
f(t,T) = \partial_T A(t,T) + \partial_T B(t,T)r(t)
\]

\[
= -\frac{\sigma^2}{2a^2} \left(1 - e^{a(t-T)}\right)^2 + \int_t^T \theta(s)e^{a(s-T)} ds + \partial_T B(t,T)r(t)
\]

\[
= -\frac{\sigma^2}{2a^2} \left(1 - e^{a(t-T)}\right)^2 + \int_t^T \partial_s \left((f_0(s) + g(s)) e^{a(s-T)}\right) ds
\]

\[
+ \partial_T B(t,T)r(t)
\]

\[
= -g(T-t) + \left[(f_0(s) + g(s)) e^{a(t-T)}\right]_{s=t} + \partial_T B(t,T)r(t)
\]

\[
= f_0(T) - f_0(t)e^{a(t-T)} - (g(T-t) - g(T) + g(t)e^{a(t-T)}) + e^{a(t-T)}r(t)
\]

\[
= f_0(T) - f_0(t)e^{a(t-T)} - \left(\frac{\sigma^2}{2a^2} \left(1 - e^{-a(T-t)}\right)^2 - \frac{\sigma^2}{2a^2} \left(1 - e^{-aT}\right)^2\right)
\]

\[
+ \frac{\sigma^2}{2a^2} \left(1 - e^{-at}\right)^2 e^{a(t-T)} + e^{a(t-T)}r(t)
\]

\[
= f_0(T) - f_0(t)e^{a(t-T)} - \frac{\sigma^2}{2a^2} \left(1 - e^{-a(T-t)}\right)^2 - \left(1 - e^{-aT}\right)^2
\]

\[
+ \left(1 - e^{-at}\right)^2 e^{a(t-T)} + e^{a(t-T)}r(t)
\]

\[
= f_0(T) - f_0(t)e^{a(t-T)} - \frac{\sigma^2}{2a^2} \left(e^{a(t-T)} - 1\right) \left(e^{a(t-T)} - e^{-a(t+T)}\right)
\]

(5.19)

What remains is a correct simulation of the short rate so that we can use its values in (5.19) to model future yield curves. To do so, we need to find an expression for \( \theta(t) \) so that the short rate \( r(t) \) matches the initial yield curve. We use the equality

\[
P(0,T) = \exp \left(-A(0,T) - B(0,T)r(0)\right).
\]

(5.20)
Taking the logarithm and writing out $A(0, T)$ and $B(0, T)$ gives:

\[
\log(P(0, T)) = -A(0, T) - B(0, T)r(0)
\]

\[
\log(P(0, T)) = -\int_0^T \theta(s)B(s, T)ds + \frac{\sigma^2}{2} \int_0^T B^2(s, T)ds - B(0, T)r(0)
\]

\[
- \int_0^T \theta(s)B(s, T)ds = \log(P(0, T)) + \frac{1}{a} \left(1 - e^{-aT}\right) r(0)
\]

\[
- \frac{\sigma^2}{2a^2} \left(T - \frac{3}{2a} + \frac{2}{a} e^{-aT} - \frac{1}{2a} e^{-2aT}\right). \tag{5.21}
\]

With this equality, we can find an explicit expression for $\theta(t)$, so that the short rate $r(t)$ can be matched to the initial yield curve. We can solve this equation for $\theta(t)$ by differentiating twice with respect to $T$, starting with the left hand side. The first derivative of the LHS is given by:

\[
\frac{\partial}{\partial T} \int_0^T \theta(s)B(s, T)ds = \theta(s)B(T, T) + \int_0^T \theta(s) \frac{\partial}{\partial T} B(s, T)ds
\]

\[
= \int_0^T \theta(s)e^{a(s-T)}ds
\]

\[
= e^{-aT} \int_0^T \theta(s)e^{as}ds. \tag{5.22}
\]

And for the second derivative:

\[
\frac{\partial^2}{\partial T^2} \int_0^T \theta(s)B(s, T)ds = -ae^{-aT} \int_0^T \theta(s)e^{as}ds + e^{-aT}\theta(T)e^{aT}
\]

\[
= -ae^{-aT} \int_0^T \theta(s)e^{as}ds + \theta(T). \tag{5.23}
\]

Substituting the result from (5.22) on the right hand side of (5.23) yields the following relation:

\[
\theta(T) = \frac{\partial^2}{\partial T^2} \int_0^T \theta(s)B(s, T)ds + ae^{-aT} \int_0^T \theta(s)e^{as}ds
\]

\[
= \frac{\partial^2}{\partial T^2} \int_0^T \theta(s)B(s, T)ds + a \frac{\partial}{\partial T} \int_0^T \theta(s)B(s, T)ds. \tag{5.24}
\]

With this relation for $\theta(T)$, we go back and calculate the first and second derivative of the right hand side of (5.21). Starting with the first derivative, we find:

\[
\frac{\partial}{\partial T} \int_0^T \theta(s)B(s, T)ds = -\frac{\partial}{\partial T} \log P(0, T) - e^{-aT}r(0) + \frac{\sigma^2}{2a^2} \left(1 + e^{-2aT} - 2e^{-aT}\right). \tag{5.25}
\]
The second derivative becomes:
\[
\frac{\partial^2}{\partial T^2} \int_0^T \theta(s) B(s, T) \, ds = -\frac{\partial^2}{\partial T^2} \log P(0, T) + ae^{-aT} r(0) + \frac{\sigma^2}{2a^2} (-2ae^{-2aT} + 2ae^{-aT}). \tag{5.26}
\]

When we substitute these two derivatives into (5.24), the relation for \( \theta(T) \) becomes:
\[
\theta(T) = -\frac{\partial^2}{\partial T^2} \log P(0, T) - a \frac{\partial}{\partial T} \log P(0, T) + \frac{\sigma^2}{2a} (1 - e^{-2aT}). \tag{5.27}
\]

Recall that
\[
f(t, T) = -\frac{\partial \log P(t, T)}{\partial T}. \tag{5.28}
\]

Using this relation, we find the final form of the above expression for \( \theta(T) \) to be
\[
\theta(T) = \frac{\partial f(0, T)}{\partial T} + af(0, T) + \frac{\sigma^2}{2a} (1 - e^{-2aT}). \tag{5.29}
\]

Finally, to apply this model, we need a discretisation of the HW short rate model \( r(t) \):
\[
dr(t) = \left( \frac{\partial f(0, t)}{\partial t} + af(0, t) + \frac{\sigma^2}{2a} (1 - e^{-2at}) - ar(t) \right) \, dt + \sigma dW^*(t). \tag{5.30}
\]

This can easily be done using an Euler scheme and we find for small time steps \( \Delta t \):
\[
r(t + \Delta t) = r(t) + \left( \frac{\partial f(0, t)}{\partial t} + af(0, t) + \frac{\sigma^2}{2a} (1 - e^{-2at}) - ar(t) \right) \Delta t + \sigma \sqrt{\Delta t} Z(t). \tag{5.31}
\]

Where \( Z(t) \) denotes a standard normally distributed random variable.
5.3 Nelson Siegel Svensson yield curve fitting

[Nelson and Siegel, 1987] present a method for term structure fitting. We will discuss the
extension of their method by [Svensson, 1995]. The BIS reports that ‘currently nine out of
thirteen central banks which report their curve estimation methods to the BIS use either the
Nelson-Siegel or the Svensson model to construct zero-coupon yield curves’ [Pooter, 2007],
[BIS: Monetary and Economic Department, 2005, p. xi - xii].

One method to calculate $\frac{\partial f(0,T)}{\partial T}$ is to interpolate the initial yield curve at the finitely
many given points. We can then define the derivative by setting:

$$\frac{\partial f(0,t)}{\partial t} := \frac{f(0,t+h) + f(0,t-h)}{2},$$

(5.32)

for small enough $h$. The alternative is to fit a continuous and differentiable function through
the coordinates of the initial yield curve. We use the Nelson-Siegel-Svensson model to ap-
proximate the yield curve. The model is described by

$$\hat{f}_{\beta,\tau}(t) = \beta_0 + \beta_1 \exp\left(-\frac{t}{\tau_1}\right) + \beta_2 \frac{t}{\tau_1} \exp\left(-\frac{t}{\tau_2}\right),$$

(5.33)

where $\beta = (\beta_0, \beta_1, \beta_2, \beta_3)$ is a set of parameters on the rates and $\tau = (\tau_1, \tau_2)$ is a set of
parameters on the maturities. We will calibrate the parameters by minimising

$$\min_{\beta,\tau} \sum_{i=1}^{n} \left(f(0,t_i) - \hat{f}_{\beta,\tau}(0,t_i)\right)^2.$$

(5.34)

We do this by splitting the minimisation in two parts:

$$\min_\tau \min_\beta \sum_{i=1}^{n} \left(f(0,t_i) - \hat{f}_{\beta,\tau}(0,t_i)\right)^2.$$

(5.35)

Given a certain set $\tau$, we can easily minimise over $\beta$. Notice that $\hat{f}_{\beta,\tau}(t)$ is linear in $\beta$ if we
fix $\tau$ and $t$. Let $f_0 = (f(0,t_1), f(0,t_2), \ldots, f(0,t_n))$ be the vector with rates from the initial
yield curve. If we fix $\tau$, the minimisation problem can be solved by solving for $\beta$

$$\begin{pmatrix} f(0,t_1) \\ f(0,t_2) \\ \vdots \\ f(0,t_n) \end{pmatrix} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 1 & g(t_1,\tau_1) & g(t_1,\tau_1) - \exp\left(-\frac{t_1}{\tau_1}\right) & g(t_1,\tau_2) - \exp\left(-\frac{t_1}{\tau_2}\right) \\ 1 & g(t_2,\tau_1) & g(t_2,\tau_1) - \exp\left(-\frac{t_2}{\tau_1}\right) & g(t_2,\tau_2) - \exp\left(-\frac{t_2}{\tau_2}\right) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & g(t_n,\tau_1) & g(t_n,\tau_1) - \exp\left(-\frac{t_n}{\tau_1}\right) & g(t_n,\tau_2) - \exp\left(-\frac{t_n}{\tau_2}\right) \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}.$$

(5.36)

Where

$$g(t_i,\tau_j) = \frac{1 - \exp(-t_i/\tau_j)}{t_i/\tau_j}.$$

(5.37)

Using Cholesky decomposition, we can find a matrix that can be used to simulate correlated
random variables in order to correctly simulate short rates.
Chapter 6

Portfolio segmentation

Dynamic versus Static segmentation

The research on dynamical segmentation consists of three parts. The first, and most interesting part is where we compare the dynamic method against the static method on a replicated and simplified mortgage portfolio. The last two parts of the research help us to understand the impact of dynamic segmentation on the measures of interest.

In the second part, we test the dynamic segmentation method on two simple contracts, leaving out all market risk factors. The goal of this part of the research is also to determine the right type of contract that we will use in the more advanced simulations. In the third part of the research, we test eight different prepayment and interest rate scenarios. We thus allow the influence of market risk factors to test the dynamic method.

To quantify the quality of the dynamic segmentation method, we must select some measures from which we can derive the quantity of the segmentation method compared to static segmentation. We will perform all contract simulations with the dynamic and static segmentation, as to be able to compare the performance measures for both the segmentation methods. In all three parts of the research on dynamical segmentation, we will focus on the stability of the NPV, BPV and earnings. In the first part, where we simulate the replication portfolio, we will also study the margin and the sharpe ratio of the earnings.
We will start by looking at the differences between dynamic and static segmentation on a replicated portfolio. We will explain the differences, based on research that has been done on the specific characteristics of dynamic segmentation. The results and explanations of that research can be found below.

The following portfolio used is a simplified version of the largest mortgage portfolio of the ING, as of May 2014. The portfolio has been stripped down so that it only contains bullet contracts. Furthermore, all contracts in the portfolio have a remaining FIRP equal to their remaining term. The contracts have been clustered based on the client rate and the remaining FIRP. The clustering is rather rough, but due to the limited available computational power, we are restricted to the following set of 23 contracts as listed in table 6.1.

Note that the client rate on the largest part of the portfolio is 0.052, and that most of the portfolio’s volume is contained in contracts with remaining FIRP that is average: three to nine years. We have used 75 pre-defined interest rate scenarios to simulate the development of the portfolio with both static and dynamic segmentation. The expected effect of using dynamic segmentation instead of static segmentation is that the development of NPV and BPV will be more stable. A more stable NPV and BPV will result in less modifications on the internal contracts, and hence, the overall earnings will probably be higher, and less influenced by the current interest rate.

The prepayment functions used to perform simulations on this portfolio are the real prepayment functions related to the original portfolio, with a slight horizontal shift. This shift is applied so that all contracts are at the money at the start of the portfolio.

There are no new contracts added to the portfolio during the simulation, so the differences between dynamic and segmentation will diminish as the total volume of the portfolio decreases.

### Table 6.1: Composition of the replicated portfolio.

<table>
<thead>
<tr>
<th>Remaining FIRP / Client rate</th>
<th>0.03</th>
<th>0.044</th>
<th>0.052</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>50,038</td>
<td>107,043</td>
<td>61,365</td>
<td>218,446</td>
</tr>
<tr>
<td>12</td>
<td>220,880</td>
<td>77,649</td>
<td>248,644</td>
<td>547,173</td>
</tr>
<tr>
<td>20</td>
<td>173,001</td>
<td>224,027</td>
<td>157,457</td>
<td>554,485</td>
</tr>
<tr>
<td>36</td>
<td>960,778</td>
<td>473,769</td>
<td>1,792,961</td>
<td>3,227,508</td>
</tr>
<tr>
<td>72</td>
<td>199,943</td>
<td>474,937</td>
<td>1,674,847</td>
<td>2,349,727</td>
</tr>
<tr>
<td>108</td>
<td>40,303</td>
<td>688,412</td>
<td>1,832,341</td>
<td>2,561,056</td>
</tr>
<tr>
<td>140</td>
<td>0</td>
<td>374,647</td>
<td>1,441,733</td>
<td>1,816,380</td>
</tr>
<tr>
<td>170</td>
<td>0</td>
<td>44,203</td>
<td>1,441,733</td>
<td>1,485,936</td>
</tr>
<tr>
<td>200</td>
<td>0</td>
<td>0</td>
<td>131,353</td>
<td>131,353</td>
</tr>
<tr>
<td>total</td>
<td>1,644,943</td>
<td>2,464,687</td>
<td>8,782,234</td>
<td></td>
</tr>
</tbody>
</table>
Remark. In the graphs below the red line represents the results of the simulations where static segmentation is applied and the blue line represents the results of the simulations where dynamic segmentation is applied.

6.1.1 Net present value

First, we look at the average net NPV, as plotted in figure [6.2]. The net NPV development under dynamic segmentation is smoother than the development under static segmentation. The bump in the red line at $t = 12$ is caused by a small amount of contracts that have reached the end of the term. The bumps at $t = 20$ and $t = 50$ are caused by parts of the portfolio that switch prepayment functions. Recall from the graph above that the expected prepayments are higher for contracts with a shorter remaining FIRP. Hence, a change in prepayment function yields a shorter expected run-time of the contract, which lowers the NPV of said contract. For $t > 50$, the changes in prepayment behaviour are minimal, and the size of the entire portfolio has decreased significantly. Therefore, the differences between dynamic and static segmentation are negligible.

The absolute size of the standard deviation is influenced by the spread of the interest rate scenarios and the volume of the portfolio. The standard deviation of both segmentation methods is plotted in [6.3]. Under both dynamic and static segmentation, the standard deviation of the NPV increases; this is due to the increasing spread of the 75 different
Figure 6.2: Development of the average of the NPV over all interest rate scenarios for dynamic and static segmentation.

Figure 6.3: Development of the standard deviation of the NPV over all interest rate scenarios for dynamic and static segmentation.
interest rate scenarios. Beyond $t = 50$, the size of the standard deviation is decreasing due to the fast decreasing volume of the entire portfolio. The two bumps in the red line at $t = 20$ and $t = 50$ are caused by a switch in prepayment function by a large share of contracts in the portfolio. The standard deviation of the NPV over all interest rate scenarios is a good measure for the stability of the NPV. We can see in the graph above that dynamic segmentation yields a lot more stability than static segmentation; the impact of the interest rate scenarios on change in prepayment functions is accounted for in advance by dynamic segmentation.

### 6.1.2 Basis point value

The figures show that the development of both the internal BPV (figure 6.4) and external BPV (figure 6.5) is smoother under dynamic segmentation than under static segmentation. The size of the BPV is not so relevant in this case, for the hedge is based on the BPV. A smoother BPV development leads to less changes to the internal contracts, and hence, a higher margin.
Figure 6.5: Average of the external BPV of all interest rate scenarios.
Chapter 7

Analysing risk factors

7.1 Model analysis

To test the influence of the change of parameters under both segmentation methods, we ran several simulations on two mortgage contracts:

We let contract 1 switch prepayment function after 37 months, i.e., in the last two years of its term. Contract 2 switches prepayment function after three months, to have a fixed prepayment function for the last five years of its term. The other risk factors, such as the yield curve, the HW-parameters and the contract rate remain fixed during the simulations. We take these two cases to determine which causes the biggest difference between the static and the dynamic method in the measures of our interest: a long period during which the prepayment functions differ at a few nodes in the HW-tree (contract 1), or a short period during which the prepayment functions differs at almost all nodes in the HW-tree (contract 2).

Running simulations on these two contracts will help us decide whether we should use contract 1 or contract 2 for further simulations. Let $ppf_{in}$ denote the initial prepayment function both contracts, and let $ppf_{sh}$ denote the shocked prepayment function for the second part of the term, after the change in prepayment function. For the initial prepayment function, we have chosen a prepayment function that is currently being used and which is interesting enough to look at. This means that $\alpha_1$ is large enough, and that its slope is averagely steep in comparison to other prepayment functions. The second prepayment function, $ppf_{sh}$ is constructed by shocking one of the parameters of $ppf_{in}$ with a certain value. In the table

<table>
<thead>
<tr>
<th>Contract</th>
<th>Notional</th>
<th>Coupon</th>
<th>FIRP</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000000</td>
<td>0.06525</td>
<td>60</td>
<td>BUL</td>
</tr>
<tr>
<td>2</td>
<td>1000000</td>
<td>0.06525</td>
<td>62</td>
<td>BUL</td>
</tr>
</tbody>
</table>

Table 7.1: Two mortgage contracts used to study two types of segmentation.
Table 7.2: Shocks that will be applied to the four parameters of the prepayment function.

below the initial parameters and the values of the shocks are displayed. Note that in every scenario, at most one parameter is shocked, so we must discern 40 scenarios per contract. For example: if we apply a 0.06 shock on the \( \alpha_1 \), this means that we have a scenario with \( ppf_{in}(x; 0.038, 0.08, 9.6, -370) \) and \( ppf_{sh}(x; 0.038, 0.14, 9.6, -370) \). The figure below shows a plot of \( ppf_{in} \). The initial position of the two contracts is marked in the function. The contract rate on both the contracts has been chosen such that the contracts are at the money\(^1\) at \( T = 0 \).

\(^1\)See list of definitions.
The shock applied to the four parameters yield the following prepayment functions:
The results of the simulations show that the differences between static and dynamic segmentation are best shown when we use a contract with a FIRP of 60 months, which changes prepayment function after 36 months. Having eliminated all external risk factors, we can clearly see the difference between dynamic and static segmentation in all these simulations if we look at the NPV and external BPV development over time.

The graphs in figure 7.6 and 7.7 show the NPV development over time for all ten scenarios where we shock the value of $\alpha_0$. The blue and red lines show the development under the dynamic and static method, respectively.

Note the sudden shocks in the NPV under the static method, when the prepayment function changes. The static segmentation method lacks the ability to anticipate for future change in prepayment behaviour. This causes a sudden change in the expected cash flow, at the moment the prepayment behaviour changes. The different effects of the segmentation methods on the two contracts is also clear. The instant shocks in the NPV immediately transfer to the external BPV, which is shown in the graphs below.

Based on these results, we choose to continue with a contract equal to contract 1. This contract shows the difference between dynamic and static segmentation method, whereas contract 2 is better for showing the difference between the various prepayment and interest rate scenarios. The next step is to test both segmentation methods in simulations where we allow market risk factors.
Figure 7.6: NPV development of contract 1.

Figure 7.7: NPV development of contract 2.
Figure 7.8: BPV development of contract 1.

Figure 7.9: BPV development of contract 2.
Table 7.3: Eight interest rate scenarios and their related prepayment functions that will be tested to visualize the market risk on a bullet contract.

<table>
<thead>
<tr>
<th>Set</th>
<th>ppf\textsubscript{int}</th>
<th>ppf\textsubscript{fin}</th>
<th>yield curve shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0.0425, 0.0814, 5.1690, −300.338)</td>
<td>(0.14, 0.1172, 7.586, −440.834)</td>
<td>None</td>
</tr>
<tr>
<td>2</td>
<td>(0.0425, 0.0814, 5.1690, −300.338)</td>
<td>(0.15, 0, 0, 0)</td>
<td>None</td>
</tr>
<tr>
<td>3</td>
<td>(0.0425, 0, 0, 0)</td>
<td>(0.15, 0, 0, 0)</td>
<td>None</td>
</tr>
<tr>
<td>4</td>
<td>(0.1043, 0.0814, 5.1690, −300.338)</td>
<td>(0.018057, 0, 0, 0)</td>
<td>None</td>
</tr>
<tr>
<td>5</td>
<td>(0.0425, 0.0814, 5.1690, −300.338)</td>
<td>(0.14, 0.1172, 7.586, −440.834)</td>
<td>+ 150 bp</td>
</tr>
<tr>
<td>6</td>
<td>(0.1043, 0.0814, 5.1690, −300.338)</td>
<td>(0.15, 0, 0, 0)</td>
<td>- 75 bp</td>
</tr>
<tr>
<td>7</td>
<td>(0.1043, 0.0814, 5.1690, −300.338)</td>
<td>(0.01801, 0.0811, 10.685, −432.417)</td>
<td>- 75 bp</td>
</tr>
<tr>
<td>8</td>
<td>(0.0425, 0, 0, 0)</td>
<td>(0.14, 0.1172, 0.945, −440.834)</td>
<td>+ 150 bp</td>
</tr>
</tbody>
</table>

7.2 Market risk analysis

7.2.1 Market risk

To compare the market risk factors of the prepayment model under dynamic and static segmentation, we performed eight sets of simulations with various interest rate and prepayment scenarios on a single bullet contract. This bullet contract has an initial notional value of €1,000,000 and a remaining FIRP of 60 months. After 36 months, we change the prepayment function from ppf\textsubscript{int} to ppf\textsubscript{fin}, and in some cases, we bump the entire yield curve by a fixed amount of basis points. Table 7.3 shows the exact details of the eight sets of simulations, with the prepayment functions represented by the respective values of their parameters (α\textsubscript{0}, α\textsubscript{1}, α\textsubscript{2}, α\textsubscript{3})\textsuperscript{2}

We will discuss the most important results of simulations per set.

Set 2 - low IRD to high CPR

Plots in figure 7.10 and figure 7.11 show the development of the net earnings on 75 different interest rate scenarios, under the dynamic and static method respectively.

The bumps in the earnings profile at time point \( T = 12, 24, 36, 48, 60 \) are caused by the bucketing method in the earnings tool. At \( T = 37 \), the static method resets its entire internal cash flow profile. Given the yield curve at \( t = 37 \) and the rise in prepayments, the earnings under the various interest rate scenarios differ more than under the dynamic segmentation method, where the change in prepayment behaviour was already foreseen. Under the dynamic method, only little adjustment has to be made to the internal cash profile at \( T = 37 \), just as at every other time step. Therefore, the earnings development under the dynamic method is relatively smooth. We also see that due to the inability of the static method to account for future changes in prepayment behaviour, the earnings profile is much more interest rate

\[\text{Graphs of the prepayment functions can be found in Appendix ??}.\]
Figure 7.10: Earnings development under the dynamic method.

Figure 7.11: Earnings development under the static method.
sensitive, because the internal cash flow profile has to be reset at the time instant at which the prepayment function changes.

**Set 3 - low IRD to high IRD**

The prepayment scenario used in this is equal to the prepayment scenario used in the hedges. It is assumed that prepayment rate goes up as the remaining FIRP remains, and that prepayment rates are interest rate dependent. In the plots in figure 7.12 and 7.13 one can see the development of the net NPV over time, and the development of the standard deviation of the net NPV over time. Again, the blue lines show the results under the dynamic method, and the red lines show the results under the static method.

The plot in figure 7.12 shows the bumps in the NPV profile under the static method at $T = 37$. Note, however, that the standard deviation of the net NPV under the dynamic method is higher before $T = 37$. This is due to the accounting for the higher prepayment rate after $T = 36$ in the cash flow calculations under the dynamic method. This causes larger NPV differences under the various interest rate scenarios.
Set 4 - high IRD to low CPR

The ability of the dynamic method to account for the future changes in prepayment behaviour yields convergent behaviour on the measures of our interest. Figure 7.14 and 7.15 show the BPV development on various interest rate scenarios under both segmentation methods.

Set 5 - low IRD to high IRD with a 150 bp interest rate bump at $T = 37$.

The upward bump in the yield curve at $T = 37$ results in a shorter expected cash flow profile, because higher interest rates yield higher prepayment rates. Hence, the internal cash flow profile shifts forward as well. The differences in the cash flow profile before and after the interest rate bump are smaller under the dynamic method, because the dynamic method was already expecting higher prepayments rates, where the static scenario was not. Hence, the impact on the internal funding profile is slightly smaller under the dynamic scenario, and thus, the impact on the earnings is lower under the dynamic method than under the static method.

A parallel shift in the yield curve does not effect the BPV very much, hence, the external BPV development is hardly affected by the interest rate bump. Under the static method,
Figure 7.14: BPV development under dynamic segmentation.

Figure 7.15: BPV development under static segmentation.
Figure 7.16: Minimum and maximum of the net earnings over time.

Figure 7.17: Standard deviation on the net earnings over time.
however, the interest rate bump comes on top of a change in prepayment behaviour, changing the expected cash flow profile rigorously. The shock in the NPV is then transferred to the BPV as well.

**Set 8 - low CPR to high IRD with a 150 bp interest rate bump at $T = 37$.**

The development of the NPV is almost the same under all interest rate scenarios. The influence of the change in the yield curve are thus very little. The dynamic method accounts for future change in prepayment behaviour, so we observe a slowly diverging NPV development on the interest rate scenarios. The impact on the NPV in the static scenario is much larger, due to the sudden change in prepayment function at the time the yield curve is shocked.
Figure 7.19: External BPV development under static segmentation.

Figure 7.20: NPV development under dynamic segmentation.
Figure 7.21: NPV development under static segmentation.
Chapter 8

Discussion

8.1 Regulatory risk management

In chapter 3 we have studied the development of regulations on risk management by banks. The Basel Committee of the BIS agreed on three influential accords that guide the risk management practices. Several papers have been published that explain the accords and their practical implications of the accords. This thesis connects the conclusions of these papers and explains whether and why the succession and order was effective. The nature of the problem we discussed; that of transparency of implementations of risk models in academic literature, and lack thereof in banks, which introduces a bias into the research. We have not considered any advantages to secrecy in financial risk management, other than competitive advantage. This thesis is no attempt to explain the causes of the financial crisis of 2007, but rather a study of the public discussion on the lack of transparency regarding financial risk modeling by banks, that ignited after the financial crisis.

8.2 Segmentation model

The analysis on the prepayment model was limited by its implementation. We did not have access to the code, and hence, were not able to alter its implementation. We have assumed that the programmers implemented the model in a correct way, but we could not verify that. The same holds for the interest rate paths we used for the simulations. They were provided by the model validation department, and hence, should suffice for testing models.

8.2.1 Scenario testing

Limited computation power forced us to test a small set of scenarios, and a simplified version of a real portfolio. We can assume that the simulations provide reasons to believe that the dynamic model outperforms the static model in terms of stability of earnings and risk, but we have not been able to prove it on a real portfolio. All experiments were performed on a run-off portfolio. This method is also used with model validation, but never happens in the
real world. Relative differences between static and dynamic segmentation will be smaller in a real world scenario.

8.2.2 Measures of interest

Each measure in itself does have some drawbacks. Studying all three measures per scenario, does allow us to draw correct conclusions on the relative performances of the static and dynamic model.

**BPV**

A major disadvantage of the BPV measure is that it implicitly assumes that the underlying yield curve shifts in parallel. According to Occam’s razor, this is the most plausible assumption. However, it is not in accordance with reality. Secondly, knowledge about the BPV does not provide information about the amount of change in the yield curve on a daily basis. It merely provides a value for the derivative at the current point in time. Uncertainty about future changes in the yield curve translates to uncertainty about the change in position.

**NPV**

The net present value depends completely on the underlying yield curve and discount rates. We have used multiple interest rate paths and corresponding yield curves to overcome this problem. A larger number of interest rate paths would yield a higher significance level, but we did not have any.

**Sharpe ratio**

The Sharpe ratio depends on the level of the mean. For small values of the mean, the Sharpe ratio might explode. Which is exactly what happens in the case of a run-off portfolio. For our simulations, the Sharpe ratio alone is not a solid measure for the quality of the model.
Chapter 9

Conclusion

In this thesis we have sought for the answers to the research questions presented in Chapter 1. We have studied the development of regulations on risk management and the reactions of the banks to these very regulations. We then did an analysis of the performance of a particular prepayment model and studied how it fits into the regulatory risk framework of ING.

We conclude that the regulations upheld by the Basel Committee, do not limit the development and quality of the credit risk models. Competitive advantages gained by secrecy on modeling techniques have shown to not benefit the general public. National banks that monitor whether banks’ implementations of credit risk models suffice the Basel Committee’s regulations, can provide a role in enhancing the transparency of risk modeling practices by publishing - possibly masked - descriptions of their nation’s banks’ implementations. The competitive advantage remains while banks can learn from each other’s successes to further improve their models. Which, in the end, benefits all stakeholders.

9.1 Conclusions on regulations

We have seen how the attempts to build a framework with two clear goals - to strengthen the soundness and stability of the international banking system and reduce the competitive inequality among internationally active banks - has developed and eventually achieved the opposite of its two goals.

One could argue that the regulations in the Accord are at least partly responsible for the financial crisis of 2008. From the complete absence of transparency about risk models used by banks we must conclude that there is no visible improvement in the competitive inequality between banks.

The power of regulation, which should be at the supervisory authorities, has shifted back to the banks which, since 2004, are allowed to use their own internal risk models to set regulatory capital.

Several studies show that the internal ratings systems are inconsistent between banks and could vary even depending on the current state of the credit cycle. The procyclicality of
the Basel II could worsen the economy in times of recession. For these reasons alone, strict requirements on internal credit risk rating systems should be maintained, if we decide that they should still be allowed without publishing the details.

The measures set by the BIS in the first Accord and the amendment to it, are ambiguous. They are easy to manipulate and do not capture the right amount of risk in an underlying portfolio. Credit risk models are hard to evaluate. Firstly, because of the lack of data, secondly because implementation differs so much amongst banks that comparison is hard.

Some banks have been labeled ‘too big to fail’, which shows the responsibility that banks have in today’s society. The entire economy depends on the banks, as private companies. The lack of transparency, especially in credit risk management, is not in accordance with the enormous responsibility and influence that major banks have.

9.2 Conclusions on segmentation model

We have measured the performances of the dynamic and static model, by analysing the stability of the NPV and BPV of portfolios. The Sharpe ratio provides a measure that clearly expresses the relative variance of changes in the value of the BPV and NPV to the mean. It thus provides a normalized basis for comparison of the two models. The simulations on the replicated portfolio, presented in chapter 6, shows that the dynamic segmentation model is more stable than the static model, in terms of NPV and BPV. Over time, the dynamic model does require less adaptations to the outstanding hedge to remain a good hedge to the mortgage portfolio. The simulations in chapter 7 show that the dynamic model recovers faster when a shock is applied to the interest rate path, i.e. the dynamic model responds better to instabilities in the market rate, than the static model does. In terms of earnings, we can conclude that the losses incurred due to mis-predictions on the prepayments are significantly smaller with the dynamic model than the static model, on the portfolios we tested. It is safe to assume that similar results will be achieved on real portfolios.

It is hard to make a prediction about the long term drawbacks of increased transparency. The short term drawback would be that some banks will lose their competitive advantage that they now have in terms of sophisticated models. In the long term, an equal market with very little competitive inequality could have a large influence on the loan supply. It will benefit society if banks are more transparent about their models. It will show the true incentive of banks, improve the quality of models, be more fair to investors and borrowers, and it will be easier for supervisory authorities to monitor the current state of the financial market and intervene if necessary.

9.3 Recommendations for additional research

The performance of the two presented segmentation models is expressed in the measures NPV \[\text{Definition 2.2}\] and BPV \[\text{Definition 2.1}\]. By studying the moving average and variance of these measures during several interest rate scenarios, we are able to compare the stability of
the models. The dynamic segmentation model outperforms the static segmentation model in every setting of our tests. It shows an improved stability in NPV and BPV under various interest rate scenarios, resulting in a larger margin of earnings. With a test on real data, we studied the dynamical model on a run-off portfolio. A more extensive study on a continuous portfolio should show that the relative differences between the static model and the dynamic model decrease, since the ratio of insecurity decreases when a portfolio is refilled with long running mortgages. The implementation of the dynamic model requires significantly more computing power than the implementation of the static model. To increase the computation time of a hedge on the real portfolio, a different method than Hull-White interest rate trees would be useful to research.

9.4 Additional comments

During my internship at the Market Risk Management department of ING Bank, I have experienced the competitive environment that is risk management. Supervisory authorities in the Netherlands closely monitor and validate the models that are used for risk assessment, but banks do not know the details of other banks’ models. It is common practice that employees have to sign a contract of secrecy; making the sharing of information about competitor’s models illegal.
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