Where does quantum theory comes from*

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Introduction

- Quantum theory describes a large number of different experiments very well

WHY ?

- Can we give an explanation that goes beyond "because that is just the way it is" ?

Main message of this talk

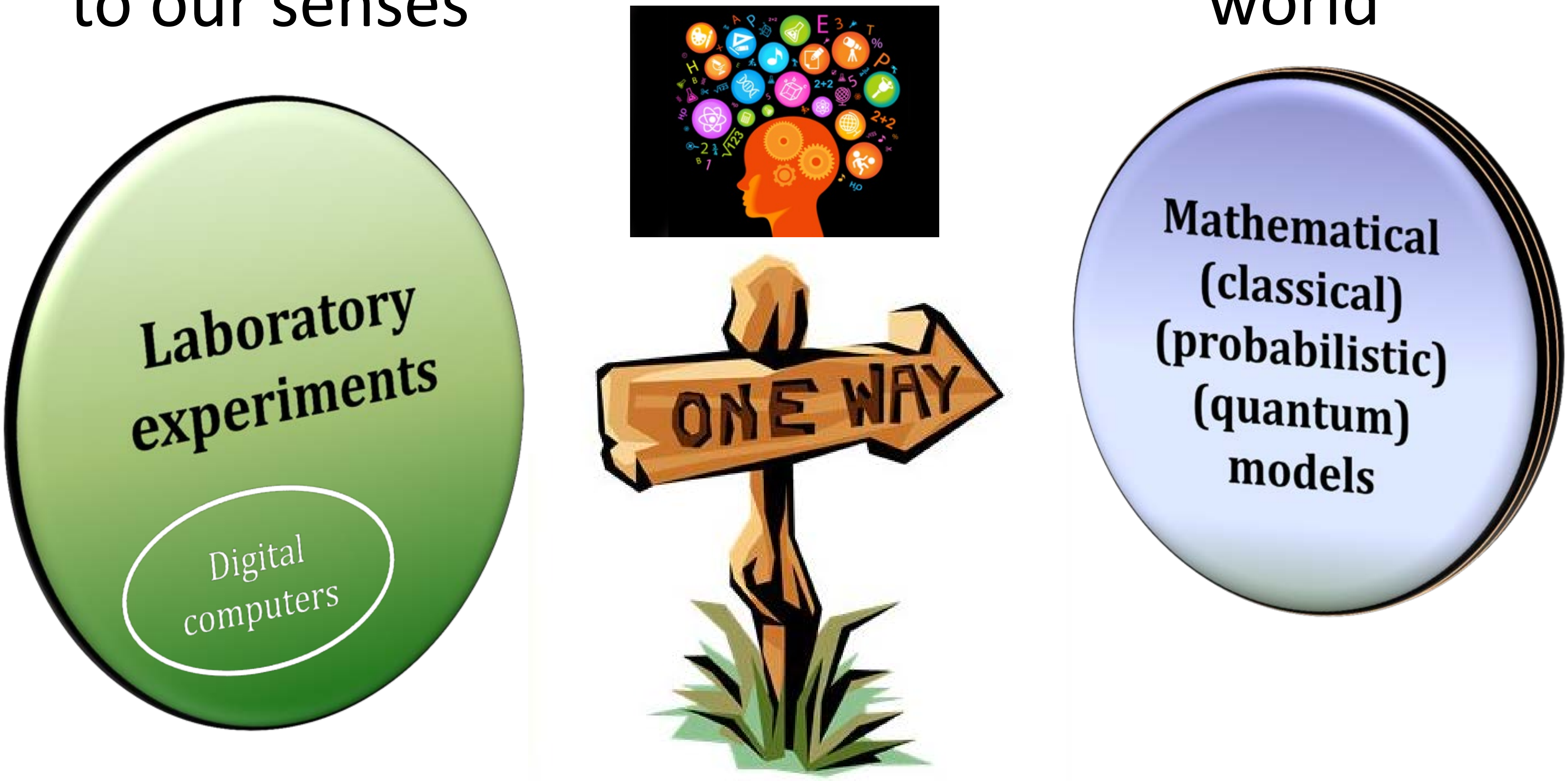
- Logical inference applied to experiments for which
  - There is uncertainty about each individual event
  - The frequencies of observed events are robust with respect to small changes in the conditions

➔ Derivation of basic equations of quantum theory from general principles of rational reasoning

- Derivation based on elementary principles of human reasoning and perception
  - Not another interpretation but a derivation
  - No ad-hoc quantization rules, no Born rule, no wavefunction collapse, no particle-wave duality, no measurement problem,…

World accessible to our senses

Mathematical world

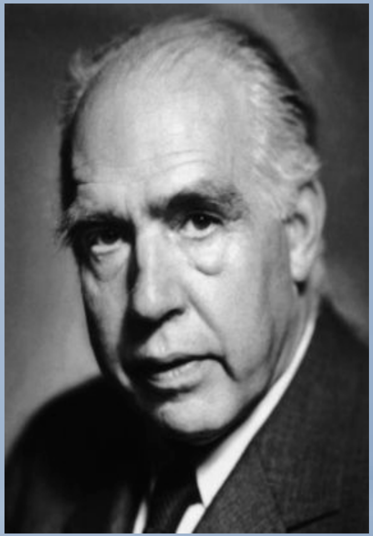


Father of quantum theory

There is no quantum world. There is only an abstract physical description.

It is wrong to think that the task of physics is to find out how Nature is.

Physics concerns what we can say about nature...



Niels Bohr

Niels Bohr, in "The philosophy of Niels Bohr" by Aage Petersen, in the *Bulletin of the Atomic Scientists* Vol. 19, No. 7 (September 1963)

Establishing a bridge between the world as we experience it and the world as we imagine it

- We can never know "everything" ➔ deal with this uncertainty from the start
- Describing data of laboratory experiments is a problem of inference, not a problem of deduction
- Can we construct a mathematical model for the cognitive processes that humans use to infer a descriptive model (theory) from observed data?

Plausible reasoning
(G. Polyá, R.T. Cox, E.T. Jaynes, …)

- Polyá: Patterns of plausible reasoning
  - Introduce the concept of a plausibility that a proposition *A* is true conditional on proposition *Z* being true
- Plausible reasoning, logical inference, inductive logic, rational reasoning:
  1. *Plausibilities must exhibit agreement with rationality*
  2. *All rules relating plausibilities must be consistent*
  3. *Plausibilities are represented by real numbers*

Plausible reasoning ➔ logical inference

- It follows (by calculation) that the plausibility $P(A|Z)$ that a proposition A (B) is true given that proposition Z is true must satisfy the rules*

a. $0 \leq P(A|Z) \leq 1$

b. $P(A|Z) + P(\bar{A}|Z) = 1 \quad ; \quad \bar{A} = \text{NOT } A$

c. $P(AB|Z) = P(A|BZ)P(B|Z) \quad ; \quad AB = A \text{ AND } B$

  - The rules (a-c) are unique.
  - Given the same data, any reasoning yielding a result that is at odds with the one obtained through rules (a-c) necessarily violates plausible reasoning

*R.T. Cox, E.T. Jaynes

Logical inference

- Algebra of logical inference

a. $0 \leq P(A|Z) \leq 1$

b. $P(A|Z) + P(\bar{A}|Z) = 1 \quad ; \quad \bar{A} = \text{NOT } A$

c. $P(AB|Z) = P(A|BZ)P(B|Z) \quad ; \quad AB = A \text{ AND } B$

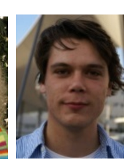
- Extension of Boolean logic, applicable to situations in which there is uncertainty about some but not all aspects

  – Kolmogorov’s probability theory is an example which complies with the rules of rational reasoning

  – Bayesian analysis

  – Quantum theory



Subjective and objective

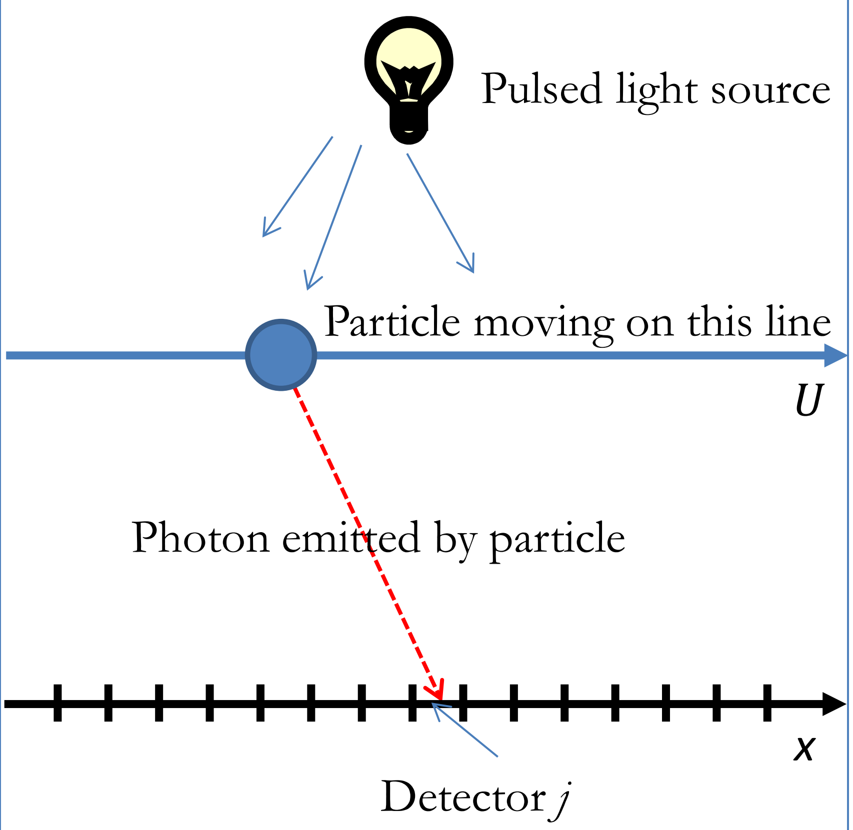
- Plausibility
  - Is an intermediate mental construct to carry out logical inference, inductive logic, rational reasoning
    1. May express a degree of believe (subjective)
    2. May be used to describe phenomena independent of individual subjective judgment ➔ this is our goal

Physics is to be regarded not so much as the study of something a priori given, but rather as the development of methods of ordering and surveying human experience. In this respect our task must be to account for such experience in a manner independent of individual subjective judgment and therefore objective in the sense that it can be unambiguously communicated in ordinary human language (N. Bohr, “XV. The Unity of Human Knowledge,” in *Complementarity Beyond Physics (1928 –1962)*)

Logical inference ➔ Schrödinger equation

- Generic procedure:
- Experiment ➔➔ ➔➔
- The "true" position U of the particle is uncertain and remains unknown
- Plausibility that the particle activates the detector at position j :

$$P(j\,|\,U,Z)$$



Plausibility of the data

- We repeat the experiment *N* times. The number of times that D_j clicks is k_j . The set of data collected is

$$Data = \{k_j \mid N = \sum_{j=-K}^{K} k_j\}$$

- Assume that events are independent ➔ the plausibility to observe k_j clicks of detector j is

$$P(Data \mid U, N, Z) = N! \prod_{j=-K}^{K} \frac{P(j \mid U, Z)^{k_j}}{k_j!}$$

Robust experiments

- Hypothesis H_0: given U we observe *Data*
- Hypothesis H_1: given $U + \varepsilon$ we observe *Data*
  - N is fixed and the same for both hypotheses H_0 and H_1
- The evidence $\mathrm{Ev}(H_1/H_0)$ is defined as

$$\mathrm{Ev}(H_1 / H_0) = \ln \frac{P(Data \mid U + \varepsilon, N, Z)}{P(Data \mid U, N, Z)} = \sum_{j=-K}^{K} n_j \ln \frac{P(j \mid U + \varepsilon, Z)}{P(j \mid U, Z)}$$

$$= \sum_{j=-K}^{K} n_j \left\{ \epsilon \frac{P'(j \mid U, Z)}{P(j \mid U, Z)} - \frac{\epsilon^2}{2} \left(\frac{P'(j \mid U, Z)}{P(j \mid U, Z)} \right)^2 + \frac{\epsilon^2}{2} \frac{P''(j \mid U, Z)}{P(j \mid U, Z)} \right\} + O(\epsilon^3)$$

- We require that *Data* is robust with respect to small changes in U ➔ minimize, in absolute value, the coefficients of ϵ, ϵ^2, for all possible U

Remove dependence on ϵ (1)

$$\mathrm{Ev}(H_1/H_0) = \sum_{j=-K}^{K} n_j \left\{ \epsilon \frac{P'(j|U,Z)}{P(j|U,Z)} - \frac{\epsilon^2}{2}\left(\frac{P'(j|U,Z)}{P(j|U,Z)}\right)^2 + \frac{\epsilon^2}{2}\frac{P''(j|U,Z)}{P(j|U,Z)} \right\} + O(\epsilon^3)$$

- Choosing $P(j|U,Z) = \frac{n_j}{N}$ removes the 1st and 3rd term
  - Frequency n_j/N is "objective" knowledge (= counts)
    - Subjective aspect of plausibility has been eliminated
  - We recover the intuitive procedure of assigning to the plausibility of the individual event, the frequency which maximizes the plausibility to observe the whole data set

Remove dependence on ϵ (2)

$$\mathrm{Ev}(H_1/H_0) = \sum_{j=-K}^{K} n_j \left\{ \epsilon \frac{P'(j|U,Z)}{P(j|U,Z)} - \frac{\epsilon^2}{2}\left(\frac{P'(j|U,Z)}{P(j|U,Z)}\right)^2 + \frac{\epsilon^2}{2}\frac{P''(j|U,Z)}{P(j|U,Z)} \right\} + O(\epsilon^3)$$

- Most robust solution: Minimize the 2nd term (Fisher information) for all U:

$$I_F = \sum_{j=-K}^{K} \frac{1}{P(j|U,Z)}\left(\frac{\partial P(j|U,Z)}{\partial U}\right)^2$$

- Continuum limit:

$$I_F = \int dx \frac{1}{P(x|U,Z)}\left(\frac{\partial P(x|U,Z)}{\partial U}\right)^2$$

Extra assumption

- It does not matter if we repeat the experiment somewhere else (translational invariance) ➔

$$P(x|U,Z) = P(x+\zeta|U+\zeta,Z) \quad ; \quad \zeta \text{ arbitrary}$$

- Condition for robust frequency distribution ⬄ minimize the functional (Fisher information)

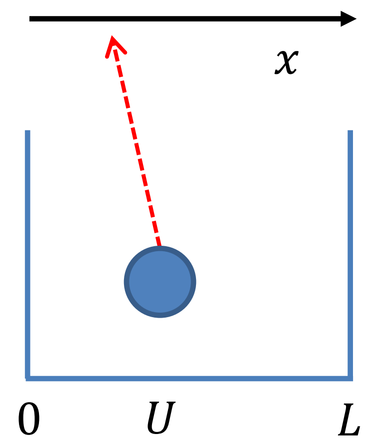
$$I_F = \int dx \frac{1}{P(x|U,Z)} \left(\frac{\partial P(x|U,Z)}{\partial x} \right)^2$$

with respect to $P(x|U,Z)$ for all U simultaneously

What if **all** we know for certain is that the particle is somewhere in a box?

- Particle in a 1D box, detectors on $[0, L]$

$$I_F = \int_0^L dx \frac{1}{P(x|U,Z)} \left(\frac{\partial P(x|U,Z)}{\partial x} \right)^2$$



- Minimize for all *U* ➔

$$P(x|U,Z) = \frac{2}{L} \left(\sin \frac{n\pi x}{L} \right)^2 \quad , \quad n = 1, 2, \ldots$$

- **We recover the result of quantum theory for a particle in a box (up to units)**

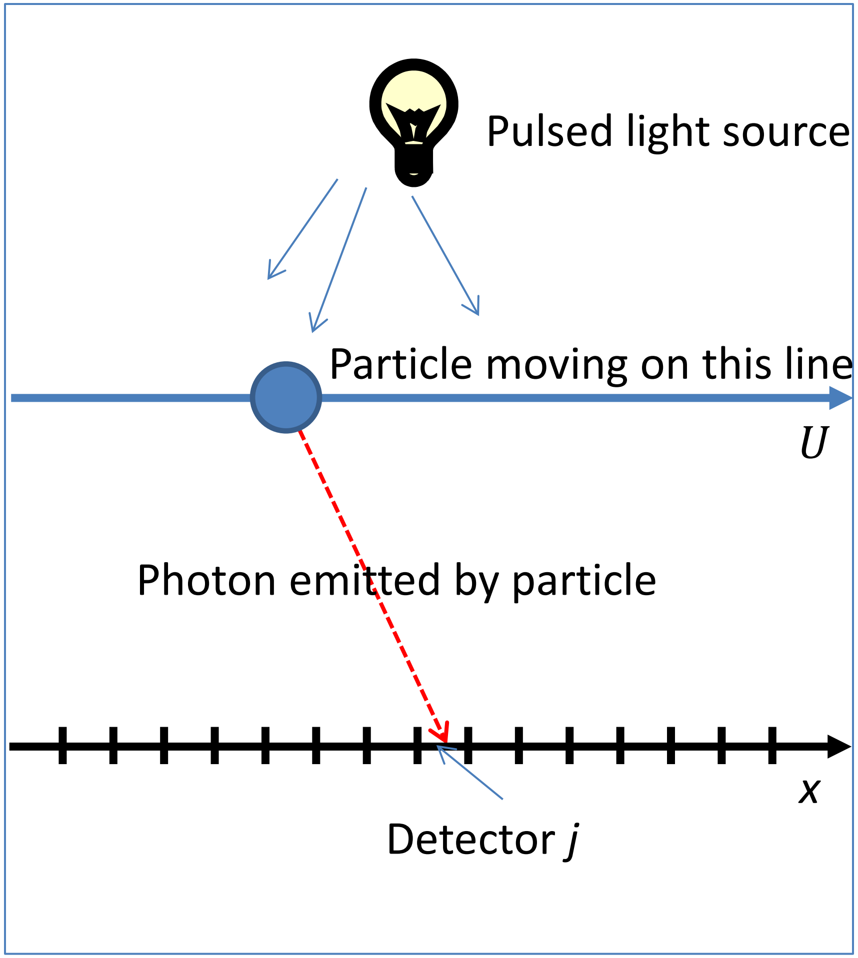
$$I_F = \frac{4n^2\pi^2}{L^2} \propto E_n$$

Schrödinger equation: Particle moving in a potential

- We have no knowledge about the motion of the particle at unknown position U other than through the observed detector clicks at j
- Hypothesis: there exists a function $f(x(t),t)$ such that

$$\frac{dx(t)}{dt} = f(x(t),t)$$

➔ Time-dependent ordinary differential equation ("flow")



Particle moving in a potential (2)

- Consequences of the "flow" hypothesis

$$\frac{d^2x(t)}{dt^2} = \frac{\partial f(x(t),t)}{\partial t} + \frac{dx(t)}{dt}\nabla f(x(t),t)$$
$$= \frac{\partial f(x(t),t)}{\partial t} + f(x(t),t)\nabla f(x(t),t)$$

- Helmholtz decomposition of a vector field

$$\mathbf{f} = \nabla S + \nabla \times \mathbf{A}$$

  – In 1D: $f(x,t) = \frac{\partial S(x,t)}{\partial x}$

Particle moving in a potential

- Consequences of the "flow" hypothesis

$$\frac{d^2x}{dt^2} = \frac{\partial}{\partial x}\left[\frac{\partial S(x,t)}{\partial t} + \frac{1}{2}\left(\frac{\partial S(x,t)}{\partial x}\right)^2\right]$$

- Newton: $\frac{d^2x}{dt^2} = \frac{1}{m}F(x,t) = -\frac{1}{m}\frac{\partial}{\partial x}V(x,t)$

➔ $$\frac{\partial S(x,t)}{\partial t} + \frac{1}{2}\left(\frac{\partial S(x,t)}{\partial x}\right)^2 + \frac{1}{m}V(x,t) = 0$$

➔ Hamilton-Jacobi equation

J. Ralston, Proc. of SPIE Vol. 8832 88320W-12 (2013)

Particle moving in a potential

- But x is NOT the particle position, U is!
- Therefore (following Schrödinger) we require that

$$\int P(x|U,t,Z)\left[\frac{\partial S(x,t)}{\partial t}+\frac{1}{2}\left(\frac{\partial S(x,t)}{\partial x}\right)^2+\frac{1}{m}V(x,t)\right]dxdt=0$$

➔ if x and U are "close" (little uncertainty) we "observe" Newtonian dynamics

Particle moving in a potential

- Putting all pieces together: Robustness ➔ minimize, for all (U, t) the Fisher information

$$I_F = \int \frac{1}{P(x|U,t,Z)} \left(\frac{\partial P(x|U,t,Z)}{\partial x} \right)^2 dxdt$$

subject to the constraint

$$\int P(x|U,t,Z) \left[\frac{\partial S(x,t)}{\partial t} + \frac{1}{2} \left(\frac{\partial S(x,t)}{\partial x} \right)^2 + \frac{1}{m} V(x,t) \right] dxdt = 0$$

Robustness + “classical limit”

- Minimize the functional

$$Q=\int\left\{\frac{1}{P(x|U,t,Z)}\left(\frac{\partial P(x|U,t,Z)}{\partial x}\right)^2+\lambda P(x|U,t,Z)\left[\frac{\partial S(x,t)}{\partial t}+\frac{1}{2}\left(\frac{\partial S(x,t)}{\partial x}\right)^2+\frac{1}{m}V(x,t)\right]\right\}dxdt$$

  for all U

  - λ = Lagrange multiplier to deal with the constraints

➔ Coupled set of nonlinear equations for $P(x|U,t,Z)$ and $S(x,t)$

= Madelung’s hydrodynamic form of TDSE (1927)

= Bohm’s formulation of TDSE (1952)

Solving the nonlinear equations

- Nonlinear equations for $P(x|U,t,Z)$ and $S(x,t)$ become linear by substituting*

$$\psi(x\,|\,U,t,Z) = \sqrt{P(x\,|\,U,t,Z)}\,e^{iS(x,t)\sqrt{\lambda}/2}$$

- Identifying $\lambda = 4\hbar^{-2}$ we find

$$i\hbar\frac{\partial\psi(x\,|\,U,t,Z)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x\,|\,U,t,Z)}{\partial x^2} + V(x,t)\psi(x\,|\,U,t,Z)$$

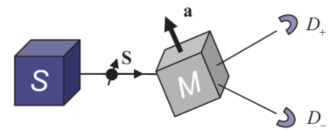
➔ Time-dependent Schrödinger equation

- Born rule $P(x|U,t,Z) = |\psi(x|U,t,Z)|^2$ is build in, no need to postulate
- No quantization rules, no particle-wave duality, no measurement problem, no wave function collapse, no interpretations of wave functions, …, wave function is a "tool"

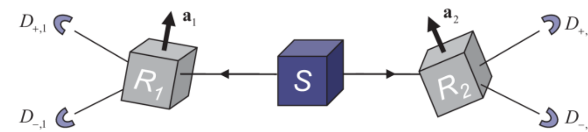
*E. Madelung, "Quantentheorie in hydrodynamischer Form," Z. Phys. 40, 322 – 326 (1927)

Successful derivations so far

- Stern-Gerlach experiment



- Einstein-Podolsky-Rosen-Bohm experiment (e.g. singlet state)



- Time-independent Schrödinger equation
- Time-dependent Schrödinger equation for particle in electromagnetic field
- Pauli equation for charged spin ½
- Pauli equation for neutral spin ½ particles
- Klein-Gordon equation
- Dirac equation without EM potentials

Where does quantum theory comes from?

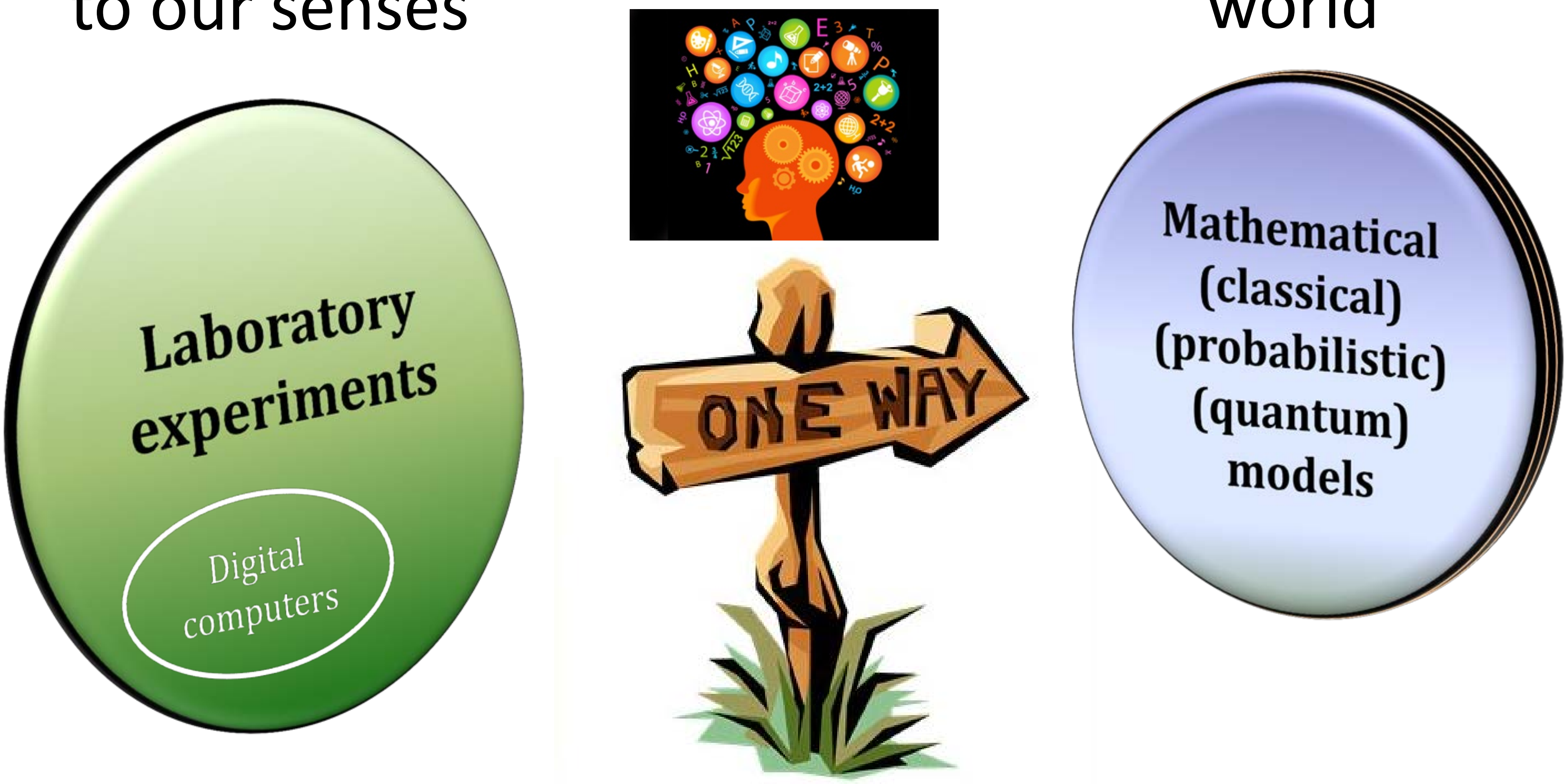
- Quantum theory is logical inference, that is common sense reasoning, applied to reproducible and robust experimental data.
  - ➔ quantum theory is a phenomenological theory which can be derived from a set of simple general principles in a way that is independent of any (strictly speaking, unknown) “more microscopic” level of description
  - As a solution of an optimization problem quantum theory is “the best” probabilistic description and hard to beat…

➔ WHY QT is so powerful

- A parallel with Einstein’s view on thermodynamics: a theory of principles based on empirically observed properties of phenomena, independent of a particular underlying model rather than a constructive theory, an attempt to build a picture of complex phenomena out of some relatively simply propositions

World accessible to our senses

Mathematical world



THANK YOU