A Derivation of Minimal Softly Broken Supersymmetric Quantum Electrodynamics

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1 Introduction

The subject of this thesis is a derivation of minimal softly broken Supersymmetric Quantum Electrodynamics (SQED). Conventional derivations are of course already known. There one starts by imposing supersymmetry and one builds a Lagrangian subject to this constraint. Only after the derivation is complete one can see which mass eigenstates (particles) and which interactions for these particles one has constructed. Also only then the advantages of the supersymmetric theory one has constructed become visible. It then appears that a supersymmetric theory has no quadratic divergences.

What we are interested in is a derivation which is more direct. This means we want to avoid the situation that one starts building and one only sees afterwards what one has built. Such derivations already exist for example for the Electroweak Standard Model. These can be seen in an article by R. Kleiss [1] and in a paper by M. van Kessel [2]. In these derivations one starts by postulating all particles of the theory. Then one starts building the couplings (vertices) between these particles. The need for the vertices one introduces is always clear, its introduction may be on empirical grounds or on consistency grounds. From this latter category mostly unitarity is used. The form of the vertices are also fixed on direct grounds, like unitarity, gauge invariance and Lorentz invariance. In this way one avoids diagonalizing mass matrices and one is always dealing with the real particles. Also one can see whether one is led uniquely to the theory to be constructed, or if there are other possibilities which also explain all experiments and are consistent.

Such a direct derivation is what we want to present for a supersymmetric theory, in this case Supersymmetric Quantum Electrodynamics. We also start by postulating the particle content and continue to introduce vertices where their need should always be clear on empirical or consistency grounds. The form of the vertices and the coupling constants we also want to fix on grounds of unitarity, gauge invariance and Lorentz invariance as in the case of the Electroweak Standard Model. We will then see if we can fix supersymmetric theories in the same way as ordinary theories via this more direct path. Also it is interesting to see if one is led uniquely to the supersymmetric theory (or some broken version of it) with our more physical constraints. However it becomes clear early in our derivation that with only the constraints used in a direct derivation of ordinary theories one cannot get very far. We will need another constraint which also is reasonable to impose.

This constraint will become the absence of quadratic divergences, which is the great advantage of supersymmetric theories over other theories. But why is this absence an advantage? To see this consider the mass of the Higgs particle in the Standard Model. At every loop order this mass receives a correction of magnitude Λ^2, where Λ is the cut-off momentum. Such an infinity is then absorbed into the bare mass to keep the physical mass finite. We know already that the Standard Model is not a good model all the way up to infinite momenta. For very large momenta some physical coupling constants really blow up which is disastrous for the Standard Model as a really fundamental theory. Also gravitational effects come into play at momenta around the Planck scale so that also from this argument it is clear the the Standard Model cannot be considered as a real fundamental theory. We should thus cut off the momentum integrals at some large value anyway. Still then the Higgs mass receives corrections of magnitude Λ^2 at every loop order, where Λ now is some very large cut-off, but doesn't approach infinity eventually. To keep the Higgs mass around its real, physical, value we see that we have to make huge corrections to the bare mass. The corrections may be between 10 or 20 orders of magnitude larger than the physical mass itself. So the bare mass should be fine-tuned very well to keep the physical mass on its actual (experimental) value. This seems very unnatural, therefore this problem is called the fine-tuning or naturalness problem.

Other masses in the Standard Model do not suffer from these problems. The fermion masses for example get corrections proportional to $m \ln \Lambda$ at every loop order. These corrections aren't that large because of the fermion mass m in front and because $\ln \Lambda$ is still not very large, even if we take the Planck mass for Λ. So one wants a protection mechanism for the Higgs too. Supersymmetric theories give such a protection mechanism. Roughly speaking contributions from boson loops cancel partially against contributions from fermion loops only leaving logarithmic divergences.

Supersymmetry does have more advantages however. We will not all treat them here since these advantages are not of importance to us and our derivation. What is important is that we see the advantage of the absence of all quadratic divergences and how it is a reasonable constraint.

With all the constraints mentioned thus far we will try to derive minimal softly broken SQED. The advantages of such an approach are the same as in the case of ordinary, not supersymmetric, theories. In this case of supersymmetric theories it is especially interesting how far one can get fixing the theory. We know that we can completely fix the Electroweak Standard Model with the direct derivation described above. The question is whether the same holds in the case of supersymmetric theories. If there appear many other theories with no quadratic divergences one immediately knows that absence of quadratic divergences is not a very good reason to introduce supersymmetry. If, on the other hand, we are led uniquely to a supersymmetric theory we know that it is a good reason to introduce supersymmetry.

From our calculations, which will go up to 2-loop order, it will appear that we can fix a large part of the theory. Which vertices exist and the form of these vertices will be completely fixed. Only some coupling constants will remain unfixed, which indicates that there many theories which have the same good properties as minimal softly broken SQED up to 2-loop order.

2 The conventional derivation

In this section we will present the conventional derivation of minimal softly broken Supersymmetric Quantum Electrodynamics (SQED). First we will derive unbroken SQED. This means we will search for the simplest Lagrangian L which is supersymmetric and contains the usual QED terms. This means the Lagrangian we are searching for should be supersymmetric, gauge invariant and of course Lorentz invariant. After this we shall break supersymmetry, but in such a way that quadratic divergences remain absent from our theory. This we will do in a minimal way. Then we shall read off the Feynman rules for the theory we have constructed. These Feynman rules are what we want to reproduce in later sections concerning a different derivation of this same theory.

2.1 The supertransformation

To derive SQED we must first know what "supersymmetric" actually means. A theory is called supersymmetric when its Lagrangian L is invariant under the supertransformation. How this supertransformation operates on an arbitrary field X we will explain below. This action is mostly written as follows. The infinitesimal change $\delta_\varepsilon X$ in the general field X when the supertransformation operates on it is given by

$$\delta_\varepsilon X = (\varepsilon Q + \bar{Q}\bar{\varepsilon})X. \tag{1}$$

This expression calls for some explanation. In this expression ε is an infinitesimal anti commuting parameter of the supertransformation. It behaves like a lefthanded spinor under Lorentz transformations and thus also has an index α, which can be 0 or 1. The word "anti commuting" means it anti commutes with all anti commuting objects in the theory including itself. Thus the infinitesimal change in ε_α under a Lorentz transformation is

$$\delta_\Delta \varepsilon_\alpha = \Delta_\mu \sigma^{\mu}{}_{\alpha}{}^{\beta} \varepsilon_\beta. \tag{2}$$

In this case Δ is the infinitesimal parameter of the Lorentz transformation. σ^0 Is numerically equal to the 2-dimensional unity matrix and the expressions σ^i, where i can be 1, 2 or 3, are numerically equal to the Pauli matrices. With help of the two dimensional tensor $\epsilon^{\alpha\beta}$ with $\epsilon^{\alpha\beta} = -\epsilon^{\beta\alpha}$ and $\epsilon^{12} = 1$ we can also define an ε with an upper index:

$$\varepsilon^\alpha = \epsilon^{\alpha\beta}\varepsilon_\beta. \tag{3}$$

Now we can finally explain what we meant by εQ in (1). By convention this product is

$$\varepsilon Q = \varepsilon^\alpha Q_\alpha. \tag{4}$$

The Q in (1) is, like ε, also a lefthanded spinor and is one of the generators of the supertransformation. What algebra this Q has to satisfy will be treated later. The $\bar{\varepsilon}$ in (1) is also an infinitesimal anti commuting parameter of the supertransformation but behaves like a righthanded spinor under Lorentz transformations:

$$\delta_\Delta \bar{\varepsilon}_{\dot{\alpha}} = \Delta_\mu \sigma^{\mu}{}_{\dot{\alpha}}{}^{\dot{\beta}} \bar{\varepsilon}_{\dot{\beta}}. \tag{5}$$

Notice that we have labeled the indices of righthanded spinors with dots. This is a convention mostly used in literature which we also adopt. The σ^0 with the lower index $\dot{\alpha}$ and the upper index $\dot{\beta}$ is numerically equal to minus the 2-dimensional unity matrix. The expressions σ^i with the lower index $\dot{\alpha}$ and the upper index $\dot{\beta}$ are numerically equal to minus the complex conjugated Pauli matrices. Now we can also say what is meant by the product $\bar{Q}\bar{\varepsilon}$ in (1):

$$\bar{Q}\bar{\varepsilon} = \bar{Q}_{\dot{\alpha}}\bar{\varepsilon}^{\dot{\alpha}}. \tag{6}$$

Thus the two terms in the supertransformation (1) both are Lorentz invariant.

Now we have cleared up the meaning of (1) we can finally say something about the algebra which Q and $\bar{Q}$ have to satisfy. This algebra is:

$$\begin{aligned}
[Q_\alpha, P_\mu] &= 0 \\
\{Q_\alpha, Q_\beta\} &= 0 \\
\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} &= 0 \\
\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} &= 2\sigma^\mu{}_{\alpha\dot{\beta}} P_\mu.
\end{aligned} \tag{7}$$

Here P^μ is the momentum operator. This algebra can be derived from the very definition of supersymmetry and can be found in many introductory texts concerning supersymmetry. (See for example reference [3].)

2.2 Superspace

Now we know what algebra Q and $\bar{Q}$ satisfy. To find a supersymmetric Lagrangian L it would however be handy to know Q and $\bar{Q}$ explicitly. Then we could check whether certain expressions of fields were suited for our supersymmetric Lagrangian.

Such an explicit realization can be made rather easily if we introduce superspace. Superspace is a space in which there are the usual coordinates x^μ plus the coordinates θ_α and $\bar{\theta}_{\dot{\alpha}}$. The coordinates θ_α anti commute with all anti commuting objects in the theory and transform like lefthanded spinors under Lorentz transformations. The coordinates $\bar{\theta}_{\dot{\alpha}}$ are also anti commuting but transform like righthanded spinors. On this superspace we have so called superfields. These fields depend on the usual coordinates x^μ and also on θ_α and $\bar{\theta}_{\dot{\alpha}}$. Their dependence on θ_α and $\bar{\theta}_{\dot{\alpha}}$ must be understood in terms of their power series expansion in θ_α and $\bar{\theta}_{\dot{\alpha}}$. This expansion only has a finite number of terms because terms with more than two θ_α's or more than two $\bar{\theta}_{\dot{\alpha}}$'s vanish because of the anti commutativity:

$$\theta_\alpha\theta_\beta\theta_\gamma = -\theta_\beta\theta_\alpha\theta_\gamma = -\theta_\alpha\theta_\beta\theta_\gamma = 0. \tag{8}$$

The first step can be taken because of the anti commutativity, the second step because always 2 indices have the same value (0 or 1) and the θ's having these indices can be swapped. By anti commuting we can then get the sequence $\theta_\alpha\theta_\beta\theta_\gamma$ back. This process does not give an extra minus sign since it always takes an even number of anti commutations to get the desired sequence back. The most general Lorentz invariant superfield S thus has the form

$$\begin{aligned}
S(x,\theta,\bar{\theta}) &= a(x) + \theta b(x) + \bar{\theta}\bar{c}(x) + \theta\theta d(x) + \bar{\theta}\bar{\theta}\bar{e}(x) + \\
&\quad \theta\sigma^\mu\bar{\theta} f_\mu(x) + \theta\theta\bar{\theta}\bar{g}(x) + \bar{\theta}\bar{\theta}\theta h(x) + \theta\theta\bar{\theta}\bar{\theta} i(x).
\end{aligned} \tag{9}$$

Here we left out all the spinor indices. How these should be placed is dictated by Lorentz invariance. One of the advantages of this superspace formalism is that a Q and a $\bar{Q}$ satisfying the supersymmetry algebra (7) can be explicitly constructed. In fact there are more than one set of a Q and a $\bar{Q}$ which would satisfy (7). At first we shall use the following explicit form of Q and $\bar{Q}$:

$$\begin{aligned}
Q_\alpha &= \partial_\alpha \\
\bar{Q}_{\dot{\alpha}} &= -\bar{\partial}_{\dot{\alpha}} - 2i\theta^\beta\sigma^\mu{}_{\beta\dot{\alpha}}\partial_\mu.
\end{aligned} \tag{10}$$

Here new kinds of derivatives are introduced. These are defined as

$$\partial_\alpha = \frac{\partial}{\partial\theta^\alpha}, \quad \bar{\partial}_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}. \tag{11}$$

These objects also anti commute with all anti commuting objects in the theory. One can explicitly calculate that the Q and $\bar{Q}$ given in (10) satisfy the supersymmetry algebra (7). It is not hard to see that they obey the first three relations of the supersymmetry algebra (10). The fourth relationship can be verified in the following way.

$$\begin{aligned}
\{Q_\alpha, \bar{Q}_{\dot\beta}\} &= \partial_\alpha(-\bar\partial_{\dot\beta} - 2i\theta^\gamma \sigma^\mu{}_{\gamma\dot\beta}\partial_\mu) + (-\bar\partial_{\dot\beta} - 2i\theta^\gamma\sigma^\mu{}_{\gamma\dot\beta}\partial_\mu)\partial_\alpha \\
&= -2i\delta^\gamma_\alpha \sigma^\mu{}_{\gamma\dot\beta}\partial_\mu + 2i\theta^\gamma\sigma^\mu{}_{\gamma\dot\beta}\partial_\mu\partial_\alpha - 2i\theta^\gamma\sigma^\mu{}_{\gamma\dot\beta}\partial_\mu\partial_\alpha \\
&= -2i\sigma^\mu{}_{\alpha\dot\beta}\partial_\mu = 2\sigma^\mu{}_{\alpha\dot\beta}P_\mu
\end{aligned} \tag{12}$$

In the second step in this calculation one should remind that we get a minus sign from pulling ∂_α through θ^γ.

Now we are ready to see the advantages of the superspace formalism. When we have a certain superfield we can let the supertransformation (1) operate on it. This gives us a new superfield. This transformation on the superfield induces a transformation on the component fields. Such a transformation on the physical fields, which only depend on the coordinates x^μ, is what we are actually looking for. The fact that the transformation on the superfield obeys the supersymmetry algebra ensures that also this induced transformation obeys the supersymmetry algebra. The advantage of the superspace formalism is that the supertransformation on the superfields is algebraically much simpler than the supertransformation on the physical component fields.

There are however more advantages of the superspace formalism. If we only look at a certain class of superfields given by a certain constraint and if this class is an invariant subspace under the supertransformation we can say the following. The physical component of the term of highest order in θ and $\bar\theta$ is suited to be included in a supersymmetric Lagrangian L. That this is indeed true can be seen from the form of Q and $\bar{Q}$ in (10). After the supertransformation has operated the highest order component can only be changed by a term with a space time derivative ∂_μ in it. The terms in Q and $\bar{Q}$ in (10) with a derivative to θ or $\bar\theta$ in them can only diminish the order of θ or $\bar\theta$ and thus not give a highest order term after the operation. The change in the highest order term thus is a space time derivative. So if the physical component of this highest order term is included in the Lagrangian density $\mathcal{L}$ the change under the supertransformation vanishes in the complete Lagrangian L. Thus to find terms suited for our desired supersymmetric Lagrangian we must search for invariant subsuperspaces and find the terms of highest order in θ and $\bar\theta$ in these subsuperspaces.

2.3 Scalar superfields

We will now proceed to do this. As said we must find constraints that give a subsuperspace invariant under the action of the supertransformation. For this to be true the constraint must be invariant and if we write the constraint as

$$OX = 0, \tag{13}$$

where O is an operator and X is a superfield, the operator O must commute with the supertransformation. It is not very hard to see that the following operator commutes with our supertransformation.

$$\bar{D}_{\dot\alpha} = -\bar\partial_{\dot\alpha} \tag{14}$$

$$\begin{aligned}
\bar{D}_{\dot\alpha}(\varepsilon^\beta Q_\beta + \bar{Q}_{\dot\beta}\bar\varepsilon^{\dot\beta}) &= -\bar\partial_{\dot\alpha}(\varepsilon^\beta\partial_\beta - \bar\partial_{\dot\beta}\bar\varepsilon^{\dot\beta} - 2i\theta^\gamma\sigma^\mu{}_{\gamma\dot\beta}\partial_\mu\bar\varepsilon^{\dot\beta}) \\
&= \varepsilon^\beta\partial_\beta(-\bar\partial_{\dot\alpha}) - \bar\partial_{\dot\beta}\bar\varepsilon^{\dot\beta}(-\bar\partial_{\dot\alpha}) - 2i\theta^\gamma\sigma^\mu{}_{\gamma\dot\beta}\partial_\mu\bar\varepsilon^{\dot\beta}(-\bar\partial_{\dot\alpha}) \\
&= (\varepsilon^\beta Q_\beta + \bar{Q}_{\dot\beta}\bar\varepsilon^{\dot\beta})\bar{D}_{\dot\alpha}
\end{aligned} \tag{15}$$

The subspace of superfields X obeying

$$\bar{D}_{\dot\alpha}X = 0 \tag{16}$$

is also easy to construct. A superfield in this subspace always has the form

$$\phi(x,\theta,\bar{\theta}) = \varphi(x) + \sqrt{2}\theta\psi(x) + \theta^2 F(x) \tag{17}$$

since such a superfield should simply have no dependence on $\bar{\theta}$. The $\sqrt{2}$ in front of the term $\theta\psi(x)$ is conventional. A superfield like (17) is called a scalar superfield. We can now show how the supertransformation operates on this superfield and how it induces the supertransformation for the component fields.

$$\begin{aligned}
\delta_\varepsilon\phi &= (\varepsilon^\alpha\partial_\alpha - \bar{\partial}_{\dot{\alpha}}\bar{\varepsilon}^{\dot{\alpha}} - 2i\theta^\gamma\sigma^\mu_{\ \gamma\dot{\alpha}}\partial_\mu\bar{\varepsilon}^{\dot{\alpha}})(\varphi(x) + \sqrt{2}\theta\psi(x) + \theta^2F(x)) \\
&= -2i\theta^\gamma\sigma^\mu_{\ \gamma\dot{\alpha}}\bar{\varepsilon}^{\dot{\alpha}}\partial_\mu\varphi(x) + \sqrt{2}\varepsilon\psi(x) - 2\sqrt{2}i\theta^\gamma\sigma^\mu_{\ \gamma\dot{\alpha}}\bar{\varepsilon}^{\dot{\alpha}}\theta\partial_\mu\psi(x) + 2\varepsilon\theta F(x) \\
&= -2i\theta\sigma^\mu\bar{\varepsilon}\partial_\mu\varphi(x) + \sqrt{2}\varepsilon\psi(x) + \sqrt{2}i\sigma^{\mu\beta}_{\ \ \ \dot{\alpha}}\theta^2\bar{\varepsilon}^{\dot{\alpha}}\partial_\mu\psi_\beta(x) + 2\varepsilon\theta F(x) \\
&= -2i\theta\sigma^\mu\bar{\varepsilon}\partial_\mu\varphi(x) + \sqrt{2}\varepsilon\psi(x) + i\sqrt{2}\theta^2\partial_\mu\psi(x)\sigma^\mu\bar{\varepsilon} + 2\varepsilon\theta F(x)
\end{aligned} \tag{18}$$

In the third step we used the identity

$$\theta^\gamma\sigma^\mu_{\ \gamma\dot{\alpha}}\theta^\beta = \frac{1}{2}\sigma^{\mu\beta}_{\ \ \ \dot{\alpha}}\theta^2. \tag{19}$$

From this expression we can read off the action of the supertransformation on the physical component fields.

$$\begin{aligned}
\delta_\varepsilon\varphi &= \sqrt{2}\varepsilon\psi(x) \\
\delta_\varepsilon\psi &= \sqrt{2}\varepsilon F(x) - \sqrt{2}i\sigma^\mu\bar{\varepsilon}\partial_\mu\varphi(x) \\
\delta_\varepsilon F &= i\sqrt{2}\partial_\mu\psi(x)\sigma^\mu\bar{\varepsilon}
\end{aligned} \tag{20}$$

That one should be very careful with minus signs can be seen in the calculation of $\varepsilon^\alpha\partial_\alpha\theta^2$ which also has to be calculated in the above calculation.

$$\begin{aligned}
\varepsilon^\alpha\partial_\alpha\theta^2 &= \varepsilon^\alpha\frac{\partial}{\partial\theta^\alpha}\theta^\beta\theta_\beta = \varepsilon^\alpha\delta^\beta_\alpha\theta_\beta - \varepsilon^\alpha\theta^\beta\frac{\partial}{\partial\theta^\alpha}\theta_\beta = \varepsilon\theta - \varepsilon^\alpha\theta^\beta\epsilon_{\delta\beta}\frac{\partial}{\partial\theta^\alpha}\theta^\delta = \\
&\quad \varepsilon\theta - \varepsilon^\alpha\theta^\beta\epsilon_{\delta\beta}\delta^\delta_\alpha = \varepsilon\theta + \varepsilon^\alpha\theta^\beta\epsilon_{\beta\alpha} = 2\varepsilon\theta
\end{aligned} \tag{21}$$

We see that the supertransformation indeed transforms fermions into bosons and bosons into fermions.

We should now proceed to find terms for our Lagrangian density. As we deduced earlier the coefficient of θ^2 in the chiral superfield is a candidate for appearing in the Lagrangian density. The question arises however what the field F is in terms of fundamental fields. Let's begin with a superfield of the form (17) where the field ψ is the fundamental field for the lefthanded part of the electron, which we call e_L. We know that the field φ is the field into which e_L transforms and that also φ transforms into e_L. This field φ thus is the superpartner of the lefthanded electron, called the "left" selectron, as a field denoted by $\tilde{e}_L$. The term "left" should not be considered as indicating a chirality as in the case with the electron, since a scalar field cannot have a chirality, it merely indicates that $\tilde{e}_L$ is the superpartner of the lefthanded electron. The field F in the superfield (17) we will call F_L where also in this case the L denotes that F_L is a field belonging to the supermultiplet containing the lefthanded electron. Later on it will appear that F_L is a non-dynamical field and can be eliminated from the Lagrangian. The superfield with all these component fields we will call ϕ_L. Now we can tell whether the component F, or in this case F_L, is suited for our Lagrangian density. This Lagrangian density must be Lorentz invariant and superinvariant, which are both by construction true. It must however also be gauge invariant if we really want to build SQED. We know that e_L transforms as follows under a gauge transformation.

$$e_L(x) \to e^{ie\Lambda(x)}e_L(x) \tag{22}$$

Here the function Λ is of course real. This suggests that we might define the gauge transformation on the superfield ϕ_L as:

$$\phi_L(x,\theta,\bar{\theta}) \to e^{ie\Lambda(x)}\phi_L(x,\theta,\bar{\theta}) \tag{23}$$

Then we see that also $\tilde{e}_L$ and F_L transform as (22) under a gauge transformation. So we see that if we take $F_L(x)$ as a term in our Lagrangian density this Lagrangian density will be Lorentz invariant, supersymmetric, but not gauge invariant. It would also not be Hermitian, this would however easily be solved by including a term $F_L^\dagger(x)$.

To construct a term which is also gauge invariant it turns out we need another superfield of the form (17), this time containing the righthanded part of the electron. For the field ψ in this new superfield we will take e_R. Notice that this is not equal to the righthanded part of the electron, which is $\bar{e}_R$ since it has to transform like a righthanded spinor, but it is the complex conjugate of the righthanded part. For this reason e_R also has charge $+e$ and not $-e$ like e_L. So under a gauge transformation e_R transforms like:

$$e_R(x) \to e^{-ie\Lambda(x)} e_R(x). \tag{24}$$

The new superfield containing this righthanded part of the electron we shall call ϕ_R, the components other than e_R we will name $\tilde{e}_R^\dagger$ and $F_R^\dagger$. Here we have included the daggers in the definitions so that the fields $\tilde{e}_R$ and F_R have charge $-e$. Thus also the fields $\tilde{e}_R^\dagger$ and $F_R^\dagger$ transform like (24). If we now multiply the superfield ϕ_R with ϕ_L and take the highest order component in θ we see that we have a term for the Lagrangian density $\mathcal{L}$ which is Lorentz invariant, superinvariant and gauge invariant. To make $\mathcal{L}$ Hermitian we should also include the Hermitian conjugated of the term just mentioned. So we get the following term suited for $\mathcal{L}$:

$$\begin{aligned} m\big(\phi_R\phi_L|_{\theta^2} + \phi_R^\dagger\phi_L^\dagger|_{\bar\theta^2}\big) &= m\tilde{e}_R^\dagger F_L + m\tilde{e}_L F_R^\dagger - m e_R e_L + \\ &\quad m\tilde{e}_R F_L^\dagger + m\tilde{e}_L^\dagger F_R - m\bar{e}_R\bar{e}_L. \end{aligned} \tag{25}$$

For the part with the spinors we used the identity:

$$\begin{aligned} 2\theta e_R \theta e_L &= -2\theta^\alpha\theta^\beta e_{R\alpha} e_{L\beta} = -2(\theta^0\theta^1 e_{R0}e_{L1} + \theta^1\theta^0 e_{R1}e_{L0}) \\ &= -2\theta^0\theta^1(e_{R0}e_{L1} - e_{R1}e_{L0}) = -\theta^2 e_R e_L. \end{aligned} \tag{26}$$

Here m is just a constant, later it will appear to be the mass of the electron, the left and right selectron.

2.4 Vector superfields

Before we can find more terms suited for the SQED Lagrangian density we have to introduce another kind of superfield. Also this superfield obeys a certain constraint. For this new kind of superfield it is however convenient to introduce a different realization of the supersymmetry algebra (7). It appears that the following Q and $\bar{Q}$ also satisfy this algebra.

$$\begin{aligned} Q_\alpha &= \partial_\alpha + i\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu \\ \bar{Q}_{\dot\alpha} &= -\bar\partial_{\dot\alpha} - i\theta^\beta\sigma^\mu_{\beta\dot\alpha}\partial_\mu \end{aligned} \tag{27}$$

For this realization of the supersymmetry algebra the same story as for the previous realization holds. So we need an invariant constraint giving a subspace of fields and the highest order components may be suited for the Lagrangian density. The new kind of subsuperspace is now given by the constraint:

$$X^\dagger = X \tag{28}$$

where X is any superfield in the subspace. This constraint is indeed invariant under the supertransformation. Would we have kept working with the old realization (10) of the supersymmetry algebra a suitable constraint would not have looked like (28) but we would need a constraint which is much more complicated algebraically. A general superfield obeying (28) looks like:

$$V(x,\theta,\bar\theta) = -\theta\sigma_\mu\bar\theta V^\mu(x) + i\theta^2\bar\theta\bar\lambda(x) - i\bar\theta^2\theta\lambda(x) + \frac{1}{2}\theta^2\bar\theta^2 D(x) + \ldots. \tag{29}$$

Such a superfield is called a vector superfield. The dots represent higher order terms which are unimportant. Why this is will be clarified in a moment. All component fields in (29) should be real to satisfy (28). Also in this case we have to say how the component fields are related to the fundamental fields and how a gauge transformation operates on our vector superfield. Let's first take a superfield of the form (29) where the field V^μ is the photon field A^μ. When we let the supertransformation operate on (29) it appears that V^μ transforms into an expression with λ. Thus the particle belonging to the field λ is the superpartner of the photon, called the photino. We shall henceforth call this field $\tilde{A}$. The field D will turn out to be a non dynamical field when we have constructed the complete Lagrangian density and this field can later be eliminated from $\mathcal{L}$. Under a gauge transformation A^μ changes as:

$$A^\mu(x) \to A^\mu(x) - \partial^\mu \Lambda(x). \tag{30}$$

The gauge transformation for the complete vector superfield is much more difficult to find. It appears that our superfield V transforms as:

$$V(x,\theta,\bar{\theta}) \to V(x,\theta,\bar{\theta}) + \phi(x,\theta,\bar{\theta}) + \phi^\dagger(x,\theta,\bar{\theta}) \tag{31}$$

under a gauge transformation. Here ϕ and $\phi^\dagger$ are scalar superfields which we encountered earlier. They don't look like (17) anymore since we now work in a different realization of the supersymmetry algebra. The sum $\phi + \phi^\dagger$ is given by:

$$\begin{aligned}\phi + \phi^\dagger \quad = \quad & \varphi + \varphi^* + \sqrt{2}(\theta\psi + \bar{\theta}\bar{\psi}) + \theta^2 F + \bar{\theta}^2 F^\dagger + i\theta\sigma^\mu\bar{\theta}\partial_\mu(\varphi - \varphi^*) + \\ & \frac{i}{\sqrt{2}}\theta^2\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi + \frac{i}{\sqrt{2}}\bar{\theta}^2\theta\sigma^\mu\partial_\mu\bar{\psi} + \frac{1}{4}\theta^2\bar{\theta}^2\Box(\varphi - \varphi^*).\end{aligned} \tag{32}$$

In this expression we recognize the familiar gauge transformation of the photon field A^μ where Λ and φ are related as:

$$\Lambda(x) = i(\varphi(x) - \varphi^*(x)) \tag{33}$$

Now we can also clarify why the terms represented by the dots in (29) are unimportant. There is a special gauge that we can take where all these terms are zero. This gauge however does not fix the photon field, photino field and D-field. It also leaves room for the ordinary gauge transformation of the photon field. This gauge is called the Wess Zumino gauge.

Before we can continue finding more terms for our Lagrangian we should remark something else. Since we are now working with a different realization of the supersymmetry algebra the scalar superfields we saw earlier also change. Their structure becomes more complicated and their transformation under a gauge transformation is no longer given by:

$$\phi_L \to e^{ie\Lambda(x)}\phi_L. \tag{34}$$

This gauge transformation is now given by:

$$\phi_L \to e^{e\phi(x,\theta,\bar{\theta})}\phi_L. \tag{35}$$

Of course the e_L-component of ϕ_L still transforms in the ordinary way (22). The gauge transformation of ϕ_R is now given by:

$$\phi_R \to e^{-e\phi(x,\theta,\bar{\theta})}\phi_R. \tag{36}$$

Now we can continue finding terms for our Lagrangian. With the new vector superfield and with the other superfields ϕ_L and ϕ_R we can now build two superfields which satisfy (28) and which are gauge invariant. These superfields are:

$$\phi_L^\dagger e^{-eV}\phi_L, \quad \phi_R^\dagger e^{eV}\phi_R. \tag{37}$$

Here the exponential should be understood as its power series in θ and $\bar{\theta}$, thus the exponentials in the above expression only have a finite number of terms. So we have two more terms suited for our SQED Lagrangian density:

$$\begin{aligned}
&\phi_L^\dagger e^{-eV}\phi_L|_{\theta^2\bar\theta^2} + \phi_R^\dagger e^{eV}\phi_R|_{\theta^2\bar\theta^2} = \\
&F_L^\dagger F_L - \tilde e_L^\dagger \Box \tilde e_L - i\partial_\mu \bar e_L \bar\sigma^\mu e_L - eA^\mu \bar e_L \bar\sigma_\mu e_L + ieA^\mu \tilde e_L^\dagger \partial_\mu \tilde e_L - \\
&ieA^\mu \partial_\mu \tilde e_L^\dagger \tilde e_L + \sqrt{2} ie \tilde e_L \bar e_L \bar{\tilde A} - \sqrt{2} ie \tilde e_L^\dagger e_L \tilde A - eD\tilde e_L^\dagger \tilde e_L + e^2 A^2 \tilde e_L^\dagger \tilde e_L + \\
&F_R^\dagger F_R - \tilde e_R^\dagger \Box \tilde e_R - i\partial_\mu \bar e_R \bar\sigma^\mu e_R + eA^\mu \bar e_R \bar\sigma_\mu e_R + ieA^\mu \tilde e_R^\dagger \partial_\mu \tilde e_R - \\
&ieA^\mu \partial_\mu \tilde e_R^\dagger \tilde e_R - \sqrt{2} ie \tilde e_R^\dagger \bar e_R \bar{\tilde A} + \sqrt{2} ie \tilde e_R e_R \tilde A + eD\tilde e_R^\dagger \tilde e_R + e^2 A^2 \tilde e_R^\dagger \tilde e_R.
\end{aligned} \tag{38}$$

It appears we need two more terms to complete the construction of our SQED Lagrangian density. The first one of these terms satisfies the constraint:

$$(-\bar\partial_{\dot\alpha} + i\theta^\beta \sigma^\mu_{\ \beta\dot\alpha}\partial_\mu)X = \bar D_{\dot\alpha} X = 0. \tag{39}$$

This constraint is the analog of (16) but with a different $\bar D_{\dot\alpha}$. This because we are now working with a different realization of the supersymmetry algebra. This new constraint is also invariant under the new supertransformation, like the old constraint (16) was invariant under the old supertransformation. A superfield satisfying (39) is given by:

$$W_\alpha = -\frac{1}{4}\bar D \bar D D_\alpha V \tag{40}$$

where D is defined as

$$D_\alpha = \partial_\alpha - i\sigma^\mu_{\ \alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu. \tag{41}$$

The expression W_α is also gauge invariant which can be verified by an explicit calculation. It is however not Lorentz invariant. The product $W^\alpha W_\alpha$ is Lorentz invariant, so the highest order component of this superfield is suited for our Lagrangian density since it is supersymmetric, Lorentz invariant and gauge invariant. To make $\mathcal{L}$ Hermitian also the Hermitian conjugated of $W^\alpha W_\alpha$ has to be included. So again we have two more terms for our SQED Lagrangian density:

$$\frac{1}{4}(W^\alpha W_\alpha|_{\theta^2} + \bar W_{\dot\alpha}\bar W^{\dot\alpha}|_{\bar\theta^2}) = \frac{1}{2}D^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\tilde A \sigma^\mu \partial_\mu \bar{\tilde A} \tag{42}$$

where $F^{\mu\nu}$ is defined in the familiar way:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu. \tag{43}$$

It will appear that with the terms for $\mathcal{L}$ we have constructed thus far we have enough terms to construct the supersymmetric extension of QED. For a more detailed derivation of this SQED Lagrangian density we refer to an article from Bagger and Wess [4].

2.5 The Lagrangian density

At last we can write down the complete Lagrangian density $\mathcal{L}$.

$$\begin{aligned}
\mathcal{L} \ = \ & \frac{1}{2}D^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\tilde A\sigma^\mu\partial_\mu \bar{\tilde A} + \\
& F_L^\dagger F_L - \tilde e_L^\dagger \Box \tilde e_L + F_R^\dagger F_R - \tilde e_R^\dagger \Box \tilde e_R + \\
& -i\partial_\mu \bar e_L \bar\sigma^\mu e_L - i\partial_\mu \bar e_R \bar\sigma^\mu e_R + \\
& -eA^\mu \bar e_L \bar\sigma_\mu e_L + eA^\mu \bar e_R \bar\sigma_\mu e_R + \\
& ieA^\mu \tilde e_L^\dagger \partial_\mu \tilde e_L - ieA^\mu \partial_\mu \tilde e_L^\dagger \tilde e_L + ieA^\mu \tilde e_R^\dagger \partial_\mu \tilde e_R - ieA^\mu \partial_\mu \tilde e_R^\dagger \tilde e_R + \\
& \sqrt{2} ie \tilde e_L \bar e_L \bar{\tilde A} - \sqrt{2} ie \tilde e_L^\dagger e_L \tilde A - \sqrt{2} ie \tilde e_R^\dagger \bar e_R \bar{\tilde A} + \sqrt{2} ie \tilde e_R e_R \tilde A + \\
& -eD\tilde e_L^\dagger \tilde e_L + eD \tilde e_R^\dagger \tilde e_R + e^2 A^2 \tilde e_L^\dagger \tilde e_L + e^2 A^2 \tilde e_R^\dagger \tilde e_R + \\
& m\tilde e_R^\dagger F_L + m\tilde e_L F_R^\dagger + m\tilde e_R F_L^\dagger + m\tilde e_L^\dagger F_R - me_R e_L - m\bar e_R \bar e_L
\end{aligned} \tag{44}$$

Now we can also see that the fields F_L, F_R and D are not dynamical, since no space time derivatives of these fields occur in $\mathcal{L}$. With help of the Euler-Lagrange equations for a field X,

$$\frac{d}{dt}\frac{\delta\mathcal{L}}{\delta\dot{X}} - \frac{\delta\mathcal{L}}{\delta X} = 0, \tag{45}$$

these auxiliary fields can be eliminated from the Lagrangian density. When we use the Euler-Lagrange equations for F_L and F_R we get

$$\begin{aligned} F_L^\dagger + m\tilde{e}_R^\dagger = 0 \quad &\Rightarrow \quad F_L = -m\tilde{e}_R \\ F_R^\dagger + m\tilde{e}_L^\dagger = 0 \quad &\Rightarrow \quad F_R = -m\tilde{e}_L. \end{aligned} \tag{46}$$

When we use the Euler-Lagrange equations for D we get

$$D - e\tilde{e}_L^\dagger\tilde{e}_L + e\tilde{e}_R^\dagger\tilde{e}_R = 0 \Rightarrow D = e\tilde{e}_L^\dagger\tilde{e}_L - e\tilde{e}_R^\dagger\tilde{e}_R. \tag{47}$$

Now we can write down the final form of the SQED Lagrangian density. We substitute the above expressions for F_L, F_R and D in (44). Also we write the terms with left- or righthanded electron fields in a more familiar fashion, using the four-component spinor ψ_e, which, in terms of the two-component spinors of the electron, is given by

$$\psi_e = \begin{pmatrix} e_L \\ \bar{e}_R \end{pmatrix}. \tag{48}$$

The same thing we do for the photino, here we introduce the following four-component spinor:

$$\psi_{\tilde{\gamma}} = \begin{pmatrix} \tilde{A} \\ \bar{\tilde{A}} \end{pmatrix}. \tag{49}$$

Here we see the difference between the electron and the photino. The electron has four independent complex components, whereas the photino only has two, the last two components of the photino are related to the first two. A fermion like the photino is called a Majorana fermion. Such a fermion also has special Feynman rules, which we will encounter below. We also use the following relations:

$$\begin{aligned} \bar{\psi}_e &= \psi_e^\dagger\gamma^0 = (e_R, \bar{e}_L), \\ \gamma^0 &= \begin{pmatrix} \emptyset & \mathbf{I} \\ \mathbf{I} & \emptyset \end{pmatrix}, \\ \gamma^\mu &= \begin{pmatrix} \emptyset & \sigma^\mu \\ \bar{\sigma}^\mu & \emptyset \end{pmatrix}, \\ \gamma^5 &= \begin{pmatrix} \mathbf{I} & \emptyset \\ \emptyset & -\mathbf{I} \end{pmatrix}, \\ P_L\begin{pmatrix} e_L \\ \bar{e}_R \end{pmatrix} &= \frac{1}{2}(1+\gamma^5)\begin{pmatrix} e_L \\ \bar{e}_R \end{pmatrix} = \begin{pmatrix} e_L \\ 0 \end{pmatrix}, \\ P_R\begin{pmatrix} e_L \\ \bar{e}_R \end{pmatrix} &= \frac{1}{2}(1-\gamma^5)\begin{pmatrix} e_L \\ \bar{e}_R \end{pmatrix} = \begin{pmatrix} 0 \\ \bar{e}_R \end{pmatrix}. \end{aligned} \tag{50}$$

The final Lagrangian density $\mathcal{L}$ is now:

$$\mathcal{L} = \mathcal{L}_K + \mathcal{L}_I. \tag{51}$$

$$\begin{aligned} \mathcal{L}_K = & -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}i\bar{\psi}_{\tilde{\gamma}}\gamma^\mu\partial_\mu\psi_{\tilde{\gamma}} - \tilde{e}_L^\dagger(\Box + m^2)\tilde{e}_L - \tilde{e}_R^\dagger(\Box + m^2)\tilde{e}_R + \\ & i\bar{\psi}_e\gamma^\mu\partial_\mu\psi_e - m\bar{\psi}_e\psi_e \end{aligned}$$

$$
\begin{aligned}
\mathcal{L}_I &= -eA^\mu \bar{\psi}_e \gamma_\mu \psi_e + \\
&ieA^\mu(\tilde{e}_L^\dagger \partial_\mu \tilde{e}_L - \partial_\mu \tilde{e}_L^\dagger \tilde{e}_L) + ieA^\mu(\tilde{e}_R^\dagger \partial_\mu \tilde{e}_R - \partial_\mu \tilde{e}_R^\dagger \tilde{e}_R) + \\
&e^2 A^2 \tilde{e}_L^\dagger \tilde{e}_L + e^2 A^2 \tilde{e}_R^\dagger \tilde{e}_R + \\
&\frac{1}{\sqrt{2}} ie\tilde{e}_L \bar{\psi}_e (1-\gamma^5)\psi_{\tilde{\gamma}} - \frac{1}{\sqrt{2}} ie\tilde{e}_L^\dagger \bar{\psi}_{\tilde{\gamma}} (1+\gamma^5)\psi_e + \\
&\frac{1}{\sqrt{2}} ie\tilde{e}_R \bar{\psi}_e (1+\gamma^5)\psi_{\tilde{\gamma}} - \frac{1}{\sqrt{2}} ie\tilde{e}_R^\dagger \bar{\psi}_{\tilde{\gamma}} (1-\gamma^5)\psi_e + \\
&-\frac{1}{2} e^2 \tilde{e}_L^\dagger \tilde{e}_L \tilde{e}_L^\dagger \tilde{e}_L - \frac{1}{2} e^2 \tilde{e}_R^\dagger \tilde{e}_R \tilde{e}_R^\dagger \tilde{e}_R + e^2 \tilde{e}_L^\dagger \tilde{e}_L \tilde{e}_R^\dagger \tilde{e}_R
\end{aligned} \tag{52}
$$

2.6 Soft breaking terms

From the Lagrangian density above we see that the electron and both the selectrons have the same mass m. The same holds for the photon and the photino, they both have mass zero. This cannot be true in reality of course, since then also the selectrons and the photinos would have been detected long ago. If we want SQED to have anything to do with reality these masses can't stay this way.

It appears there is a way to maintain the great advantage of supersymmetric theories, namely the absence of quadratic divergences, and at the same time break supersymmetry so that the masses of the selectrons and photino can, in theory, be very large. In this way one can account for the fact that these superparticles haven't been seen yet. The extra terms we can introduce for our theory are:

$$
m_{ij}\varphi_i\varphi_j^\dagger, \quad \alpha_{ij}\varphi_i\varphi_j + \text{H.c.}, \quad \beta_{ijk}\varphi_i\varphi_j\varphi_k + \text{H.c.}, \quad \mu(\lambda\lambda + \bar{\lambda}\bar{\lambda}) \tag{53}
$$

where the fields φ are scalar fields (normal scalar fields, not superfields) and the field λ is a (two-component) gaugino, the superpartner of a gauge boson. Note that when we add these terms we do have to see to it that we still have gauge invariance. Because of this the only possible terms we can add in our theory are:

$$
m_{LL}\tilde{e}_L\tilde{e}_L^\dagger, \quad m_{LR}\tilde{e}_L\tilde{e}_R^\dagger, \quad m_{RL}\tilde{e}_R\tilde{e}_L^\dagger, \quad m_{RR}\tilde{e}_R\tilde{e}_R^\dagger, \quad \mu\bar{\psi}_{\tilde{\gamma}}\psi_{\tilde{\gamma}}. \tag{54}
$$

The other terms are simply not gauge invariant. To get an Hermitian Lagrangian density m_{LR} should equal m_{RL}. The term with the $\psi_{\tilde{\gamma}}$ simply is a mass term for the photino, which is exactly what we needed. The terms with the m_{LL} and the m_{RR} are mass terms for both selectrons, which we also needed to be able to let these masses become large. The terms with the m_{LR} and m_{RL} are mixing terms which make that the fields $\tilde{e}_L$ and $\tilde{e}_R$ are no longer mass eigenstates. If we would also introduce these additional terms we would have to diagonalize the mass matrix of the selectrons and also the interactions for the actual particles (mass eigenstates) will become much more complicated. Because we want to look at the minimal version of softly broken SQED we will therefore not include these terms. So the extra soft breaking terms we introduce are:

$$
\mathcal{L}_{S.B.} = m_{LL}\tilde{e}_L\tilde{e}_L^\dagger + m_{RR}\tilde{e}_R\tilde{e}_R^\dagger + \mu\bar{\psi}_{\tilde{\gamma}}\psi_{\tilde{\gamma}}. \tag{55}
$$

All these terms only contribute to the masses of the superparticles. These total masses we will henceforth name m_L for the left selectron, m_R for the right selectron and $m_{\tilde{\gamma}}$ for the photino.

2.7 The Feynman rules

From the Lagrangian density we have constructed we can extract the Feynman rules in the familiar way. These Feynman rules are listed in figures (1), (2) and (3). We will denote electron lines as solid lines with arrows on them, selectron lines as solid lines with a label L or R denoting which selectron is represented, photon lines as wiggly lines and photino lines as a solid line with a wiggly line. Something should be said here about the Feynman rules for the photino. Because of the self-conjugacy of the photino, which is therefore called a Majorana fermion, the multiplication order in a certain Feynman diagram is not unambiguous, as for example in the case of pure QED. Also fermion number violation is possible so that one cannot unambiguously fix a fermion flow and define a multiplication order.

In an article by Denner, Eck, Hahn and Küblbeck [5] this problem is treated. With their results one can treat Feynman diagrams containing Majorana fermions quite easily. According to their results one can treat these diagrams as follows. Electron lines should carry arrows, as usual. Photino lines should not carry arrows. In every fermion chain one should choose an arbitrary orientation for the fermion flow, which will be denoted by an arrow next to the fermion line. Multiplication should now be performed opposite to this chosen fermion flow, as usual. The expressions for all propagators are the same ones as usual, one should however count the momentum occurring in propagators along the chosen fermion flow (and not necessarily along the arrow of the electron line). The vertices with fermions flowing through them should be treated in the following way. If one encounters a vertex with one (or two) electrons where the arrow of the electron points along the chosen fermion flow one should use the usual vertex expression, which we denote here as $i\Gamma$. If, on the other hand, the arrow of the electron points opposite to the fermion flow one should choose the following expression for the vertex:

$$i\Gamma' = iC\Gamma^T C^{-1}. \tag{56}$$

Here T denotes transposition and C is the charge-conjugation matrix. The expression on the right hand side is equal to:

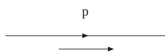
$$iC\Gamma^T C^{-1} = i\eta\Gamma \tag{57}$$

where η is given by:

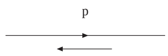
$$\eta = \begin{cases} 1 \quad \text{for} \quad \Gamma = \mathbf{I}, i\gamma^5, \gamma^\mu\gamma^5 \\ -1 \quad \text{for} \quad \Gamma = \gamma^\mu, \sigma^{\mu\nu}. \end{cases} \tag{58}$$

Fermi minus signs and combinatorial factors are handled in the ordinary way.

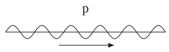
Figure 1: The Feynman rules for the propagators in minimal softly broken SQED. The indicated momenta flow from left to right. The arrows next to the fermion lines represent the chosen fermion flow.



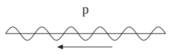
$\leftrightarrow \quad \frac{i(\not p+m_e)}{p^2-m_e^2+i\varepsilon}$



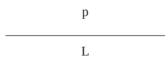
$\leftrightarrow \quad \frac{i(-\not p+m_e)}{p^2-m_e^2+i\varepsilon}$



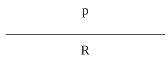
$\leftrightarrow \quad \frac{i(\not p+m_{\tilde\gamma})}{p^2-m_{\tilde\gamma}^2+i\varepsilon}$



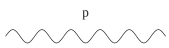
$\leftrightarrow \quad \frac{i(-\not p+m_{\tilde\gamma})}{p^2-m_{\tilde\gamma}^2+i\varepsilon}$



$\leftrightarrow \quad \frac{i}{p^2-m_L^2+i\varepsilon}$

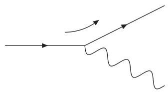


$\leftrightarrow \quad \frac{i}{p^2-m_R^2+i\varepsilon}$

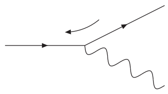


$\leftrightarrow \quad \frac{-ig_{\mu\nu}}{p^2+i\varepsilon}$

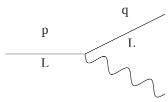
Figure 2: The Feynman rules for the 3-vertices in minimal softly broken SQED. The indicated momenta flow from left to right. The arrows next to the fermion lines represent the chosen fermion flow. Notice that the electron-selectron-photino vertices where the fermion flow is opposite to the electron arrow are not explicitly given, these vertices have the same expression as their counterparts where the fermion flow is along the electron arrow.



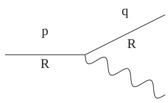
$\leftrightarrow \quad -ie\gamma^\mu$



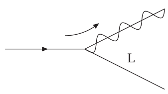
$\leftrightarrow \quad ie\gamma^\mu$



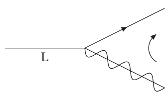
$\leftrightarrow \quad -ie(p+q)^\mu$



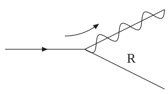
$\leftrightarrow \quad -ie(p+q)^\mu$



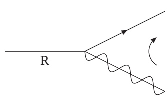
$\leftrightarrow \quad \frac{e}{\sqrt{2}}(\mathbf{I}+\gamma^5)$



$\leftrightarrow \quad -\frac{e}{\sqrt{2}}(\mathbf{I}-\gamma^5)$

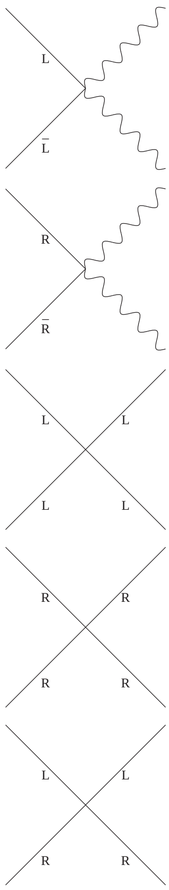


$\leftrightarrow \quad \frac{e}{\sqrt{2}}(\mathbf{I}-\gamma^5)$



$\leftrightarrow \quad -\frac{e}{\sqrt{2}}(\mathbf{I}+\gamma^5)$

Figure 3: The Feynman rules for the 4-vertices in minimal softly broken SQED



$$\leftrightarrow \quad 2ie^2 g^{\mu\nu}$$

$$\leftrightarrow \quad 2ie^2 g^{\mu\nu}$$

$$\leftrightarrow \quad -2ie^2$$

$$\leftrightarrow \quad -2ie^2$$

$$\leftrightarrow \quad ie^2$$

3 A different derivation

3.1 Introduction

In this section we will give a different derivation of minimal softly broken supersymmetric QED (SQED). This is the main topic of this thesis. In this different derivation we will not start by imposing supersymmetry on a theory. So we will have nothing to do with the supersymmetry algebra introduced in the previous section. What we will do is start with a theory having the same particle content as SQED. Then we impose on this theory that it has all the good things minimal softly broken SQED also has. With "good things" we mean things like unitarity, gauge invariance and absence of quadratic divergences. By imposing constraints like these we will try to fix all the couplings between the particles in the theory. We can then also see whether or not we are automatically led to the couplings of minimal softly broken SQED. The advantage of this approach is that we fix the theory with constraints that are much more physical than the constraint of supersymmetry. This latter constraint is mostly introduced in an ad hoc fashion, one can only see the advantages of it later on when the theory is constructed.

3.2 The 3-vertices

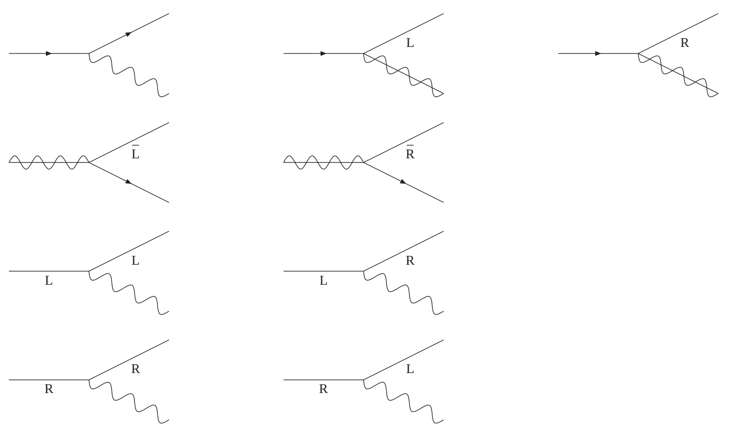
In the previous section we saw that we had the following particles in SQED:

$$
\begin{array}{ll}
e & \tilde{e}_L, \tilde{e}_R \\
\gamma & \tilde{\gamma}.
\end{array}
\tag{59}
$$

Here we have grouped the particles in such a way that the supersymmetric pairing between the particles can be seen.

The particle content (59) is the basis of the different derivation we want to present. This particle content and the identity of the particles is what we shall assume in the rest of this derivation. This last remark means that we shall assume that the electron e is a fermion with spin $\frac{1}{2}$, mass m_e and charge $-e$, that the left selectron $\tilde{e}_L$ is a boson with spin 0, mass m_L and charge $-e$, that the right selectron $\tilde{e}_R$ is a boson with spin 0, mass m_R and charge $-e$, that the photon γ is a boson with spin 1, mass 0 and charge 0 and finally that the photino $\tilde{\gamma}$ is a Majorana fermion with spin $\frac{1}{2}$, mass $m_{\tilde{\gamma}}$ and charge 0. So for the photino the special Feynman rules from Denner, Eck, Hahn and Küblbeck [5] as mentioned in the previous section have to be used.

We can now wonder what couplings exist between these 5 particles. We shall first draw all possible 3-vertices with these 5 particles:



All these 3-vertices can be found in a systematic way. The electron can only evolve into a particle with charge $-e$ plus a particle with charge 0. If we pick another electron for the particle with charge $-e$ then the particle with charge 0 must be a photon. If we pick the left or right selectron for the particle with charge $-e$ the particle with charge 0 must be the photino, else we would not have another spinor index to contract with the spinor index of the electron. This gives us the first 3 vertices.

The photino can only evolve into two uncharged particles or two charged ones, one with charge $-e$ and one with charge $+e$. When we consider the first possibility the two uncharged particles can only be a photino and a photon. As we know however a photon does not couple to an uncharged particle like the photino, making such a vertex impossible. In the second possibility the particle with charge $-e$ must be an electron, a left or a right selectron. In the case of the electron the particle with charge $+e$ must be a left or right anti-selectron. In the case with the left or right selectron the particle with charge $+e$ must be an anti-electron. Two of these four new vertices we have already encountered when we looked at what particles the electron could evolve to.

The left selectron can only evolve into two particles where one has charge $-e$ and one has charge 0. If we pick the electron for the one with charge $-e$ we only get vertices which we already had. If we pick the left or right selectron for the particle with charge $-e$ the uncharged particle must be a photon and we get the sixth and the seventh vertex of the vertices drawn above.

Considering what particles the right selectron can evolve into gives only the last two of the drawn vertices as new ones. It is easy to see that considering what particles the photon can evolve into will not give any new vertices.

Now we have constructed all at first sight possible 3-vertices. We can immediately discard two of them. These are the seventh and ninth vertex. These two vertices are strange since we know from experiment that the photon cannot change the identity of a particle. Of course we have never seen the left or right selectron in an experiment but they do couple to the observable electron. If the photon can change the identity of the selectrons and assuming the left and right selectron couple in a different way to the electron the photon can then also have influences on the electron which contradict experiment. On could argue however that such an effect can be made very small. There is however a better argument to discard these two vertices. We should remember that we are searching for a minimal theory having all the advantages minimal softly broken SQED has. So if we don't really need vertices to obtain the desired advantages we shouldn't build them into our theory. If we would there would also be no chance of getting to the interaction vertices we found in the conventional derivation of minimal softly broken SQED (since also there we kept things minimal), which is what we aim for. For now we shall thus discard them. If we would need them again later on (which will not be the case) we could always reintroduce them.

The other 7 3-vertices we will not discard. From later calculations it will appear that we really need all of them to obtain all desired advantages.

After discarding 2 vertices there are some more degrees of freedom which we can eliminate. It appears that the introduced 3-vertices are not all independent. This can be seen in the following way. The transition amplitude for a system to go from a state $|\text{in}\rangle$ at time $t = -\infty$ to a state $|\text{out}\rangle$ at time $t = \infty$ is given in the familiar way by:

$$\langle \text{in}|S|\text{out}\rangle. \tag{60}$$

The transition probability thus is:

$$|\langle \text{in}|S|\text{out}\rangle|^2 = \langle \text{in}|S|\text{out}\rangle\langle \text{out}|S^\dagger|\text{in}\rangle. \tag{61}$$

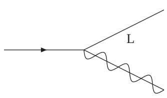
If we now "sum" over all possible out-states this probability should be one. The term "sum" is of course meant symbolically here, it might also involve integrations. This gives us:

$$\sum_{\text{out}} \langle \text{in}|S|\text{out}\rangle\langle \text{out}|S^\dagger|\text{in}\rangle = \langle \text{in}|SS^\dagger|\text{in}\rangle = 1. \tag{62}$$

This should be true for any in-state so that we can say:

$$SS^\dagger = S^\dagger S = \mathbf{I}. \tag{63}$$

Since the S-matrix must be unitary, the Hamiltonian H and with it the Lagrangian L should be Hermitian. Now consider the following vertex.



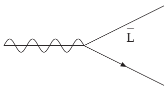
This vertex belongs to the following term in the Lagrangian density $\mathcal{L}$.

$$\bar{\psi}_{\tilde{\gamma}}(-i)\mathbf{B}_1\psi_e\tilde{e}_L^\dagger \tag{64}$$

Here we have called the vertex expression belonging to the vertex above $\mathbf{B}_1$. Because this term is present in $\mathcal{L}$ the term

$$\bar{\psi}_e i\mathbf{B}_1^\dagger\psi_{\tilde{\gamma}}\tilde{e}_L \tag{65}$$

should also be present to make $\mathcal{L}$ Hermitian. This term corresponds to the vertex



.

If we call the vertex expression belonging to the vertex above $\mathbf{B}_2$ these two vertices are related as

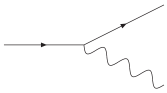
$$\mathbf{B}_2 = -\mathbf{B}_1^\dagger. \tag{66}$$

The same derivation can of course be given for the two vertices with the right selectron.

So now we are left with 5 independent 3-vertices which we have to fix. The fact that we need them all to obtain the desired advantages which minimal softly broken SQED also has is not clear yet. This will become clear when we proceed further and actually demand from our Feynman rules that certain things (like unitarity, gauge invariance and absence of quadratic divergences) are true.

3.3 The vertex expression for the electron photon interaction

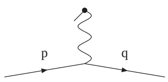
Now we have found all possible 3-vertices in our theory we can begin trying to find the expressions belonging to these vertices. This we will first do for the vertex

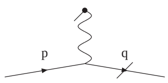
 .

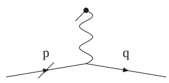
Of course we know already from experiments that this vertex should be:

$$-ie\gamma^{\mu}. \tag{67}$$

This is however also the only possibility if we insist that our theory is gauge invariant. This we will now show. From gauge invariance the Ward Takahashi relation can be derived. Written in Feynman diagram language this relation reads:



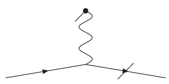
$=$


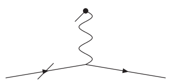
$-$


. (68)

Here the two electron legs should be considered internal and the photon leg should be considered as external but off-shell. Also the polarization vector of the photon is meant to be longitudinal, with the momentum counted inward, thus equal to $q-p$. This we indicated with the little handle at the end of the photon line. Furthermore the following extra Feynman rules are understood.

 $\leftrightarrow$ i

 $\leftrightarrow$ $-ie$

 $\leftrightarrow$ $-ie$ (69)

Let's now start with a general expression for our desired vertex and impose on it the relation (68). A general form for our vertex should have one Lorentz index and two Dirac indices. This leaves us with

$$E^{\mu} = S^{\mu}\mathbf{I} + V^{\mu}_{\alpha}\gamma^{\alpha} + T^{\mu}_{\alpha\beta}\sigma^{\alpha\beta} + A^{\mu}_{\alpha}\gamma^5\gamma^{\alpha} + P^{\mu}\gamma^5 \tag{70}$$

as the most general expression E^{μ} for our vertex. Here the tensor $\sigma^{\alpha\beta}$ is defined as

$$\sigma^{\alpha\beta} = \frac{i}{2}(\gamma^{\alpha}\gamma^{\beta} - \gamma^{\beta}\gamma^{\alpha}). \tag{71}$$

Note that all the objects in (70) which have Dirac indices are independent. Now we impose (68) on E^{μ}.

$$\frac{i(\not{q} + m_e)}{q^2 - m_e^2} E^{\mu}(q-p)_{\mu} \frac{i(\not{p} + m_e)}{p^2 - m_e^2} = e\frac{i(\not{p} + m_e)}{p^2 - m_e^2} - e\frac{i(\not{q} + m_e)}{q^2 - m_e^2} \tag{72}$$

First have a look at what the second term in the general expression (70) gives when we substitute it for E^{μ}. For V^{μ}_{α} we use the following general expression:

$$V^{\mu}_{\alpha} = Ag^{\mu}_{\alpha} + Bp^{\mu}p_{\alpha} + Cq^{\mu}q_{\alpha} + Dp^{\mu}q_{\alpha} + Eq^{\mu}p_{\alpha} + F\epsilon^{\mu}_{\alpha\nu\beta}p^{\nu}q^{\beta}. \tag{73}$$

Here the objects A, B, C, D, E and F are constants, since we don't want any form factors in our vertex expressions. First we only use the first term of V^μ_α and see what this term gives. So for the moment we substitute for E^μ:

$$E^\mu = A g^\mu_\alpha \gamma^\alpha = A\gamma^\mu. \tag{74}$$

This gives

$$\begin{aligned}
& \frac{i(\not{q}+m_e)}{q^2-m_e^2} E^\mu (q-p)_\mu \frac{i(\not{p}+m_e)}{p^2-m_e^2} \\
= \quad & A\frac{i(\not{q}+m_e)}{q^2-m_e^2}(\not{q}-\not{p})\frac{i(\not{p}+m_e)}{p^2-m_e^2} \\
= \quad & A\frac{i(\not{q}+m_e)}{q^2-m_e^2}((\not{q}-m_e)-(\not{p}-m_e))\frac{i(\not{p}+m_e)}{p^2-m_e^2} \\
= \quad & iA\frac{i(\not{p}+m_e)}{p^2-m_e^2} - iA\frac{i(\not{q}+m_e)}{q^2-m_e^2}.
\end{aligned} \tag{75}$$

We could choose $A = -ie$ to let this last expression be equal to the desired expression

$$e\frac{i(\not{p}+m_e)}{p^2-m_e^2} - e\frac{i(\not{q}+m_e)}{q^2-m_e^2}. \tag{76}$$

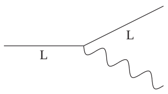
So a possible choice for our vertex if we insist on gauge invariance is

$$-ie\gamma^\mu. \tag{77}$$

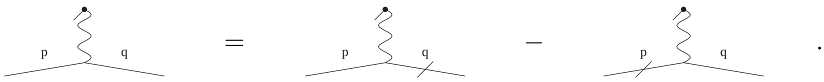
We could however also have chosen other terms from our general expression (70) or other terms from V^μ_α to be present in this vertex expression. They would however never have given the desired expression, since all the terms in the general expression and in V^μ_α are independent. So also by insisting on gauge invariance we can get to the well known expression (67) for the electron photon vertex.

3.4 The vertex expression for the selectron photon interaction

We now look at the vertex



This vertex we can fix in the same way as the electron photon vertex in the previous subsection. So we will again use a Ward Takahashi identity like (68), this time of course for our selectron coupling to the photon. The Ward Takahashi identity then looks like:



(78)

Here the solid lines indicate either the left or the right selectron. The special Feynman rules are defined analogous to the case of the electron.

The general expression for the vertex expression E^μ is much simpler in this case. The expression should have no Dirac indices and one Lorentz index and can only depend on the momenta p and q (the third momentum is always dependent on these two), thus we get

$$E^\mu = Ap^\mu + Bq^\mu \tag{79}$$

where A and B are again constants. When we translate the pictorial Ward Takahashi identity (78) to an equation we get:

$$\frac{i}{p^2 - m_L^2} E^\mu (q-p)_\mu \frac{i}{q^2 - m_L^2} = e\frac{i}{p^2 - m_L^2} - e\frac{i}{q^2 - m_L^2}. \tag{80}$$

When we substitute the general expression (79) in the left hand side we get:

$$\begin{aligned}
& \frac{i}{p^2 - m_L^2}(-Ap^2 + Bq^2 + (A-B)p\cdot q)\frac{i}{q^2 - m_L^2} \\
= \;& \frac{i}{p^2 - m_L^2}(-A(p^2 - m_L^2) + B(q^2 - m_L^2) + (A-B)p\cdot q)\frac{i}{q^2 - m_L^2} \\
= \;& -iA\frac{i}{q^2 - m_L^2} + iB\frac{i}{p^2 - m_L^2} + (A-B)p\cdot q\frac{i}{p^2 - m_L^2}\frac{i}{q^2 - m_L^2}.
\end{aligned} \tag{81}$$

To satisfy the Ward Takahashi identity we see that we should choose:

$$A = -ie, \quad B = -ie. \tag{82}$$

Of course this calculation done for the right selectron would have given exactly the same constants A and B. So for our vertex which gives the coupling of the left or the right selectron to the photon we get:

$$-ie(p+q)^\mu. \tag{83}$$

Here p is the momentum of the incoming selectron, directed into the vertex and q is the momentum of the outgoing selectron directed out of the vertex.

3.5 A 4-vertex

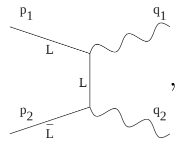
In the last section we saw that the coupling of the left and right selectron to the photon should be of the form (83) to get gauge invariance. This we saw by imposing the Ward Takahashi identity for the 3-vertex where the left (or right) selectron couples to the photon. This identity is however only one of the consequences of gauge invariance. It is not so that if this identity is satisfied one automatically has gauge invariance. This is the case for the electron but not for scalar particles like the left and right selectron, as we will see in this section.

To see that we still do not have gauge invariance have a look at the process

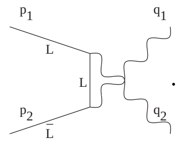
$$\tilde{e}_L \bar{\tilde{e}}_L \to \gamma\gamma. \tag{84}$$

With the 3-vertices we have constructed thus far the diagrams contributing to the matrix element in lowest order are

$M_1 =$



, $M_2 =$



. (85)

The indicated momenta flow from left to right. The matrix element of the first diagram is:

$$M_1 = -ie(2p_1 - q_1)^\mu \frac{i}{(p_1 - q_1)^2 - m_L^2}(-ie)(q_2 - 2p_2)^\nu \varepsilon_{1\mu}\varepsilon_{2\nu} \tag{86}$$

and the matrix element belonging to the second diagram is:

$$M_2 = -ie(2p_1 - q_2)^\nu \frac{i}{(q_1 - p_2)^2 - m_L^2}(-ie)(q_1 - 2p_2)^\mu \varepsilon_{1\mu}\varepsilon_{2\nu}. \tag{87}$$

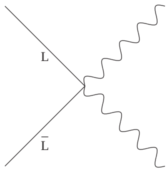
As a consequence of gauge invariance we should have that the total matrix element is zero when we take one of the photons to be longitudinally polarized and off-shell. The other particles should be on-shell. Such a longitudinal polarization vector is proportional to the momentum of the photon. So if we take the momentum q_1 for the polarization vector ε_1 of photon 1 the total matrix element should vanish for photon 1 off-shell and the other particles on-shell. If we do this M_1 and M_2 become:

$$\begin{aligned}
M_1 &= -ie(2p_1 - q_1)\cdot q_1 \frac{i}{(p_1 - q_1)^2 - m_L^2}(-ie)(q_2 - 2p_2)\cdot\varepsilon_2 \\
&= -ie(2p_1\cdot q_1 - q_1^2)\frac{i}{-2p_1\cdot q_1 + q_1^2}(-ie)(-2p_2)\cdot\varepsilon_2 = -2ie^2 p_2\cdot\varepsilon_2, \\
M_2 &= -ie(2p_1 - q_2)\cdot\varepsilon_2 \frac{i}{(q_1 - p_2)^2 - m_L^2}(-ie)(q_1 - 2p_2)\cdot q_1 \\
&= -ie2p_1\cdot\varepsilon_2 \frac{i}{-2q_1\cdot p_2 + q_1^2}(-ie)(-2p_2\cdot q_1 + q_1^2) = -2ie^2 p_1\cdot\varepsilon_2.
\end{aligned} \tag{88}$$

So the total matrix element is not zero but

$$M_T = -2ie^2(p_1 + p_2)\cdot\varepsilon_2 = -2ie^2 q_1\cdot\varepsilon_2. \tag{89}$$

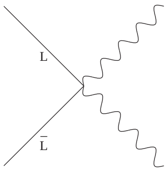
This means we need an extra diagram and since we have taken along all diagrams we can build with the vertices we constructed thus far it means we need a new vertex. We have constructed all possible 3-vertices (except the ones where a photon makes a left/right selectron go into a right/left selectron, these are however not going to help here, so there is still no need to introduce them), so when looking for a new vertex this must be a 4-vertex. Since we are doing our calculations at tree-level and thus want no loops the only possibility for a new vertex is:



A general expression for this vertex can again easily be written down. One of the terms in this general expression would of course be

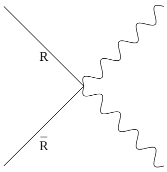
$$Ag^{\mu\nu}. \tag{90}$$

If we now choose $A = 2ie^2$ we see that the total matrix element does vanish because of the third diagram we get on account of the new 4-vertex. We also see that other terms of the general expression, which contain two momenta coming into the vertex, could never give minus (89). (Only if we introduce undesired form factors in the new 4-vertex this would be possible.) So we have the following new 4-vertex:



$$= 2ie^2 g^{\mu\nu}. \tag{91}$$

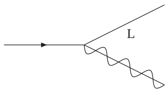
Again a similar derivation for the right selectron shows that we also need a new 4-vertex with right selectrons:

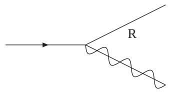


$$= 2ie^2 g^{\mu\nu}. \tag{92}$$

3.6 The vertex expression for the electron selectron photino vertex

In the previous sections we have found the expressions belonging to 3 of our 5 independent 3-vertices. This leaves us with the following 3-vertices.





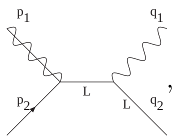
We cannot fix these vertices with the Ward Takahashi identity, since there is no gauge boson present, but we can try to fix them by imposing that no longitudinally polarized photons are produced, like we did when fixing the latest 4-vertices. So we need a process where a photon and the vertex to be fixed occurs. If we only consider processes with 2 in going and 2 outgoing particles the only processes that have these properties are:

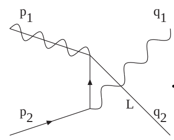
$$e\tilde{\gamma} \to \tilde{e}_L \gamma \tag{93}$$

and

$$e\tilde{\gamma} \to \tilde{e}_R \gamma. \tag{94}$$

Let's first have a look at the first process. The contributing diagrams are:

$M_1 =$ 

, $M_2 =$ 

. (95)

The vertex to be fixed we call B. This B should have 2 Dirac indices because two fermions coincide in the vertex. The two matrix elements for the process are then:

$$M_1 = \bar{v}(p_1)Bu(p_2)\frac{i}{(p_1+p_2)^2 - m_L^2}(-ie)(p_1+p_2+q_2)\cdot\varepsilon \tag{96}$$

and

$$M_2 = \bar{v}(p_1)B\frac{i(\not{p}_2 - \not{q}_1 + m_e)}{(p_2-q_1)^2 - m_e^2}(-ie\gamma^\mu)u(p_2)\varepsilon_\mu. \tag{97}$$

Now we substitute q_1 for ε. Again the photon is off-shell whereas the other particles are on-shell.

$$\begin{aligned}
M_1 &= \bar{v}(p_1)Bu(p_2)\frac{i}{(q_1+q_2)^2 - m_L^2}(-ie)(q_1+2q_2)\cdot q_1 \\
&= e\bar{v}(p_1)Bu(p_2)\frac{1}{2q_1\cdot q_2 + q_1^2}(2q_2\cdot q_1 + q_1^2) \\
&= e\bar{v}(p_1)Bu(p_2) \\
M_2 &= \bar{v}(p_1)B\frac{i(\not{p}_2 - \not{q}_1 + m_e)}{(p_2-q_1)^2 - m_e^2}(-ie\not{q}_1)u(p_2) \\
&= e\bar{v}(p_1)B\frac{\not{p}_2\not{q}_1 - q_1^2 + m_e\not{q}_1}{(p_2-q_1)^2 - m_e^2}u(p_2) \\
&= e\bar{v}(p_1)B\frac{2p_2\cdot q_1 - q_1^2}{-2p_2\cdot q_1 + q_1^2}u(p_2) \\
&= -e\bar{v}(p_1)Bu(p_2)
\end{aligned} \tag{98}$$

The sum $M_1 + M_2$ is automatically zero. So we see that imposing gauge invariance on this process does not give a condition on B, we can still choose for B whatever we want.

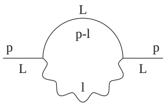
Again the calculation for the process with the right selectron is similar, also this calculation gives no constraint for the electron right selectron photino vertex.

The processes above are the only $2 \to 2$ processes which might fix B by imposing the impossibility of producing longitudinally polarized photons. Thus we could hope to fix B by looking at processes with more than 4 particles. In the next section we will see however that there is another way to proceed fixing B.

3.7 Quadratic divergences

As already mentioned one of the advantages of SQED is that it has no quadratic divergences. In this section we will make precise the meaning of a quadratic divergence. Also we begin imposing the condition that there are no quadratic divergences in the theory we are constructing out of our 5 elementary particles.

First we will give an example that shows what quadratic divergences are and how they enter into our theory. Consider the propagator of the left selectron at order e^2. Of this propagator consider only the following 1-loop diagram.

$P =$ 

(99)

The indicated external momentum p and the loop momentum l again flow from left to right. We consider this diagram as a contribution to the propagator of the left selectron, thus the incoming and outgoing momentum p is off shell. Now we write down the expression P for this diagram. We leave out the 2 factors $\frac{i}{p^2-m_L^2}$ for the in going and outgoing leg.

$$P = \frac{1}{(2\pi)^4}\int d^4l(-ie)(2p-l)^\mu \frac{i}{(p-l)^2-m_L^2}\frac{-ig_{\mu\nu}}{l^2}(-ie)(2p-l)_\nu \tag{100}$$

There are two remarks to be made before we can continue. Firstly we should think of the ε-prescription to make the integral in (100) well defined. Thus we should add a term $i\varepsilon$ in the 2 denominators and when the integral over the loop momentum l is done we should let ε approach zero (from above). Secondly we should think of the possible divergence related to the masslessness of the photon. If the loop momentum l becomes zero the denominator from the photon propagator blows up. The possible divergence coming from this singularity is called an infrared divergence, because the divergence comes from the low energy area of the complete integration area. Technically one can introduce a photon mass m_γ to regularize this kind of divergence. At the end of all calculations one should then let m_γ approach zero and hope that no infrared divergences occur anymore. For us infrared divergences are not of interest however. As we will see in the coming calculation we can discard all terms having such an infrared divergence since such a divergence has nothing to do with a quadratic divergence in which we are only interested. As an illustration we will however use the regularization mass m_γ in the coming calculation. In all later calculations we will not and we will just discard infinities coming from the infrared integration area.

Now our complete expression P should thus be

$$P = \frac{1}{(2\pi)^4}\int d^4l(-ie)(2p-l)^\mu \frac{i}{(p-l)^2-m_L^2+i\varepsilon}\frac{-ig_{\mu\nu}}{l^2-m_\gamma^2+i\varepsilon}(-ie)(2p-l)_\nu. \tag{101}$$

Now we can further calculate P.

$$\begin{aligned}
P &= \frac{-e^2}{(2\pi)^4}\int d^4l(2p-l)^2\frac{1}{(p-l)^2-m_L^2+i\varepsilon}\frac{1}{l^2-m_\gamma^2+i\varepsilon} \\
&= \frac{-e^2}{(2\pi)^4}\int d^4l(2(p-l)^2+2p^2-l^2)\frac{1}{(p-l)^2-m_L^2+i\varepsilon}\frac{1}{l^2-m_\gamma^2+i\varepsilon} \\
&= \frac{-e^2}{(2\pi)^4}\int d^4l\big(2((p-l)^2-m_L^2+i\varepsilon)-(l^2-m_\gamma^2+i\varepsilon)+ \\
&\quad (2p^2+2m_L^2-m_\gamma^2-i\varepsilon)\big)\frac{1}{(p-l)^2-m_L^2+i\varepsilon}\frac{1}{l^2-m_\gamma^2+i\varepsilon} \\
&= \frac{-2e^2}{(2\pi)^4}\int d^4l\frac{1}{l^2-m_\gamma^2+i\varepsilon}+\frac{e^2}{(2\pi)^4}\int d^4l\frac{1}{(p-l)^2-m_L^2+i\varepsilon}+ \\
&\quad \frac{-e^2}{(2\pi)^4}\int d^4l(2p^2+2m_L^2-m_\gamma^2-i\varepsilon)\big)\frac{1}{(p-l)^2-m_L^2+i\varepsilon}\frac{1}{l^2-m_\gamma^2+i\varepsilon}
\end{aligned} \tag{102}$$

The only thing we are interested in is the quadratically divergent part of this expression. Before we can isolate this part we must know what we mean by quadratically divergent. For this we have to regularize the divergent integrals in (102) by introducing a cut-off momentum Λ. This means we only integrate the loop momentum l over a finite area given by:

$$|l_E| \leq \Lambda \tag{103}$$

where l_E is the Euclidean momentum belonging to the momentum l in Minkowski space. The degree of divergence of an integral is now defined as the highest power of Λ present when the integral is done. An integral is quadratically divergent when it's degree of divergence is 2. An integral is called quartically divergent when it's degree of divergence is 4. An integral is said to be logarithmically divergent when the worst behaving term is proportional to $\ln \Lambda$.

The degree of divergence of an integral can be determined naively by counting the powers of the loop momentum in the integrand. For example the integral

$$\int d^4l \frac{1}{l^2 - m^2 + i\varepsilon} \tag{104}$$

has 4 powers of l in the numerator and 2 powers of l in the denominator. Then one expects the worst behavior of the integral to be proportional to Λ^2. Such a naive approach is not always correct as can be seen in the following case:

$$\int d^4l \frac{l \cdot p}{l^2 - m^2 + i\varepsilon}. \tag{105}$$

This integral is zero because the integrand changes sign under $l \to -l$. Thus this integral does not have degree of divergence 1.

Some integrals of interest to us are calculated in appendix 1. Now we can determine what the quadratically divergent part of P, given in (102), is. We see, with the knowledge of appendix 1, that the first integral indeed is quadratically divergent. The second integral is of a form we have not calculated in appendix 1 but can be reduced to such forms by making an expansion in the momentum p around $p = 0$:

$$\int d^4l \frac{1}{(p-l)^2 - m_L^2 + i\varepsilon} = \int d^4l \frac{1}{l^2 - m_L^2 + i\varepsilon} - \int d^4l \frac{p^2}{(l^2 - m_L^2 + i\varepsilon)^2} + \dots. \tag{106}$$

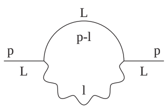
Thus we see that also this integral has a quadratically divergent part. It also has a logarithmically divergent part. The third and last integral in P only has a logarithmically divergent part which can also be seen by expanding the integrand around $p = 0$. The last integral in (102) can thus be discarded if we are only interested in quadratic divergences. With help of appendix 1 we see that P has the following quadratically divergent part:

$$\frac{-e^2}{(2\pi)^4}(-i\pi^2\Lambda^2) = \frac{ie^2}{16\pi^2}\Lambda^2. \tag{107}$$

Now we have seen what quadratic divergences actually are and now we have calculated divergent integrals with help of a cut-off momentum Λ we can turn to the theory we are building again. In the next sections we will impose one our theory that quadratical divergences are absent from our theory, beginning with imposing this on the selectron propagators. Our hope is to fix the vertex expression B for the vertex we considered in the previous section with this constraint.

3.8 The left selectron propagator

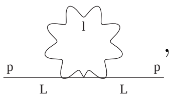
In this section we will have a look at the propagator of the left selectron. We will impose that there are no quadratic, or even worse, divergences in this propagator. In the previous section we already saw that one of the contributing diagrams was quadratically divergent:

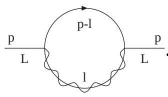
$P_1 =$ 

$$= \frac{ie^2}{16\pi^2}\Lambda^2. \tag{108}$$

On the righthandside we have only written the quadratically divergent part of P_1, since we are only interested in this part. Henceforth all =-signs occurring in calculations where we are only interested in the worst divergence of an expression should be interpreted in this way.

The quadratical divergence in (108) should now be canceled by quadratical divergences in other contributing diagrams. There are two such diagrams:

$P_2 = \frac{1}{2}$ 

, $P_3 =$ 

. (109)

Again the indicated momenta flow from left to right. First we calculate P_2.

$$\begin{aligned} P_2 &= \frac{1}{2}\frac{1}{(2\pi)^4}\int d^4l 2ie^2 g^{\mu\nu}\frac{-ig_{\mu\nu}}{l^2+i\varepsilon} = 4\frac{e^2}{(2\pi)^4}\int d^4l\frac{1}{l^2+i\varepsilon} \\ &= -4\frac{ie^2}{16\pi^2}\Lambda^2 \end{aligned} \tag{110}$$

Notice the symmetry factor $\frac{1}{2}$ we get for this diagram. Also notice that also here we have left out the propagators for the in going and outgoing line.

Now we should calculate P_3. This diagram contains the unknown vertex B_L which we hope to fix in this section. In terms of this unknown expression B_L, P_3 is:

$$\begin{aligned} P_3 &= -\frac{1}{(2\pi)^4}\int d^4l \mathrm{Tr}\Big(B_L(p-l,-l)\frac{i(\not{p}-\not{l}+m_e)}{(p-l)^2-m_e^2+i\varepsilon} \\ &\quad (-1)B_L^\dagger(p-l,-l)\frac{i(-\not{l}+m_{\tilde{\gamma}})}{l^2-m_{\tilde{\gamma}}^2+i\varepsilon}\Big) \\ &= -\frac{1}{(2\pi)^4}\int d^4l\frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_{\tilde{\gamma}}^2+i\varepsilon} \\ &\quad \mathrm{Tr}\big(B_L(p-l,-l)(\not{p}-\not{l}+m_e)B_L^\dagger(p-l,-l)(-\not{l}+m_{\tilde{\gamma}})\big). \end{aligned} \tag{111}$$

Notice the extra minus sign we get because of the fermion loop in P_3. The dependence on the momenta of the expressions B_L and $B_L^\dagger$ we have also included. The first argument of B_L indicates the momentum in the electron line, counted along the electron arrow. The second argument indicates the momentum in the photino line, directed out of the vertex. The first argument of $B_L^\dagger$ indicates the momentum in the electron line also counted along the electron arrow, which is now directed out of the vertex. The second argument indicates the momentum in the photino line directed into the vertex. We now use the following general expression for $B_L(p-l,-l)$:

$$B_L = S\mathbf{I} + V_\alpha\gamma^\alpha + T_{\alpha\beta}\sigma^{\alpha\beta} + A_\alpha\gamma^5\gamma^\alpha + P\gamma^5 \tag{112}$$

where $\sigma^{\alpha\beta}$ is defined as:

$$\sigma^{\alpha\beta} = \frac{i}{2}(\gamma^\alpha\gamma^\beta - \gamma^\beta\gamma^\alpha). \tag{113}$$

Since we do not want any form factors in our fundamental vertices S and P should be constants. For V_α, A_α and $T_{\alpha\beta}$ we can write down the general expressions

$$\begin{aligned} V_\alpha &= v_1(p-l)_\alpha + v_2 l_\alpha \\ A_\alpha &= a_1(p-l)_\alpha + a_2 l_\alpha \\ T_{\alpha\beta} &= t_1(p-l)_\alpha l_\beta + t_2(p-l)_\beta l_\alpha + t_3\epsilon_{\alpha\beta\delta\gamma}(p-l)^\delta l^\gamma \end{aligned} \tag{114}$$

where also v_1, v_2, a_1, a_2, t_1, t_2 and t_3 are constants. Notice we haven't included a term proportional to $g_{\alpha\beta}$ in $T_{\alpha\beta}$ since

$$g_{\alpha\beta}\sigma^{\alpha\beta} = 0 \tag{115}$$

because $\sigma^{\alpha\beta}$ is antisymmetric in its indices and $g_{\alpha\beta}$ is symmetric.

The general expression for B_L should now be substituted in P_3. The calculation of the trace is rather lengthy and can best be done by computer. For this we wrote a small program in `FORM` which is presented, together with its output, in appendix 2. This `FORM`-program uses the following identity for the ϵ-tensor:

$$\epsilon^{\alpha\beta\mu\nu}\epsilon^{\delta\gamma}{}_{\mu\nu} = 2(g^{\alpha\gamma}g^{\beta\delta} - g^{\alpha\delta}g^{\beta\gamma}) \tag{116}$$

so that

$$\epsilon^{\alpha\beta\mu\nu}\epsilon^{\delta\gamma}{}_{\mu\nu}p_\alpha l_\beta p_\delta l_\gamma = 2((p\cdot l)^2 - p^2 l^2). \tag{117}$$

This identity is proven in the following way. The expression $\epsilon^{\alpha\beta\mu\nu}\epsilon^{\delta\gamma}{}_{\mu\nu}$ should be antisymmetric under $\alpha \leftrightarrow \beta$ and $\delta \leftrightarrow \gamma$. Thus our expression can only be something of the form

$$A\epsilon^{\alpha\beta\delta\gamma} + B(g^{\alpha\delta}g^{\beta\gamma} - g^{\alpha\gamma}g^{\beta\delta}). \tag{118}$$

To fix the constants A and B we consider the following cases.

$$\begin{aligned} \alpha=\delta=0,\ \beta=\gamma=1 \quad &: \quad \epsilon^{01\mu\nu}\epsilon^{01}{}_{\mu\nu} = \\ & \quad\quad \epsilon^{0123}\epsilon^{01}{}_{23} + \epsilon^{0132}\epsilon^{01}{}_{32} = 2 = -B \\ \alpha=0,\ \beta=1,\ \delta=2,\ \gamma=3 \quad &: \quad \epsilon^{01\mu\nu}\epsilon^{23}{}_{\mu\nu} = \\ & \quad\quad \epsilon^{0123}\epsilon^{23}{}_{23} + \epsilon^{0132}\epsilon^{23}{}_{32} = 0 = -A \end{aligned} \tag{119}$$

So we get indeed the identity as stated above.

When we have a look at the output, which is the result of the trace in (111), we see that we get a variety of terms, of which some terms also give quartic divergences. To see which term gives which kind of divergence we make the following expansion in (111).

$$\begin{aligned} \frac{1}{(p-l)^2 - m_e^2 + i\varepsilon}\frac{1}{l^2 - m_{\tilde\gamma}^2 + i\varepsilon} &= \frac{1}{l^2 - m_e^2 + i\varepsilon}\frac{1}{l^2 - m_{\tilde\gamma}^2 + i\varepsilon} + \\ & \frac{2p\cdot l - p^2}{(l^2 - m_e^2 + i\varepsilon)^2}\frac{1}{l^2 - m_{\tilde\gamma}^2 + i\varepsilon} + \\ & \frac{4(p\cdot l)^2 - 4p^2 p\cdot l}{(l^2 - m_e^2 + i\varepsilon)^2}\frac{1}{l^2 - m_{\tilde\gamma}^2 + i\varepsilon} + \\ & \ldots \end{aligned} \tag{120}$$

Now we can tell what kind of worst divergence all the terms resulting from the trace have. In table (1) the worst divergence for each type of term in the trace is listed. With the expansion made above it is not so hard to deduce the worst divergences as given in this table. Consider for example the term $(p\cdot l)l^2$ from the trace. This term, together with the first term of the expansion (120), vanishes under the integral. The worst divergence this term causes thus comes from the second term in the expansion (120). This worst divergence is:

$$\frac{-1}{(2\pi)^4}\int d^4l \frac{2p\cdot l - p^2}{(l^2 - m_e^2 + i\varepsilon)^2}\frac{1}{l^2 - m_{\tilde\gamma}^2 + i\varepsilon}(p\cdot l)l^2$$

Table 1: The terms resulting from the trace with the worst divergence these terms give in the complete expression for P_3 (111).

type of term in the trace	resulting worst divergence
$(p \cdot l)l^2$	$\frac{1}{2}p^2 \frac{i}{16\pi^2}\Lambda^2$
$p \cdot l$	$\frac{1}{2}p^2 \frac{-i}{8\pi^2} \ln \Lambda$
l^2	$\frac{i}{16\pi^2}\Lambda^2$
$(l^2)^2$	$\frac{-i}{32\pi^2}\Lambda^4$
$(p \cdot l)^2 l^2$	$\frac{1}{4}p^2 \frac{-i}{32\pi^2}\Lambda^4$
$(p \cdot l)^2$	$\frac{1}{4}p^2 \frac{i}{16\pi^2}\Lambda^2$
$(p \cdot l)^3$	$\frac{3}{8}(p^2)^2 \frac{i}{16\pi^2}\Lambda^2$

$$
\begin{aligned}
&= \frac{-1}{8\pi^4}\int d^4l \frac{(p\cdot l)^2}{(l^2 - m_e^2 + i\varepsilon)^2} \\
&= \frac{-1}{8\pi^4} p_\mu p_\nu \int d^4l \frac{l^\mu l^\nu}{(l^2 - m_e^2 + i\varepsilon)^2} \\
&= \frac{-1}{8\pi^4} p_\mu p_\nu \int d^4l \frac{1}{4} g^{\mu\nu} \frac{l^2}{(l^2 - m_e^2 + i\varepsilon)^2} \\
&= \frac{-1}{32\pi^4} p^2 \int d^4l \frac{1}{l^2 - m_e^2 + i\varepsilon} \\
&= \frac{1}{2}p^2 \frac{i}{16\pi^2}\Lambda^2.
\end{aligned} \tag{121}
$$

Here we again left out all terms not contributing to the worst, in this case quadratic, divergence.

Also consider the term $(l^2)^2$. This term, as can be seen in table 1, gives a quartic divergence. That this is the case is also not hard to see. The term $(l^2)^2$ together with the first term in the expansion (120) give:

$$
\begin{aligned}
&\frac{-1}{(2\pi)^4}\int d^4l \frac{1}{l^2 - m_e^2 + i\varepsilon}\frac{1}{l^2 - m_{\tilde{\gamma}}^2 + i\varepsilon}(l^2)^2 \\
&= \frac{-1}{(2\pi)^4}\int d^4l \\
&= \frac{-i}{16\pi^4}\int d^4l_E \\
&= \frac{-i}{16\pi^4} 2\pi^2 \int d|l_E||l_E|^3 \\
&= \frac{-i}{8\pi^2}[\frac{1}{4}|l_E|^4]_0^\Lambda \\
&= \frac{-i}{32\pi^2}\Lambda^4.
\end{aligned} \tag{122}
$$

As a last example consider the term $(p\cdot l)^3$. Because this term is odd in l the worst divergence resulting from this term comes from the second term in the expansion (120):

$$
\frac{-1}{(2\pi)^4}\int d^4l \frac{2p\cdot l - p^2}{(l^2 - m_e^2 + i\varepsilon)^2}\frac{1}{l^2 - m_{\tilde{\gamma}}^2 + i\varepsilon}(p\cdot l)^3
$$

$$
\begin{aligned}
&= \frac{-1}{8\pi^4}\int d^4l \frac{(p\cdot l)^4}{(l^2-m_e^2+i\varepsilon)^2}\frac{1}{l^2-m_{\tilde{\gamma}}^2+i\varepsilon} \\
&= \frac{-1}{8\pi^4}p_\mu p_\nu p_\alpha p_\beta \int d^4l \frac{l^\mu l^\nu l^\alpha l^\beta}{(l^2-m_e^2+i\varepsilon)^2}\frac{1}{l^2-m_{\tilde{\gamma}}^2+i\varepsilon} \\
&= \frac{-1}{8\pi^4}\frac{3}{16}(p^2)^2\int d^4l \frac{1}{l^2-m_e^2+i\varepsilon} \\
&= \frac{3}{8}(p^2)^2\frac{i}{16\pi^2}\Lambda^2. \qquad (123)
\end{aligned}
$$

Here we have used

$$
\begin{aligned}
\int d^4l \frac{l^\mu l^\nu l^\alpha l^\beta}{(l^2-m_1^2+i\varepsilon)(l^2-m_2^2+i\varepsilon)(l^2-m_3^2+i\varepsilon)} = \\
A g^{\mu\nu}g^{\alpha\beta} + B g^{\mu\alpha}g^{\nu\beta} + C g^{\mu\beta}g^{\nu\alpha} \qquad (124)
\end{aligned}
$$

with

$$
A = B = C = \frac{1}{16}\int d^4l \frac{(l^2)^2}{(l^2-m_1^2+i\varepsilon)(l^2-m_2^2+i\varepsilon)(l^2-m_3^2+i\varepsilon)}. \qquad (125)
$$

Now we know how the results in table (1) can be calculated we can start using them. As we know the other two diagrams P_1 and P_2 only contain quadratically divergent terms which do not contain a dependence on the momentum p. Our vertex expression B_L should be chosen in such a way that P_3 cancels the quadratic divergences we got thus far. We should not get any new kind of quadratic, or even quartic, divergences. Thus we should now impose that the expression of coupling constants in front of these new kind of divergences vanish. Also we should impose that the expression of coupling constants in front of the p-independent quadratic divergence is such that quadratic divergences from P_1 and P_2 are canceled. The first constraint gives us 4 complex equations. Now we know what divergences all the terms of the trace give we can read off these equations from the output of our program, given in appendix 2. The equation coming from the quartically divergent terms proportional to p^2 is:

$$
\begin{aligned}
& (-4|t_1|^2 + 4t_1t_2^* + 4t_1^*t_2 - 4|t_2|^2 - 16|t_3|^2) + \\
& \frac{1}{4}(4|t_1|^2 - 4t_1t_2^* - 4t_1^*t_2 + 4|t_2|^2 + 16|t_3|^2) = 0 \qquad (126) \\
\Rightarrow \quad & -3|t_1-t_2|^2 - 12|t_3|^2 = 0 \\
\Rightarrow \quad & t_1 = t_2, \quad t_3 = 0. \qquad (127)
\end{aligned}
$$

Because $t_1 = t_2$ and $t_3 = 0$ the expression in front of the tensor $\sigma^{\alpha\beta}$ becomes completely symmetric under $\alpha \leftrightarrow \beta$. Since $\sigma^{\alpha\beta}$ is antisymmetric the total term vanishes. Thus we cannot have a term proportional to the tensor $\sigma^{\alpha\beta}$ in our vertex expression!

Now look at the quartically divergent terms with no factor p^2 in front. Imposing that also these terms vanish gives:

$$
\begin{aligned}
& 4|v_1|^2 - 4v_1v_2^* - 4v_1^*v_2 + 4|v_2|^2 + 4|a_1|^2 - 4a_1a_2^* - 4a_1^*a_2 + 4|a_2|^2 = 0 \\
\Rightarrow \quad & 4|v_1-v_2|^2 + 4|a_1-a_2|^2 = 0 \\
\Rightarrow \quad & v_1 = v_2, \quad a_1 = a_2. \qquad (128)
\end{aligned}
$$

If we now consider the terms with a quadratic divergence and a factor $(p^2)^2$ we discover that this gives again an equation similar to (126).

Finally also the terms which are quadratically divergent and have a factor p^2 should vanish. As we can see in appendix 2 the equation coming from this restraint is rather lengthy. It is therefore handy first to substitute the results we got thus far into this equation. Then we get:

$$
\begin{aligned}
& (-4|v_1|^2 - 4|a_1|^2) + \frac{1}{4}(8|v_1|^2 + 8|a_1|^2) = 0 \\
\Rightarrow \quad & v_1 = 0, \quad a_1 = 0 \\
\Rightarrow \quad & v_2 = 0, \quad a_2 = 0. \qquad (129)
\end{aligned}
$$

So we see that also the terms with the γ^α and the $\gamma^5\gamma^\alpha$ in the general expression (112) should vanish. This leaves us with:

$$B_L = S_L\mathbf{I} + P_L\gamma^5. \tag{130}$$

The trace finally becomes:

$$4l^2(|S_L|^2 + |P_L|^2) - 4p\cdot l(|S_L|^2 + |P_L|^2) + 4m_e m_{\tilde{\gamma}}(|S_L|^2 - |P_L|^2). \tag{131}$$

For S_L and P_L we still get a restraint from the demand that the terms in P_3 giving a quadratic divergence independent of p should cancel the quadratic divergences in P_1 and P_2. This gives:

$$\begin{aligned} & \frac{ie^2}{16\pi^2}\Lambda^2 - 4\frac{ie^2}{16\pi^2}\Lambda^2 + 4(|S_L|^2 + |P_L|^2)\frac{ie^2}{16\pi^2}\Lambda^2 = 0 \\ \Rightarrow \quad & -3e^2 + 4(|S_L|^2 + |P_L|^2) = 0. \end{aligned} \tag{132}$$

So finally we know the form of our vertex expression B_L. We do not yet know the coupling constants however, since equation (132) cannot fix these uniquely. To fix these we have to search for other equations, which will of course be the main subject for the coming sections.

In this section we found the form of the expression B_L belonging to the vertex where an electron, a photino an a left selectron coincide. The same calculation can be done for the vertex where instead of the left selectron we have a right selectron. It will not be a surprise that also for this vertex we get the expression

$$B_R = S_R\mathbf{I} + P_R\gamma^5 \tag{133}$$

and the equation

$$-3e^2 + 4(|S_R|^2 + |P_R|^2) = 0. \tag{134}$$

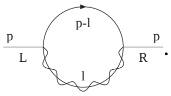
3.9 The process $\tilde{e}_L \to \tilde{e}_R$

Let's have a look at another process with only 2 external lines. Consider the process

$$\tilde{e}_L \to \tilde{e}_R \tag{135}$$

where the in going and outgoing momentum p is off-shell. Also the expression associated with this process should have no quadratic divergences. With this restraint we hope to fix S_L, P_L, S_R and P_R further. The process has only one contributing diagram at one loop order:

$$P_1 = \tag{136}$$



Note that there would have been more contributing diagrams if we included 3-vertices where a left/right selectron evolved to a right/left selectron and a photon. In this section we will indeed see that these vertices are not needed to remove all quadratic divergences from the current process.

The matrix element belonging to our only diagram is:

$$\begin{aligned}
P_1 &= -\frac{1}{(2\pi)^4}\int d^4l \mathrm{Tr}\Big(B_R \frac{i(\not{p}-\not{l}+m_e)}{(p-l)^2-m_e^2+i\varepsilon}(-1)B_L^\dagger \frac{i(-\not{l}+m_{\tilde{\gamma}})}{l^2-m_{\tilde{\gamma}}^2+i\varepsilon}\Big)\\
&= \frac{-1}{(2\pi)^4}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_{\tilde{\gamma}}^2+i\varepsilon}\\
&\quad \mathrm{Tr}((S_R+P_R\gamma^5)(\not{p}-\not{l}+m_e)(S_L^*-P_L^*\gamma^5)(-\not{l}+m_{\tilde{\gamma}}))\\
&= \frac{-1}{(2\pi)^4}m_e m_{\tilde{\gamma}}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_{\tilde{\gamma}}^2+i\varepsilon}\\
&\quad \mathrm{Tr}((S_R+P_R\gamma^5)(S_L^*-P_L^*\gamma^5)) +\\
&\quad \frac{1}{(2\pi)^4}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_{\tilde{\gamma}}^2+i\varepsilon}\\
&\quad \mathrm{Tr}((S_R+P_R\gamma^5)(\not{p}-\not{l})(S_L^*-P_L^*\gamma^5)\not{l})\\
&= \frac{-1}{(2\pi)^4}m_e m_{\tilde{\gamma}}4(S_L^*S_R-P_L^*P_R)\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_{\tilde{\gamma}}^2+i\varepsilon}+\\
&\quad \frac{1}{(2\pi)^4}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_{\tilde{\gamma}}^2+i\varepsilon}\\
&\quad \mathrm{Tr}((S_R+P_R\gamma^5)(S_L^*+P_L^*\gamma^5)(\not{p}-\not{l})\not{l})\\
&= \frac{-1}{(2\pi)^4}m_e m_{\tilde{\gamma}}4(S_L^*S_R-P_L^*P_R)\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_{\tilde{\gamma}}^2+i\varepsilon}+\\
&\quad \frac{1}{(2\pi)^4}(S_L^*S_R+P_L^*P_R)\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_{\tilde{\gamma}}^2+i\varepsilon}\\
&\quad \mathrm{Tr}((\not{p}-\not{l})\not{l})\\
&= \frac{-1}{(2\pi)^4}m_e m_{\tilde{\gamma}}4(S_L^*S_R-P_L^*P_R)\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_{\tilde{\gamma}}^2+i\varepsilon}+\\
&\quad \frac{1}{(2\pi)^4}4(S_L^*S_R+P_L^*P_R)\int d^4l \frac{(p\cdot l-l^2)}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_{\tilde{\gamma}}^2+i\varepsilon}.
\end{aligned} \tag{137}$$

The first term is of course only logarithmically divergent. The part $p\cdot l$ in the second term also only gives a logarithmic divergence. This we know form the previous section (see table (1)). To see this one might also rewrite $p\cdot l$ as

$$p\cdot l = -\frac{1}{2}(p-l)^2+\frac{1}{2}p^2+\frac{1}{2}l^2. \tag{138}$$

Both the terms $(p-l)^2$ and l^2 give the same quadratic divergence, so that they cancel each other in the total expression because of the relative minus sign of the coefficients. The part l^2 in the second term does give a quadratic divergence. So imposing that this divergence is absent gives the restraint:

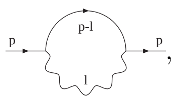
$$S_L^* S_R + P_L^* P_R = 0. \tag{139}$$

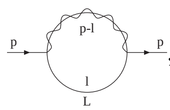
Naturally a similar calculation for the process $\tilde{e}_R \rightarrow \tilde{e}_L$ would have given the same restraint (139).

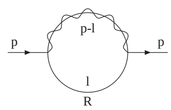
We see that we can still not fix the complex constants S_L, P_L, S_R and P_R uniquely. To do this we must find more constraints.

3.10 The electron propagator

In this section we will see what constraint we get from imposing that there are no quadratic divergences in the electron propagator. There are 3 diagrams contributing to this propagator at one loop order (order e^2).

$P_1 =$ 

, $P_2 =$ 

, $P_3 =$ 

(140)

Let's calculate the diagrams. Again we will leave out the (lowest order) propagators for the in going and outgoing legs, like we did in the cases with the selectrons.

$$\begin{aligned}
P_1 &= \frac{1}{(2\pi)^4}\int d^4l(-ie)\gamma^\mu \frac{i(\not{p}-\not{l}+m_e)}{(p-l)^2-m_e^2+i\varepsilon}(-ie)\gamma^\nu \frac{-ig_{\mu\nu}}{l^2+i\varepsilon} \\
&= \frac{-e^2}{(2\pi)^4}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2+i\varepsilon}\gamma^\mu(\not{p}-\not{l}+m_e)\gamma_\mu \\
&= \frac{-e^2}{(2\pi)^4}4m_e\mathbf{I}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2+i\varepsilon} + \\
&\quad \frac{-e^2}{(2\pi)^4}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2+i\varepsilon}\gamma^\mu(-\gamma_\mu(\not{p}-\not{l})+2(p-l)_\mu) \\
&= \frac{-4e^2}{(2\pi)^4}m_e\mathbf{I}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2+i\varepsilon} + \\
&\quad \frac{2e^2}{(2\pi)^4}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2+i\varepsilon}(\not{p}-\not{l})
\end{aligned} \tag{141}$$

For the term with the $\not{l}$ we use the following:

$$\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2+i\varepsilon}\not{l} = A\not{p} \tag{142}$$

with A given by

$$\begin{aligned}
A &= \frac{1}{4}\frac{1}{p^2}\mathrm{Tr}\Big(\not{p}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2+i\varepsilon}\not{l}\Big) \\
&= \frac{1}{p^2}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2+i\varepsilon}p\cdot l.
\end{aligned} \tag{143}$$

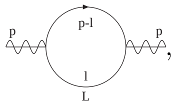
From the previous sections we know that this A is not quadratically divergent, thus also the term with the $\not{l}$ in (141) has no quadratic divergence. Neither the term proportional to m_e and the term proportional to $\not{p}$ in (141) have quadratic divergences. So we see that P_1 is free of quadratic divergences.

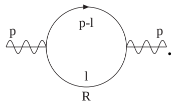
This could also be expected if we recall normal QED. The diagram P_1 is the only contributing diagram at order e^2 in QED. We also know that in QED the renormalization of the electron mass needs no quadratically divergent terms. In QED the electron mass is protected against such terms because of the chiral symmetry of the massless theory.

When one calculates P_2 and P_3 it appears that also in these diagrams there occur no quadratic divergences at all. So from demanding there are no quadratic divergences in the electron propagator we get no constraints for our coupling constants.

3.11 The photino propagator

In this section we will have a look at the photino propagator at 1 loop order. At this order there are 2 contributing diagrams:

$P_1 =$ 

, $P_2 =$ 

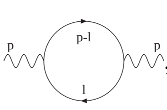
. (144)

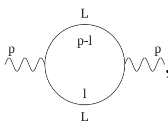
Also in these diagrams there occur no quadratic divergences at all, as in the case with the electron propagator. Again this is because of the fermionic character of the photino. So also here we find no constraints.

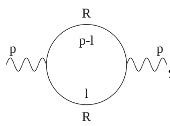
3.12 The photon propagator or vacuum polarization

The last 2-point function we can look at at order e^2 is the photon propagator, also called vacuum polarization. There are no other 2-point functions in our theory, so if we still have to find more constraints after this calculation we either have to consider processes with more than 2 legs or we have to consider the 2-point functions again, this time at higher loop order.

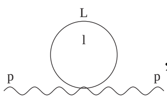
There are 5 diagrams for the photon propagator at 1 loop order:

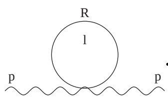
$P_1 =$ 

, $P_2 =$ 

, $P_3 =$ 

,

$P_4 =$ 

, $P_5 =$ 

. (145)

Immediately we see that the vertices we want to fix do not occur in the photon propagator at 1-loop order. Still we will do the calculation as a check that with the vertex expressions we have fixed thus far the quadratic divergences indeed vanish. Also this calculation gives a nice illustration of how supersymmetry cancels all quadratic divergences.

The first diagram $P_1^{\mu\nu}$ is:

$$\begin{aligned} P_1^{\mu\nu} &= -\frac{1}{(2\pi)^4}\int d^4l \mathrm{Tr}\Big((-ie)\gamma^\nu \frac{i(\not{p}-\not{l}+m_e)}{(p-l)^2-m_e^2+i\varepsilon}(-ie)\gamma^\mu \frac{i(-\not{l}+m_e)}{l^2-m_e^2+i\varepsilon}\Big) \\ &= -\frac{e^2}{(2\pi)^4}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_e^2+i\varepsilon} \\ &\quad \mathrm{Tr}\big(\gamma^\nu(\not{p}-\not{l}+m_e)\gamma^\mu(-\not{l}+m_e)\big). \end{aligned} \tag{146}$$

This expression we can always write in the form

$$P_1^{\mu\nu} = A_1\big(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\big) + B_1 g^{\mu\nu} \tag{147}$$

with

$$\begin{aligned} A_1 &= \frac{1}{3}P_{1\ \mu}^{\ \mu} - \frac{4}{3}\frac{p_\mu p_\nu}{p^2}P_1^{\mu\nu}, \\ B_1 &= \frac{p_\mu p_\nu}{p^2}P_1^{\mu\nu}. \end{aligned} \tag{148}$$

The first part in this expression is of course the transversal part of the propagator. This part is zero if we attach polarization vectors to this propagator and take one of the polarization vectors longitudinal:

$$\begin{aligned} &\varepsilon_{\mathrm{in}\mu}\big(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\big)\varepsilon_{\mathrm{out}\nu} \sim \\ &\varepsilon_{\mathrm{in}\mu}\big(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\big)p_\nu = \\ &\varepsilon_{\mathrm{in}\mu}(p^\mu - p^\mu) = 0. \end{aligned} \tag{149}$$

Now let's calculate A_1 and B_1 for our first diagram.

$$\begin{aligned} A_1 &= -\frac{1}{3}\frac{e^2}{(2\pi)^4}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_e^2+i\varepsilon} \\ &\quad \mathrm{Tr}\big(\gamma_\mu(\not{p}-\not{l}+m_e)\gamma^\mu(-\not{l}+m_e)\big) + \end{aligned}$$

$$
\begin{aligned}
& \frac{4}{3}\frac{e^2}{(2\pi)^4}\frac{1}{p^2}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_e^2+i\varepsilon} \\
& \mathrm{Tr}\big(\not{p}(\not{p}-\not{l}+m_e)\not{p}(-\not{l}+m_e)\big) \\
= \quad & -\frac{1}{3}\frac{e^2}{(2\pi)^4}16m_e^2\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_e^2+i\varepsilon} + \\
& \frac{1}{3}\frac{e^2}{(2\pi)^4}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_e^2+i\varepsilon} \\
& \mathrm{Tr}\big((-\gamma_\mu\gamma^\mu(\not{p}-\not{l})\not{l}+2(\not{p}-\not{l})\not{l}\big) + \\
& \frac{4}{3}\frac{e^2}{(2\pi)^4}4m_e^2\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_e^2+i\varepsilon} + \\
& -\frac{4}{3}\frac{e^2}{(2\pi)^4}\frac{1}{p^2}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_e^2+i\varepsilon} \\
& \mathrm{Tr}\big(-\not{p}\not{p}(\not{p}-\not{l})\not{l}+2p\cdot(p-l)\not{p}\not{l}\big) \\
= \quad & -\frac{8}{3}\frac{e^2}{(2\pi)^4}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_e^2+i\varepsilon}(p-l)\cdot l + \\
& -\frac{4}{3}\frac{e^2}{(2\pi)^4}\frac{1}{p^2}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_e^2+i\varepsilon} \\
& 4(-p^2(p-l)\cdot l+2p\cdot(p-l)p\cdot l)
\end{aligned} \tag{150}
$$

The quadratically divergent part of this expression is:

$$
\begin{aligned}
& -\frac{8}{3}\frac{e^2}{(2\pi)^4}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_e^2+i\varepsilon}l^2 + \\
& \frac{32}{3}\frac{e^2}{(2\pi)^4}\frac{1}{p^2}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_e^2+i\varepsilon}(p\cdot l)^2 \\
= \quad & -\frac{8}{3}\frac{e^2}{(2\pi)^4}\int d^4l \frac{1}{l^2-m_e^2+i\varepsilon} + \\
& \frac{32}{3}\frac{e^2}{(2\pi)^4}\frac{1}{p^2}\int d^4l \frac{1}{(l^2-m_e^2+i\varepsilon)^2}(p\cdot l)^2 \\
= \quad & -\frac{8}{3}\frac{e^2}{(2\pi)^4}\int d^4l \frac{1}{l^2-m_e^2+i\varepsilon} + \\
& \frac{32}{3}\frac{e^2}{(2\pi)^4}\frac{p^\mu p^\nu}{p^2}\int d^4l \frac{l_\mu l_\nu}{(l^2-m_e^2+i\varepsilon)^2} \\
= \quad & -\frac{8}{3}\frac{e^2}{(2\pi)^4}\int d^4l \frac{1}{l^2-m_e^2+i\varepsilon} + \\
& \frac{32}{3}\frac{e^2}{(2\pi)^4}\frac{p^\mu p^\nu}{p^2}\frac{1}{4}\int d^4l \frac{g_{\mu\nu}}{l^2-m_e^2+i\varepsilon} = 0.
\end{aligned} \tag{151}
$$

Here we have expanded the expression around $p = 0$ in the first step and only kept terms which could give quadratic divergences. We see that there are no quadratic divergences in the transversal part of P_1. This is of course known from QED, where the transversal part of the photon propagator is protected against quadratic divergences by gauge invariance. The other part of the photon propagator does have quadratic divergences however, as we shall see below.

$$
\begin{aligned}
B_1 \quad = \quad & -\frac{e^2}{(2\pi)^4}\frac{1}{p^2}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_e^2+i\varepsilon} \\
& \mathrm{Tr}\big(\not{p}(\not{p}-\not{l}+m_e)\not{p}(-\not{l}+m_e)\big) \\
= \quad & -\frac{e^2}{(2\pi)^4}4m_e^2\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_e^2+i\varepsilon} +
\end{aligned}
$$

$$
\begin{aligned}
& \frac{e^2}{(2\pi)^4}\frac{1}{p^2}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_e^2+i\varepsilon} \\
& (-4p^2(p-l)\cdot l + 8(p-l)\cdot pp\cdot l) \\
= \; & 4\frac{e^2}{(2\pi)^4}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_e^2+i\varepsilon}l^2 + \\
& -8\frac{e^2}{(2\pi)^4}\frac{1}{p^2}\int d^4l \frac{1}{(p-l)^2-m_e^2+i\varepsilon}\frac{1}{l^2-m_e^2+i\varepsilon}(p\cdot l)^2 \\
= \; & 4\frac{e^2}{(2\pi)^4}\int d^4l \frac{1}{l^2-m_e^2+i\varepsilon} + \\
& -2\frac{e^2}{(2\pi)^4}\int d^4l \frac{1}{l^2-m_e^2+i\varepsilon} = -\frac{ie^2}{8\pi^2}\Lambda^2
\end{aligned} \tag{152}
$$

Also here we have discarded terms which are not quadratically divergent. We see that this part of the photon propagator does indeed give a quadratic divergence. This divergence should be canceled by the other diagrams in our theory, which we shall consider now.

Consider $P_2^{\mu\nu}$. Also $P_2^{\mu\nu}$ we write as (147) and we calculate A_2 and B_2.

$$
\begin{aligned}
P_2^{\mu\nu} \; = \; & \frac{1}{(2\pi)^4}\int d^4l(-ie)(p-2l)^\mu \frac{i}{(p-l)^2-m_L^2+i\varepsilon} \\
& \frac{1}{l^2-m_L^2+i\varepsilon}(-ie)(p-2l)^\nu \\
= \; & \frac{e^2}{(2\pi)^4}\int d^4l \frac{1}{(p-l)^2-m_L^2+i\varepsilon}\frac{1}{l^2-m_L^2+i\varepsilon} \\
& (p-2l)^\mu(p-2l)^\nu
\end{aligned} \tag{153}
$$

$$
\begin{aligned}
A_2 \; = \; & \frac{1}{3}\frac{e^2}{(2\pi)^4}\int d^4l \frac{1}{(p-l)^2-m_L^2+i\varepsilon}\frac{1}{l^2-m_L^2+i\varepsilon}(p-2l)^2 + \\
& -\frac{4}{3}\frac{e^2}{(2\pi)^4}\frac{1}{p^2}\int d^4l \frac{1}{(p-l)^2-m_L^2+i\varepsilon}\frac{1}{l^2-m_L^2+i\varepsilon} \\
& (p-2l)\cdot p(p-2l)\cdot p \\
= \; & \frac{4}{3}\frac{e^2}{(2\pi)^4}\int d^4l \frac{1}{(p-l)^2-m_L^2+i\varepsilon}\frac{1}{l^2-m_L^2+i\varepsilon}l^2 + \\
& -\frac{16}{3}\frac{e^2}{(2\pi)^4}\frac{1}{p^2}\int d^4l \frac{1}{(p-l)^2-m_L^2+i\varepsilon}\frac{1}{l^2-m_L^2+i\varepsilon}(p\cdot l)^2 \\
= \; & \frac{4}{3}\frac{e^2}{(2\pi)^4}\int d^4l \frac{1}{l^2-m_L^2+i\varepsilon} + \\
& -\frac{4}{3}\frac{e^2}{(2\pi)^4}\int d^4l \frac{1}{l^2-m_L^2+i\varepsilon} = 0
\end{aligned} \tag{154}
$$

$$
\begin{aligned}
B_2 \; = \; & \frac{e^2}{(2\pi)^4}\frac{1}{p^2}\int d^4l \frac{1}{(p-l)^2-m_L^2+i\varepsilon}\frac{1}{l^2-m_L^2+i\varepsilon}(p-2l)\cdot p(p-2l)\cdot p \\
= \; & 4\frac{e^2}{(2\pi)^4}\frac{1}{p^2}\int d^4l \frac{1}{(p-l)^2-m_L^2+i\varepsilon}\frac{1}{l^2-m_L^2+i\varepsilon}(p\cdot l)^2 \\
= \; & \frac{e^2}{(2\pi)^4}\int d^4l \frac{1}{l^2-m_L^2+i\varepsilon} = -\frac{ie^2}{16\pi^2}\Lambda^2
\end{aligned} \tag{155}
$$

We see that the transversal part of $P_2^{\mu\nu}$ again has no quadratic divergences. This is also known from scalar QED. There one has only $P_2^{\mu\nu}$ and $P_4^{\mu\nu}$ as contributions to the photon propagator. As we will see in a moment $P_4^{\mu\nu}$ has no transversal part at all, so that in scalar QED the total transversal part of the photon propagator is free of quadratic divergences. Again this is because of the gauge invariance of scalar QED. We see however that B_2 does have a quadratic divergence.

Now we know $P_2^{\mu\nu}$, A_2 and B_2 the quantities $P_3^{\mu\nu}$, A_3 and B_3 are easily found:

$$\begin{aligned} P_3^{\mu\nu} &= \frac{e^2}{(2\pi)^4}\int d^4l \frac{1}{(p-l)^2-m_R^2+i\varepsilon}\frac{1}{l^2-m_R^2+i\varepsilon} \\ &\quad (p-2l)^\mu(p-2l)^\nu, && (156) \\ A_3 &= 0, && (157) \\ B_3 &= -\frac{ie^2}{16\pi^2}\Lambda^2. && (158) \end{aligned}$$

Here the expression for $P_{3\mu\nu}$ is exact (at 1 loop order), but the expressions given for A_3 and B_3 again only contain the quadratically divergent parts.

Finally we calculate $P_4^{\mu\nu}$, A_4 and B_4 and $P_5^{\mu\nu}$, A_5 and B_5.

$$\begin{aligned} P_4^{\mu\nu} &= \frac{1}{(2\pi)^4}\int d^4l 2ie^2 g^{\mu\nu}\frac{i}{l^2-m_L^2+i\varepsilon} \\ &= -\frac{2e^2}{(2\pi)^4}\int d^4l \frac{1}{l^2-m_L^2+i\varepsilon}g^{\mu\nu} && (159) \\ A_4 &= 0 && (160) \\ B_4 &= -\frac{2e^2}{(2\pi)^4}\int d^4l\frac{1}{l^2-m_L^2+i\varepsilon} = \frac{ie^2}{8\pi^2}\Lambda^2 && (161) \end{aligned}$$

In this case A_4 and B_4 could simply be read off from $P_{4\mu\nu}$.

$$\begin{aligned} P_5^{\mu\nu} &= \frac{1}{(2\pi)^4}\int d^4l 2ie^2 g^{\mu\nu}\frac{i}{l^2-m_R^2+i\varepsilon} \\ &= -\frac{2e^2}{(2\pi)^4}\int d^4l \frac{1}{l^2-m_R^2+i\varepsilon}g^{\mu\nu} && (162) \\ A_5 &= 0 && (163) \\ B_5 &= -\frac{2e^2}{(2\pi)^4}\int d^4l\frac{1}{l^2-m_R^2+i\varepsilon} = \frac{ie^2}{8\pi^2}\Lambda^2 && (164) \end{aligned}$$

Now we can finally check whether the quadratic divergences actually cancel. We saw already that these quadratic divergences are only present in the B's. So their sum should be free of quadratic divergences.

$$B_1 + \ldots + B_5 = \Big(-\frac{1}{8}-\frac{1}{16}-\frac{1}{16}+\frac{1}{8}+\frac{1}{8}\Big)\frac{ie^2}{\pi^2}\Lambda^2 = 0 \qquad (165)$$

And indeed there are completely no quadratic divergences in the photon propagator. Here one can see how supersymmetry works when canceling all quadratic divergences. The quadratic divergence of the electron loop is canceled by the quadratic divergences from the 4 selectron loops. Also one sees here that one really needs 2 selectrons, called left and right, to cancel the quadratic divergence from the electron loop.

3.13 Higher order propagators

Now we have found that the constraints for our coupling constants from propagators at order e^2 are not enough to fix our theory we should wonder how to proceed. Roughly there are two possibilities which are practically possible. A first option would be considering diagrams with more than 4 external legs at tree level. Remember that we already considered all diagrams with 4 or less external legs. A second option would be considering the propagators at higher order than e^2, thus with more than one loop.

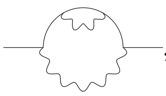
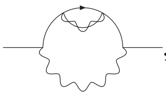
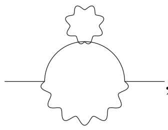
The first option might enable us to fix coupling constants with the constraint of gauge invariance. There is however not much hope here since all the vertices with a photon have already been fixed. It is far-fetched to expect we can fix other vertices with this constraint. The second option will probably give constraints if we again impose absence of quadratic divergences on all propagators. The calculations will be large however.

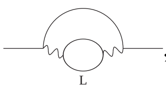
Note that considering processes with more than 2 external legs at 1-loop order and imposing there are no quadratic divergences will not give any constraints. Such processes have no quadratic divergences at all which can be verified by simple power counting. The more external legs we attach, the lesser the degree of divergence will become.

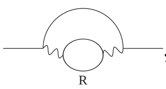
The best option thus is to consider the propagators of the previous sections again, now at higher order, and again demand that quadratic (or worse) divergences are absent. We will start again with the propagator of the left selectron.

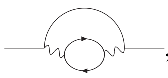
3.14 The left selectron propagator at order e^4

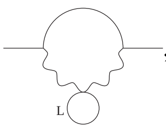
In this section we have the task of doing the rather large calculation of the left selectron propagator at two loop order. Of course we start by finding all contributing diagrams. Since there are so many of these this calls for a systematic approach. We have used the Schwinger Dyson equations to find all diagrams. With this approach one automatically finds the symmetry factors accompanying every diagram. We will not outline all the steps in this lengthy procedure here, but simply state the diagrams we found. With the systematic approach just outlined one will find all diagrams, including tadpole diagrams. These are, and should be, zero and can thus be left out. Also we will find all 1-particle reducible diagrams which can be built from diagrams of propagators at order e^2. These diagrams do contribute, but their sum will never give quadratic divergences anymore, since we already made sure these vanished at order e^2. So we only have to look at all the 1-particle irreducible diagrams. These are drawn below. Notice we did not include L's and R's on the solid lines everywhere. Only where the identity (left or right selectron) of the solid lines is not fixed uniquely we included an L or R.

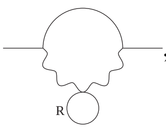
$P_1 =$ , $P_2 =$ , $P_3 = \frac{1}{2}$ ,

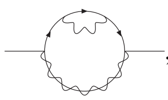
$P_4 =$ 

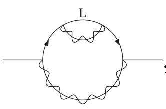
, $P_5 =$ 

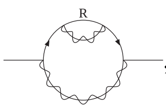
, $P_6 =$ ,

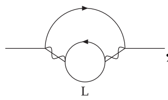
$P_7 =$ 

, $P_8 =$ 

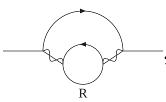
, $P_9 =$ ,

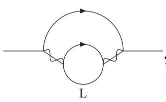
$P_{10} =$ 

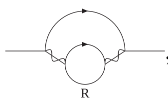
, $P_{11} =$ 

, $P_{12} =$ 

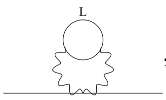
,

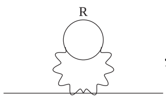
$P_{13} =$ 

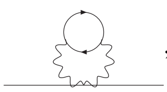
, $P_{14} =$ 

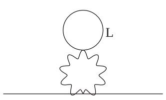
, $P_{15} =$ 

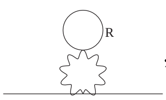
,

$P_{16} = \frac{1}{2}$ 

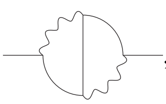
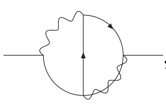
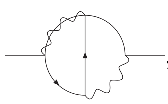
, $P_{17} = \frac{1}{2}$ 

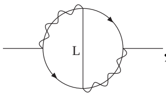
, $P_{18} = \frac{1}{2}$ ,

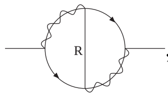
$P_{19} = \frac{1}{2}$ 

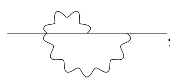
$P_{20} = \frac{1}{2}$ 

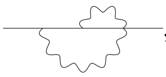
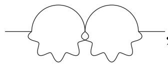
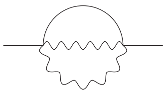
,

$P_{21} =$ , $P_{22} =$ , $P_{23} =$ ,

$P_{24} =$ 

, $P_{25} =$ 

, $P_{26} =$ ,

$P_{27} =$ , $P_{28} =$ , $P_{29} = \frac{1}{2}$  (166)

Now we should calculate the quadratically divergent parts of all these diagrams. Before we do this however we should have a closer look at all 1-particle irreducible diagrams to see if we can discard some diagrams immediately. The 1PI diagrams can be divided into two groups. One group contains all diagrams which have a 1-loop propagator diagram as a subdiagram, the other group contains the rest. In the first group of diagrams there do occur quadratic divergences, since we saw earlier that these occur in 1-loop propagator diagrams. However these quadratic divergences coming only from the loop of the subdiagram which also is a propagator diagram at 1-loop order need not be considered, since these already vanish at 1-loop order (if we impose the constraints we found). So from the first group of diagrams not all quadratic divergences are interesting to us. Only the quadratic divergences which are the result of the integration of the loop momentum of the main loop we should consider. How this works in detail can later be seen in the explicit calculations. The situation just mentioned is especially convenient in the case the subdiagram which is a propagator diagram with one loop contains only the 4-vertex where 2 selectrons couple to 2 photons. Since the loop momentum of the main loop never occurs in the expression belonging to the subdiagram for this particular vertex these 2-loop diagrams give two independent integrals. The integral over the loop momentum of the loop in the subdiagram gives the quadratic divergence already encountered at 1-loop order. The integral over the main loop momentum never gives a quadratic divergence, but at most a logarithmic one. This last allegation can be seen almost immediately by counting the powers of the main loop momentum. So the diagrams which have a propagator diagram as a subdiagram and which contain the 4-vertex can be neglected completely.

In the current calculation this amounts to discarding the diagrams P_3, P_7, P_8, P_{19} and P_{20}. In later calculations which will contain more diagrams it will become even more important. Now we can start calculating P_1.

$$
\begin{aligned}
P_1 \quad = \quad & \frac{1}{(2\pi)^8}\int d^4l_1 d^4l_2 (-ie)(p+l_1)^\mu \frac{i}{l_1^2 - m_L^2 + i\varepsilon}\frac{-ig_{\mu\nu}}{(p-l_1)^2 + i\varepsilon} \\
& (-ie)(2l_1 - l_2)^\alpha \frac{i}{(l_1-l_2)^2 - m_L^2 + i\varepsilon}\frac{-ig_{\alpha\beta}}{l_2^2 + i\varepsilon} \\
& (-ie)(2l_1 - l_2)^\beta \frac{i}{l_1^2 - m_L^2 + i\varepsilon}(-ie)(p+l_1)^\nu = \\
& \frac{ie^4}{(2\pi)^8}\int d^4l_1 d^4l_2 \frac{1}{(p-l_1)^2 + i\varepsilon}\frac{1}{(l_1^2 - m_L^2 + i\varepsilon)^2}\frac{1}{l_2^2 + i\varepsilon} \\
& \frac{1}{(l_1-l_2)^2 - m_L^2 + i\varepsilon}(p+l_1)^2(2l_1 - l_2)^2
\end{aligned} \tag{167}
$$

To find the quadratically divergent parts we expand all fractions around $p = 0$. In this way we can get rid of all p's in the denominator of the integrand. By counting the powers of the loop momentum l_1 in the integrand it is easily verified that only the first term in the expansion can possibly give a quadratic divergence resulting from the integral over the main loop momentum l_1. Higher order terms would be important if we were interested in the quadratic divergence resulting from the subdiagram. There these higher order terms contribute to the coefficient in front of such a quadratic divergence. We are however not interested in the exact form of this coefficient and will therefore discard all higher order terms. We get:

$$\begin{aligned}
P_1 &= \frac{ie^4}{(2\pi)^8}\int d^4l_1 d^4l_2 \frac{1}{l_1^2+i\varepsilon}\frac{1}{(l_1^2-m_L^2+i\varepsilon)^2}\frac{1}{l_2^2+i\varepsilon} \\
&\quad \frac{1}{(l_1-l_2)^2-m_L^2+i\varepsilon}(p+l_1)^2(2l_1-l_2)^2 \\
&= \frac{ie^4}{(2\pi)^8}p^2\int d^4l_1 d^4l_2 \frac{1}{l_1^2+i\varepsilon}\frac{1}{(l_1^2-m_L^2+i\varepsilon)^2}\frac{1}{l_2^2+i\varepsilon} \\
&\quad \frac{1}{(l_1-l_2)^2-m_L^2+i\varepsilon}(2l_1-l_2)^2+ \\
&\quad \frac{ie^4}{(2\pi)^8}\int d^4l_1 d^4l_2 \frac{1}{l_1^2+i\varepsilon}\frac{1}{(l_1^2-m_L^2+i\varepsilon)^2}\frac{1}{l_2^2+i\varepsilon} \\
&\quad \frac{1}{(l_1-l_2)^2-m_L^2+i\varepsilon}2p\cdot l_1(2l_1-l_2)^2+ \\
&\quad \frac{ie^4}{(2\pi)^8}\int d^4l_1 d^4l_2 \frac{1}{(l_1^2-m_L^2+i\varepsilon)^2}\frac{1}{l_2^2+i\varepsilon} \\
&\quad \frac{1}{(l_1-l_2)^2-m_L^2+i\varepsilon}(2l_1-l_2)^2 \\
&= \frac{ie^4}{(2\pi)^8}p^2\int d^4l_1 d^4l_2 \frac{1}{l_1^2+i\varepsilon}\frac{1}{(l_1^2-m_L^2+i\varepsilon)^2}\frac{1}{l_2^2+i\varepsilon} \\
&\quad \frac{1}{(l_1-l_2)^2-m_L^2+i\varepsilon}(2l_1-l_2)^2+ \\
&\quad \frac{ie^4}{(2\pi)^8}\int d^4l_1 d^4l_2 \frac{1}{(l_1^2-m_L^2+i\varepsilon)^2}\frac{1}{l_2^2+i\varepsilon} \\
&\quad \frac{1}{(l_1-l_2)^2-m_L^2+i\varepsilon}(2l_1-l_2)^2.
\end{aligned} \tag{168}$$

Now use:

$$(2l_1-l_2)^2 = 2\big((l_1-l_2)^2-m_L^2+i\varepsilon\big)+2(l_1^2-m_L^2+i\varepsilon)-(l_2^2+i\varepsilon)+4m_L^2-3i\varepsilon. \tag{169}$$

Then P_1 becomes:

$$\begin{aligned}
P_1 &= 2\frac{ie^4}{(2\pi)^8}p^2\int d^4l_1 d^4l_2 \frac{1}{l_1^2+i\varepsilon}\frac{1}{(l_1^2-m_L^2+i\varepsilon)^2}\frac{1}{l_2^2+i\varepsilon}+ \\
&\quad 2\frac{ie^4}{(2\pi)^8}p^2\int d^4l_1 d^4l_2 \frac{1}{l_1^2+i\varepsilon}\frac{1}{l_1^2-m_L^2+i\varepsilon}\frac{1}{l_2^2+i\varepsilon}\frac{1}{(l_1-l_2)^2-m_L^2+i\varepsilon}- \\
&\quad \frac{ie^4}{(2\pi)^8}p^2\int d^4l_1 d^4l_2 \frac{1}{l_1^2+i\varepsilon}\frac{1}{(l_1^2-m_L^2+i\varepsilon)^2}\frac{1}{(l_1-l_2)^2-m_L^2+i\varepsilon}+ \\
&\quad 4\frac{ie^4}{(2\pi)^8}m_L^2p^2\int d^4l_1 d^4l_2 \frac{1}{l_1^2+i\varepsilon}\frac{1}{(l_1^2-m_L^2+i\varepsilon)^2}\frac{1}{l_2^2+i\varepsilon} \\
&\quad \frac{1}{(l_1-l_2)^2-m_L^2+i\varepsilon}+ \\
&\quad 2\frac{ie^4}{(2\pi)^8}\int d^4l_1 d^4l_2 \frac{1}{(l_1^2-m_L^2+i\varepsilon)^2}\frac{1}{l_2^2+i\varepsilon}+ \\
&\quad 2\frac{ie^4}{(2\pi)^8}\int d^4l_1 d^4l_2 \frac{1}{l_1^2-m_L^2+i\varepsilon}\frac{1}{l_2^2+i\varepsilon}\frac{1}{(l_1-l_2)^2-m_L^2+i\varepsilon}-
\end{aligned}$$

$$\frac{ie^4}{(2\pi)^8}\int d^4l_1d^4l_2\frac{1}{(l_1^2-m_L^2+i\varepsilon)^2}\frac{1}{(l_1-l_2)^2-m_L^2+i\varepsilon}+$$
$$4\frac{ie^4}{(2\pi)^8}m_L^2\int d^4l_1d^4l_2\frac{1}{(l_1^2-m_L^2+i\varepsilon)^2}\frac{1}{l_2^2+i\varepsilon}$$
$$\frac{1}{(l_1-l_2)^2-m_L^2+i\varepsilon}. \tag{170}$$

In appendix 3 integrals like the ones occurring in (170) are calculated. Note that we rewrote all the terms to a form where there are no loop momenta present in the numerator anymore. From appendix 3 we know that the second, fourth and eighth term will never give a quadratic divergence. So these terms can be discarded. The fifth and seventh term only give a quadratic divergence due to the l_2 integral. The sum of these two gives the quadratic divergence already encountered at 1-loop order. Also the coefficient is consistent with the coefficient we found at 1-loop order. The same holds for the first and third term. Also these give rise to a quadratic divergence due to the l_2 integral. The l_1 integral is different, this comes from the fact that we have expanded around $p=0$. Thus also the first, third, fifth and seventh term are unimportant to us since the quadratically divergent part present in these terms will cancel against the quadratic divergences of subdiagrams present in diagrams similar to P_1 (like for example P_2).

Thus the only important term in P_1 is:

$$P_1=2\frac{ie^4}{(2\pi)^8}\int d^4l_1d^4l_2\frac{1}{l_1^2-m_L^2+i\varepsilon}\frac{1}{l_2^2+i\varepsilon}\frac{1}{(l_1-l_2)^2-m_L^2+i\varepsilon}. \tag{171}$$

From appendix 3 we know that this expression indeed has a quadratically divergent part which comes from both the l_1 and the l_2 integral. Also we know that the masses present in the denominators of the integrand will not be important for the quadratically divergent part and thus only the coefficient multiplying the integral over the two loop momenta is of importance to us. When we also leave out the factor $\frac{1}{(2\pi)^8}$ which is present in all diagrams the multiplying coefficient for P_1 is $2ie^4$. For convenience we will henceforth write:

$$P_1=2ie^4. \tag{172}$$

All the expressions for the diagrams can be calculated in this way. From the calculation of P_1 we see that it isn't even necessary to take along all masses and the external momentum p, since these will not occur in the quadratically divergent term. To shorten the calculations of the expressions drastically we might thus set them to zero. Of course this method is not very elegant mathematically since we get infrared divergences which aren't present in the exact expression. Since we aren't interested in these anyway and always discard such infrared divergences the method is good enough for our purposes. To see how it works let us calculate P_2 in this short manner.

$$\begin{aligned}
P_2 &= -\frac{1}{(2\pi)^8}\int d^4l_1d^4l_2(-ie)l_1^\mu\frac{i}{l_1^2}\mathrm{Tr}\Big((-1)(S_L^*-P_L^*\gamma^5)\frac{-i\not{l}_2}{l_2^2}(S_L+P_L\gamma^5)\\
&\quad \frac{i(\not{l}_1-\not{l}_2)}{(l_1-l_2)^2}\Big)\frac{i}{l_1^2}(-ie)l_1^\nu\frac{-ig_{\mu\nu}}{l_1^2}\\
&= -\frac{ie^2}{(2\pi)^8}\int d^4l_1d^4l_2\frac{1}{(l_1^2)^2}\frac{1}{l_2^2}\frac{1}{(l_1-l_2)^2}\\
&\quad \mathrm{Tr}\big((S_L^*-P_L^*\gamma^5)\not{l}_2(S_L+P_L\gamma^5)(\not{l}_1-\not{l}_2)\big)\\
&= -\frac{ie^2}{(2\pi)^8}\int d^4l_1d^4l_2\frac{1}{(l_1^2)^2}\frac{1}{l_2^2}\frac{1}{(l_1-l_2)^2}4(|S_L|^2+|P_L|^2)l_2\cdot(l_1-l_2)\\
&= 2\frac{ie^2}{(2\pi)^8}(|S_L|^2+|P_L|^2)\int d^4l_1d^4l_2\frac{1}{(l_1^2)^2}\frac{1}{l_2^2}+\\
&\quad 2\frac{ie^2}{(2\pi)^8}(|S_L|^2+|P_L|^2)\int d^4l_1d^4l_2\frac{1}{(l_1^2)^2}\frac{1}{(l_1-l_2)^2}-
\end{aligned}$$

$$2\frac{ie^2}{(2\pi)^8}(|S_L|^2+|P_L|^2)\int d^4l_1d^4l_2\frac{1}{l_1^2}\frac{1}{l_2^2}\frac{1}{(l_1-l_2)^2} \tag{173}$$

The first two terms we recognize as the terms with the quadratic divergences coming from the l_2 loop. Also here the total coefficient of these two terms matches with what we already saw at 1-loop order. The last term is the important one for us, its coefficient being $-2ie^2(|S_L|^2+|P_L|^2)$. Again, for bookkeeping purposes, we write this conveniently as:

$$P_2 = -2ie^2(|S_L|^2+|P_L|^2). \tag{174}$$

It will not surprise the reader that all the expressions can be calculated like this. We will of course not present all these calculations here, but we will give a final example. Consider P_{16}.

$$\begin{aligned}
P_{16} &= \frac{1}{(2\pi)^8}\int d^4l_1d^4l_2(-ie)(-l_1)^\mu\frac{i}{l_1^2}\frac{-ig_{\mu\nu}}{l_1^2}(-ie)(-2l_1-l_2)^\alpha\frac{i}{(l_1+l_2)^2} \\
&\quad (-ie)(-l_1-2l_2)^\nu\frac{i}{l_2^2}\frac{-ig_{\alpha\beta}}{l_2^2}(-ie)(-l_2)^\beta \\
&= \frac{ie^4}{(2\pi)^8}\int d^4l_1d^4l_2\frac{1}{(l_1^2)^2}\frac{1}{(l_2^2)^2}\frac{1}{(l_1+l_2)^2}l_1\cdot(l_1+2l_2)l_2\cdot(2l_1+l_2) \\
&= \frac{ie^4}{(2\pi)^8}\int d^4l_1d^4l_2\frac{1}{(l_1^2)^2}\frac{1}{(l_2^2)^2}l_2\cdot(2l_1+l_2) - \\
&\quad \frac{ie^4}{(2\pi)^8}\int d^4l_1d^4l_2\frac{1}{(l_1^2)^2}\frac{1}{l_2^2}\frac{1}{(l_1+l_2)^2}l_2\cdot(2l_1+l_2) \\
&= \frac{ie^4}{(2\pi)^8}\int d^4l_1d^4l_2\frac{1}{(l_1^2)^2}\frac{1}{l_2^2} - \\
&\quad \frac{ie^4}{(2\pi)^8}\int d^4l_1d^4l_2\frac{1}{(l_1^2)^2}\frac{1}{l_2^2} + \\
&\quad \frac{ie^4}{(2\pi)^8}\int d^4l_1d^4l_2\frac{1}{l_1^2}\frac{1}{l_2^2}\frac{1}{(l_1-l_2)^2} \\
&= \frac{ie^4}{(2\pi)^8}\int d^4l_1d^4l_2\frac{1}{l_1^2}\frac{1}{l_2^2}\frac{1}{(l_1-l_2)^2}
\end{aligned} \tag{175}$$

So we see:

$$P_{16} = ie^4. \tag{176}$$

We also see that there are no quadratic divergences due to subdiagrams of P_{16}. This could be expected since all subdiagrams of P_{16} have three external legs and will therefore not give any quadratic divergences.

Below we list all the coefficients in front of the important quadratically divergent terms for all the diagrams.

$$\begin{aligned}
P_1 &= 2ie^4 \\
P_2 &= -2ie^2(|S_L|^2+|P_L|^2) \\
P_3 &= 0 \\
P_4 &= 0 \\
P_5 &= 0 \\
P_6 &= 0 \\
P_7 &= 0 \\
P_8 &= 0 \\
P_9 &= -4ie^2(|S_L|^2+|P_L|^2) \\
P_{10} &= -2i\big((|S_L|^2+|P_L|^2)^2+(S_L^*P_L+S_LP_L^*)^2\big)
\end{aligned}$$

$$
\begin{aligned}
P_{11} &= -2i\big((|S_L|^2+|P_L|^2)(|S_R|^2+|P_R|^2)+(S_L^*P_L+S_LP_L^*)(S_R^*P_R+S_RP_R^*)\big)\\
P_{12} &= -2i\big((|S_L|^2+|P_L|^2)^2+(S_L^*P_L+S_LP_L^*)^2\big)\\
P_{13} &= -2i\big((|S_L|^2+|P_L|^2)(|S_R|^2+|P_R|^2)+(S_L^*P_L+S_LP_L^*)(S_R^*P_R+S_RP_R^*)\big)\\
P_{14} &= -2i\big((|S_L|^2+|P_L|^2)^2-(S_L^*P_L+S_LP_L^*)^2\big)\\
P_{15} &= -2i\big((|S_L|^2+|P_L|^2)(|S_R|^2+|P_R|^2)-(S_L^*P_L+S_LP_L^*)(S_R^*P_R+S_RP_R^*)\big)\\
P_{16} &= ie^4\\
P_{17} &= ie^4\\
P_{18} &= 4ie^4\\
P_{19} &= 0\\
P_{20} &= 0\\
P_{21} &= ie^4\\
P_{22} &= 2ie^2(|S_L|^2+|P_L|^2)\\
P_{23} &= 2ie^2(|S_L|^2+|P_L|^2)\\
P_{24} &= -4i\big((|S_L|^2-|P_L|^2)^2-(S_L^*P_L-S_LP_L^*)^2\big)\\
P_{25} &= -4i\big((|S_L|^2-|P_L|^2)(|S_R|^2-|P_R|^2)-(S_L^*P_L-S_LP_L^*)(S_R^*P_R-S_RP_R^*)\big)\\
P_{26} &= -3ie^4\\
P_{27} &= -3ie^4\\
P_{28} &= 0\\
P_{29} &= 8ie^4 \qquad (177)
\end{aligned}
$$

If we want no quadratic divergences the sum of all these coefficients should be zero. This gives us the equation:

$$
\begin{aligned}
&11ie^4-2ie^2|S_L|^2-2ie^2|P_L|^2-10i|S_L|^4-10i|P_L|^4-16i|S_L|^2|P_L|^2+\\
&2i(S_L^*)^2P_L^2+2iS_L^2(P_L^*)^2-10i|S_L|^2|S_R|^2-10i|P_L|^2|P_R|^2+\\
&-2i|S_L|^2|P_R|^2-2i|P_L|^2|S_R|^2+2iS_L^*P_LS_R^*P_R+2iS_LP_L^*S_RP_R^*+\\
&-6iS_L^*P_LS_RP_R^*-6iS_LP_L^*S_R^*P_R=0. \qquad (178)
\end{aligned}
$$

So finally we found the constraint that should be satisfied if no quadratic divergences ought to occur in the left selectron propagator. We could now proceed to consider other propagators, but, as we will see in the next section, there is first another problem to be dealt with.

3.15 A problem

Consider the equations we found by imposing that there are no quadratic divergences at 1-loop order:

$$\begin{aligned} 3e^2 - 4|S_L|^2 - 4|P_L|^2 &= 0 \\ 3e^2 - 4|S_R|^2 - 4|P_R|^2 &= 0 \\ S_L S_R^* + P_L P_R^* &= 0. \end{aligned} \tag{179}$$

From the second equation we see that

$$|S_R|^2 = \frac{3}{4}e^2 - |P_R|^2. \tag{180}$$

From the last one of the three equations it follows that

$$P_L = -\frac{S_L S_R^*}{P_R^*}. \tag{181}$$

Substituting both relations in the first equation gives:

$$\begin{aligned} 3e^2 - 4|S_L|^2\Big(1 + \frac{\frac{3}{4}e^2 - |P_R|^2}{|P_R|^2}\Big) &= 3e^2\Big(1 - \frac{|S_L|^2}{|P_R|^2}\Big) = 0 \\ &\Rightarrow |S_L| = |P_R|. \end{aligned} \tag{182}$$

From (181) it then also follows that

$$|S_R| = |P_L|. \tag{183}$$

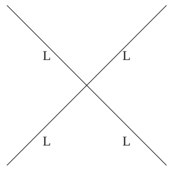
Now we use all these results in the 2-loop condition (178). The left hand side of (178) then reduces to:

$$\begin{aligned} &11ie^4 - 2ie^2(\frac{3}{4}e^2) - 10i(|S_L|^2 + |P_L|^2)^2 + 4i|S_L|^2|P_L|^2 + \\ &2i(S_L^*)^2P_L^2 + 2iS_L^2(P_L^*)^2 - 10i|P_R|^2(|S_R|^2 + |P_L|^2) - 2i|S_L|^4 - 2i|P_L|^4 + \\ &2iS_L^*S_R^*P_LP_R + 2iS_LS_RP_L^*P_R^* + 12i|P_L|^2|P_R|^2 = \\ &11ie^4 - 2ie^2(\frac{3}{4}e^2) - 10i(\frac{3}{4}e^2)^2 - 4i|S_L|^2|P_L|^2 - 2i|S_L|^4 - 2i|P_L|^4 + \\ &2i(S_L^*)^2P_L^2 + 2iS_L^2(P_L^*)^2 + 2iS_L^*S_R^*P_LP_R + 2iS_LS_RP_L^*P_R^* = \\ &11ie^4 - 2ie^2(\frac{3}{4}e^2) - 10i(\frac{3}{4}e^2)^2 - 2i(\frac{3}{4}e^2)^2 + \\ &2i(S_L^*)^2P_L^2 + 2iS_L^2(P_L^*)^2 + 2iS_L^*S_R^*P_LP_R + 2iS_LS_RP_L^*P_R^* = \\ &2\frac{3}{4}ie^4 + 2i\frac{(S_L^*)^2S_L^2(S_R^*)^2}{(P_R^*)^2} + 2i\frac{S_L^2(S_L^*)^2S_R^2}{P_R^2} - 2i\frac{S_L^*S_L(S_R^*)^2P_R}{P_R^*} - 2i\frac{S_LS_L^*S_R^2P_R^*}{P_R} = \\ &2\frac{3}{4}ie^4 + 2i|S_L|^4\Big(\frac{(S_R^*)^2}{(P_R^*)^2} + \frac{S_R^2}{P_R^2}\Big) - 2i|S_L|^2\Big(\frac{(S_R^*)^2P_R}{P_R^*} + \frac{S_R^2P_R^*}{P_R}\Big) = \\ &2\frac{3}{4}ie^4 + 2i\big((S_R^*)^2P_R^2 + S_R^2(P_R^*)^2\big) - 2i\big((S_R^*)^2P_R^2 + S_R^2(P_R^*)^2\big) = \\ &2\frac{3}{4}ie^4. \end{aligned} \tag{184}$$

This should be zero if we don't want any quadratic divergences in the left selectron propagator at 2-loop order. This is clearly impossible. A way out of this is to introduce new vertices, which will be done in the next section.

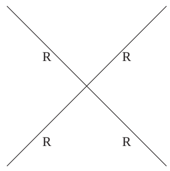
3.16 More 4-vertices

In the previous section we saw that we really need more vertices which generate extra diagrams to cancel all quadratic divergences. Since we already have taken along all possible 3-vertices we should now look for new 4-vertices. Since things went wrong in selectron propagators, as we saw above, it seems logical to introduce 4-vertices with only selectrons. There are of course many possibilities for such vertices. We should however introduce vertices which give new diagrams which contribute to the processes we considered thus far, else we can still not repair our problem. A 4-vertex with only left selectrons would certainly generate diagrams which contribute in the processes we considered. So we will introduce this vertex:



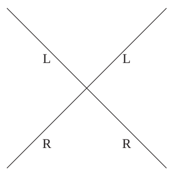
$$= V_{LLLL}.$$

Since there only coincide scalar particles in this vertex the vertex expression is just a constant, which we already called V_{LLLL}. One can expect that when one needs this vertex one also needs the same vertex, but with only right selectrons:



$$= V_{RRRR}.$$

Also we will introduce the following vertex.



$$= V_{LRLR}$$

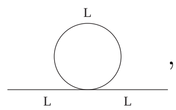
Its necessity can't be seen right away since we are dealing with rather complicated equations here. However if it appears in the future that we do not really need this vertex we can always set it to zero without any problems.

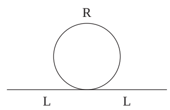
These are for the moment all the new vertices we introduce. Also we will not need more vertices as we will see below. This last allegation is of course also known from the conventional derivation. We should now see what our new constraints become. At 1-loop order it is easy to see this but at 2-loop order it is more work to do this. Also we haven't calculated the quadratically divergent parts of all propagators at 2-loop order yet. To do all these calculations by hand would be a lot of work and very sensitive to errors. In the next sections we will solve this problem by letting the computer do the calculating. We will write a `FORM` program which can calculate the quadratical divergences of all propagators at 2-loop order.

Before we do this however we can say something more about the constants V_{LLLL}, V_{RRRR} and V_{LRLR}. Because the Lagrangian density should be Hermitian these constants have to be purely imaginary. This is deduced in the same way as was done for the electron selectron photino couplings. The only difference is that there 2 different vertices were each other's Hermitian conjugated. In the case with the new 4-vertices the only possibility to make the Lagrangian density Hermitian is that all 3 4-vertices are their own Hermitian conjugated. For this reason then all the V's should be purely imaginary (not purely real, since one gets an extra i when going from the Lagrangian density to the Feynman rules).

3.17 The new constraints at 1-loop order

The extra diagrams we get because of the new 4-vertices at 1-loop order are easily found. For the left selectron propagator we get the following 2 extra diagrams.

$P_4 =$ 

, $P_5 =$ 

(185)

Also the expressions belonging to these diagrams follow immediately.

$$\begin{aligned} P_4 &= V_{LLLL}\frac{1}{(2\pi)^4}\int d^4l \frac{i}{l^2 - m_L^2 + i\varepsilon} = V_{LLLL}\frac{1}{16\pi^2}\Lambda^2 \\ P_5 &= V_{LRLR}\frac{1}{(2\pi)^4}\int d^4l \frac{i}{l^2 - m_R^2 + i\varepsilon} = V_{LRLR}\frac{1}{16\pi^2}\Lambda^2 \end{aligned} \tag{186}$$

With these 2 extra quadratically divergent terms our old constraint (132) changes to:

$$3e^2 - 4(|S_L|^2 + |P_L|^2) + iV_{LLLL} + iV_{LRLR} = 0. \tag{187}$$

The calculation for the right selectron propagator is of course similar, there we get the new constraint:

$$3e^2 - 4(|S_R|^2 + |P_R|^2) + iV_{RRRR} + iV_{LRLR} = 0. \tag{188}$$

The constraint for the process $\tilde{e}_L \to \tilde{e}_R$ does not change. It stays:

$$S_L^* S_R + P_L^* P_R = 0. \tag{189}$$

We simply cannot make new contributing diagrams from the new 4-vertices for this process (at least not at 1-loop order). Also, for the same reason, the electron propagator, the photino propagator and the photon propagator do not change. The first two still have no quadratic divergences at all, whereas quadratic divergences do occur in individual diagrams of the photon propagator, but they nicely cancel, as we saw before.

3.18 The new constraints at 2-loop order

Doing the 2-loop order calculation of the left selectron propagator again and calculating the other propagators at 2-loop order by hand is quite a lot of work and very sensitive to mistakes. Therefore, as mentioned earlier, we have written a `FORM` program which can do the calculation for us. This program, called `LOOPS.frm` can be seen in appendix 4. Here we will briefly outline how it works. A central object used is the function D. This function D is equal to the sum of all diagrams of a certain process at a certain order in the coupling constant e. With all diagrams we really mean all diagrams, the 1-particle irreducible, the 1-particle reducible and even tadpole diagrams are included in D. The first argument of D specifies the order in e. The other arguments specify the process itself. They define the external legs of the process. The x's in D are used to separate the arguments that denote the different types of external lines. The arguments after the first x for example denote the momentum, Dirac index and chosen fermion flow of electron lines with their arrow pointed into the diagram. The fermion flow argument is necessary because of the special Feynman rules for Majorana fermions like the photino.

The program starts with one function D, in which the user can specify the process he wants to calculate. This D is then iterated by the computer using the Schwinger Dyson equations. When this lengthy procedure is complete there are a lot of D's at order 0 left. These should be replaced by the proper (zero order) propagators. Also taken into account here is that external lines can be tied in several ways, giving different diagrams. Also the momenta which are then tied together should be set equal to each other, which is done with help of the function delta. After this procedure products of γ-matrices are worked out and traces are taken where needed.

The part of the program `LOOPS` described thus far is quite general. The second part is especially designed for a 2-loop process since it would be very difficult to make this part suited for arbitrary order. Here the integrals, or better said the integrands, since `LOOPS` only deals with those, are rewritten. Eventually the quadratically divergent parts can then be identified and the other parts can be discarded. In this way one rewrites all important terms to terms proportional to the standard integrals A and B, which are defined as:

$$\begin{aligned} A &= \int d^4l_1 d^4l_2 \frac{1}{l_1^2+i\varepsilon}\frac{1}{(l_1-l_2)^2+i\varepsilon}\frac{1}{l_2^2+i\varepsilon}, \\ B &= \int d^4l_1 d^4l_2 \frac{1}{(l_1^2+i\varepsilon)^2}\frac{1}{l_2^2+i\varepsilon}. \end{aligned} \tag{190}$$

The combination of constants in front of both terms should now vanish. If everything is all right at 1-loop order this should automatically happen for the constants in front of B, since the integral B is what one typically gets from quadratic divergences of subdiagrams. The combination of constants in front of A will give a new constraint.

This program `LOOPS` can be used to calculate the same propagators as we did by hand at 1-loop order, this time at 2-loop order. In appendices 5 to 12 the output is shown. Here the program code itself has been deleted and replaced by the dots. Only the interesting parts are listed, like the output and the function D we let `LOOPS` start with.

Let's first have a look at the left selectron propagator. The output of `LOOPS` in this case is given in appendix 5. As said, the combination of coefficients in front of B is not so important to us. Indeed one sees that this combination already vanishes when one uses the constraints we found at 1-loop order. The coefficient in front of the A is important. This gives us the constraint:

$$\begin{aligned} &11ie^4 - 2ie^2|S_L|^2 - 2ie^2|P_L|^2 - 10i|S_L|^4 - 10i|P_L|^4 - 16i|S_L|^2|P_L|^2 + \\ &2i(S_L^*)^2P_L^2 + 2iS_L^2(P_L^*)^2 - 10i|S_L|^2|S_R|^2 - 10i|P_L|^2|P_R|^2 + \\ &-2i|S_L|^2|P_R|^2 - 2i|P_L|^2|S_R|^2 + 2iS_L^*P_LS_R^*P_R + 2iS_LP_L^*S_RP_R^* + \\ &-6iS_L^*P_LS_RP_R^* - 6iS_LP_L^*S_R^*P_R + 2e^2V_{LLLL} + 2e^2V_{LRLR} + \\ &-2|S_L|^2V_{LLLL} - 2|P_L|^2V_{LLLL} - 2|S_R|^2V_{LRLR} - 2|P_R|^2V_{LRLR} + \\ &-\frac{1}{2}iV_{LLLL}^2 - iV_{LRLR}^2 = 0. \end{aligned} \tag{191}$$

We see indeed that if we set all V's to zero we get the constraint (178) back as it should be.

We can also let the computer calculate the right selectron propagator. The result of this calculation is given in appendix 6. The coefficient in front of B again vanishes if we use 1-loop constraints, as it should be. The coefficient in front of A gives us the constraint:

$$\begin{aligned}
&11ie^4 - 2ie^2|S_R|^2 - 2ie^2|P_R|^2 - 10i|S_R|^4 - 10i|P_R|^4 - 16i|S_R|^2|P_R|^2 + \\
&2i(S_R^*)^2P_R^2 + 2iS_R^2(P_R^*)^2 - 10i|S_R|^2|S_L|^2 - 10i|P_R|^2|P_L|^2 + \\
&-2i|S_R|^2|P_L|^2 - 2i|P_R|^2|S_L|^2 + 2iS_R^*P_RS_L^*P_L + 2iS_RP_R^*S_LP_L^* + \\
&-6iS_R^*P_RS_LP_L^* - 6iS_RP_R^*S_L^*P_L + 2e^2V_{RRRR} + 2e^2V_{LRLR} + \\
&-2|S_R|^2V_{RRRR} - 2|P_R|^2V_{RRRR} - 2|S_L|^2V_{LRLR} - 2|P_L|^2V_{LRLR} + \\
&-\frac{1}{2}iV_{RRRR}^2 - iV_{LRLR}^2 = 0.
\end{aligned} \tag{192}$$

This is exactly the constraint (191) but with all L's replaced by R's and the other way around. This is of course also what one expects.

Also we can let the computer perform the calculation for the process $\tilde{e}_L \to \tilde{e}_R$. The result can be seen in appendix 7. Again the coefficient in front of B vanishes if we use the constraint (139) we found from this process at 1-loop order. The coefficient of A gives:

$$\begin{aligned}
&-2ie^2S_L^*S_R - 2ie^2P_L^*P_R - 8i|S_L|^2P_L^*P_R - 10i|S_L|^2S_L^*S_R + \\
&2iS_L(P_L^*)^2S_R - 8i|P_L|^2S_L^*S_R - 8i|P_R|^2S_L^*S_R - 10i|S_R|^2S_L^*S_R + \\
&2iS_L^*S_R^*P_R^2 + 2i(S_L^*)^2P_LP_R - 10i|P_L|^2P_L^*P_R - 8i|S_R|^2P_L^*P_R + \\
&2iP_L^*S_R^2P_R^* - 10i|P_R|^2P_L^*P_R - 2S_L^*S_RV_{LRLR} - 2P_L^*P_RV_{LRLR} = 0.
\end{aligned} \tag{193}$$

In appendix 8 the result of the electron propagator calculation can be seen. Since the last part of the program is designed to deal with scalar expression only we didn't actually calculate the electron propagator, but:

$$\frac{1}{4}\frac{1}{p^2}\mathrm{Tr}(\not{p}P) \tag{194}$$

where P is the actual propagator. If there should be no quadratic divergences in the electron propagator itself there should also be none in this quantity. We see that this doesn't give any constraint, which could also be expected. As in the case of 1-loop order there occur no divergences at all in the electron propagator. This is because of the fermionic character of the electron.

The same holds for the photino propagator. The result of this calculation at 2-loop can be seen in appendix 9. As expected, no quadratic divergences occur.

In appendix 10 and 11 we calculate 2 quantities related to the photon propagator. They are:

$$\begin{aligned}
&g^{\mu\nu}P_{\mu\nu}, \\
&\frac{1}{p^2}p^\mu p^\nu P_{\mu\nu}
\end{aligned} \tag{195}$$

where $P_{\mu\nu}$ is the actual photon propagator. This we do again for the reason that the last part of our program is designed to handle only scalar quantities and no quantities with Lorentz indices. Again, as can be seen, in the end no quadratic divergences are left. They do occur however in the individual diagram, but they cancel as they did at 1-loop order.

So from our 2-loop order calculations we get 3 more constraints, (191), (192) and (193). In the next section we will deal with those constraints.

3.19 The final conclusions

From the previous sections it has become clear that only 6 calculations give constraints for the coupling constants we want to fix. As can be seen in appendices 5 to 11 all equations are satisfied when we substitute the known values of the coupling constants. This indeed confirms that minimal softly broken SQED, which can be derived by imposing supersymmetry, does have no quadratic divergences up to 2-loop order. Our hope is however also to fix the coupling constants as much as possible with the equations we found. Notice however that we already fixed more with the demand that no quadratic divergences occur than is obvious at first sight. The type and form of the vertices are completely fixed by this demand! In the previous sections it should have become clear that we really need all the different types of vertices we introduced. Also it should have become clear that they must necessarily be of the form which one also finds in the conventional derivation. Only the constants in these vertex expressions are still not uniquely fixed. So one only needs to consider our theory up to 2-loop order to get already the precise types and forms of all the vertices! However let us now look at how far we can get fixing also the coupling constants.

Our constants to be fixed are:

$$\begin{aligned}&S_L, P_L, S_R, P_R,\\ &V_{LLLL}, V_{RRRR}, V_{LRLR}.\end{aligned}$$

Here the first 4 constants can be complex and the V's should all be purely imaginary. This gives us 11 degrees of freedom. The following processes gave a constraint on these coupling constants:

$$\begin{array}{lcll}\tilde{e}_L & \rightarrow & \tilde{e}_L & \text{at 1-loop order,}\\ \tilde{e}_R & \rightarrow & \tilde{e}_R & \text{at 1-loop order,}\\ \tilde{e}_L & \rightarrow & \tilde{e}_R & \text{at 1-loop order,}\\ \tilde{e}_L & \rightarrow & \tilde{e}_L & \text{at 2-loop order,}\\ \tilde{e}_R & \rightarrow & \tilde{e}_R & \text{at 2-loop order,}\\ \tilde{e}_L & \rightarrow & \tilde{e}_R & \text{at 2-loop order.}\end{array}$$

From the calculations we did we know that the constraints from the first, second, fourth and fifth are purely real (or purely imaginary), so naively we can only hope to fix 4 degrees of freedom with these. The other 2 calculations, where a left selectron goes to a right one gave complex equations, so with these we expect to be able to fix another 4 degrees of freedom. Naively we thus expect to be able to fix 8 degrees of freedom, which is not enough to fix all of our coupling constants uniquely.

One could hope that the equations we found are of a special form that enables us to fix more than one degree of freedom per real equation. Upon closer inspection this doesn't appear to be the case. In fact there is even some overlap between the equations. So with the constraints we found thus far we cannot fix uniquely all of our vertices.

This also means however that minimal softly broken SQED is not the only theory which has no quadratic divergences up to 2-loop order! One can come up with many possible choices to make a theory where no quadratic divergences occur up to 2-loop order. One only has to see to it that these constants satisfy all 6 constraints (187), (188), (189), (191), (192) and (193).

We can however try to come up with some more reasonable constraints, not coming from the demand that no quadratic divergences ought to occur, so that we might fix a few or even all couplings constants. This we will try now. Let's first have a look at what the 2 complex equations of the process $\tilde{e}_L \rightarrow \tilde{e}_R$ tell us. We consider therefore equation (193) and use the 1-loop equation (189) to rewrite it to a more convenient form.

$$\begin{aligned}&-2ie^2 S_L^* S_R - 2ie^2 P_L^* P_R - 8i|S_L|^2 P_L^* P_R - 10i|S_L|^2 S_L^* S_R +\\ &2iS_L(P_L^*)^2 S_R - 8i|P_L|^2 S_L^* S_R - 8i|P_R|^2 S_L^* S_R - 10i|S_R|^2 S_L^* S_R +\\ &2iS_L^* S_R^* P_R^2 + 2i(S_L^*)^2 P_L P_R - 10i|P_L|^2 P_L^* P_R - 8i|S_R|^2 P_L^* P_R +\end{aligned}$$

$$
\begin{aligned}
&2iP_L^*S_R^2P_R^* - 10i|P_R|^2P_L^*P_R - 2S_L^*S_RV_{LRLR} - 2P_L^*P_RV_{LRLR} = \\
&-2i|S_L|^2S_L^*S_R - 2i|P_L|^2P_L^*P_R - 2i|P_R|^2P_L^*P_R - 2i|S_R|^2S_L^*S_R + \\
&2iS_LS_R(P_L^*)^2 + 2iS_L^*S_R^*P_R^2 + 2iP_LP_R(S_L^*)^2 + 2iP_L^*P_R^*S_R^2 = \\
&2i|S_L|^2S_L^*S_R\Big(\frac{S_R^2}{P_R^2} - 1\Big) + 2i|S_R|^2S_L^*S_R\Big(\frac{(S_L^*)^2}{(P_L^*)^2} - 1\Big) + \\
&2i|P_L|^2P_L^*P_R\Big(\frac{P_R^2}{S_R^2} - 1\Big) + 2i|P_R|^2P_L^*P_R\Big(\frac{(P_L^*)^2}{(S_L^*)^2} - 1\Big) = \\
&2iS_L^*S_R\Big(\frac{(P_L^*)^2}{(S_L^*)^2} - 1\Big)(|S_L|^2 - |P_R|^2) + 2iS_L^*S_R\Big(\frac{(S_L^*)^2}{(P_L^*)^2} - 1\Big)(|S_R|^2 - |P_L|^2) = 0
\end{aligned}
\tag{196}
$$

Now we can see some of the overlap that our equations have. If we would choose $|S_L| = |P_R|$ the first term in the left hand side of (196) would vanish. Because of the 1-loop restraint (189) $|S_R|$ would then also equal $|P_L|$ and the second term in (196) would also vanish. So with this choice the complex equation (196) would only let us fix one more degree of freedom and not two, as one would expect.

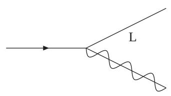
Now have a closer look at (196). It is tempting to set:

$$P_L = \pm S_L. \tag{197}$$

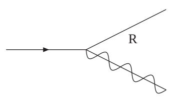
Then because of the 1-loop restraint (189) P_R would be:

$$P_R = \mp S_R. \tag{198}$$

If we pick the plus sign in (197) we get exactly the situation that a lefthanded electron can only go into a left selectron and a righthanded electron can only evolve into a right selectron. This can be seen because our vertices then have the form:



$$= \quad S_L(\mathbf{I} + \gamma^5)$$



$$= \quad S_R(\mathbf{I} - \gamma^5). \tag{199}$$

Our theory now has a chiral symmetry, if the electron selectron photino vertices weren't of the form (199) there would always be the possibility that a lefthanded electron evolved to a righthanded one (even if all masses were zero) via a selectron and a photino. One could say that this process is suppressed by the large mass of the selectron but by choosing (199) we have completely eliminated this possibility and we have satisfied two of our equations.

We see that our 2 complex equations from the process $\tilde{e}_L \to \tilde{e}_R$ are not of the same strength as the demand for chiral symmetry. This cannot be expected of course, since in the case of chiral symmetry the process $\tilde{e}_L \to \tilde{e}_R$ is completely zero, whereas we only demanded that the quadratically divergent term was zero. However our equations give an indication that there might indeed be chiral symmetry. We shall indeed assume this and see what this means for the other constraints and see what follows from them.

Our assumption (199) might also be seen as an assumption to make the theory minimal. If one considers the lefthanded and righthanded electron as different particles what we actually do by assuming that the vertices have the form (199) is discarding the 2 vertices were a lefthanded electron goes to a right selectron and were a righthanded electron goes to a left selectron. Indeed these vertices are not needed to let all quadratic divergences vanish.

Now let's see what follows from our other constraints if we make this assumption. The constraints (187) and (188) will become:

$$
\begin{aligned}
3e^2 - 8|S_L|^2 + iV_{LLLL} + iV_{LRLR} &= 0, \\
3e^2 - 8|S_R|^2 + iV_{RRRR} + iV_{LRLR} &= 0.
\end{aligned}
\tag{200}
$$

Let's now define $V_L = iV_{LLLL}$, $V_R = iV_{RRRR}$ and $V_{LR} = iV_{LRLR}$ for convenience. From these equations it then follows that:

$$\begin{aligned} V_L &= 8|S_L|^2 - 3e^2 - V_{LR}, \\ V_R &= 8|S_R|^2 - 3e^2 - V_{LR}. \end{aligned} \tag{201}$$

These relations can be used in the 2-loop constraint (191) from the left selectron propagator. This equation then becomes:

$$21\frac{1}{2}e^4 - 56e^2|S_L|^2 + 32|S_L|^4 - 16|S_L|^2|S_R|^2 + (-12|S_L|^2 + 4|S_R|^2 + 3e^2)V_{LR} + 1\frac{1}{2}V_{LR}^2 = 0. \tag{202}$$

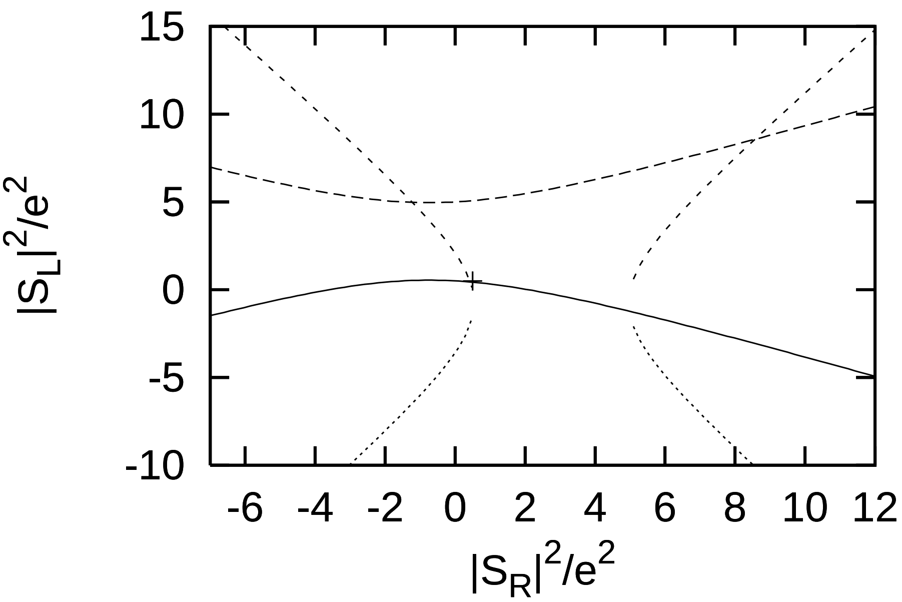
From the 2-loop constraint (192) from the right selectron propagator follows the same equation but with the L's and R's swapped:

$$21\frac{1}{2}e^4 - 56e^2|S_R|^2 + 32|S_R|^4 - 16|S_R|^2|S_L|^2 + (-12|S_R|^2 + 4|S_L|^2 + 3e^2)V_{LR} + 1\frac{1}{2}V_{LR}^2 = 0. \tag{203}$$

Now we have used all equations we had. We see that we still have 3 degrees of freedom, $|S_L|$, $|S_R|$ and V_{LR} and only 2 equations. Notice that the complex phase of S_L and S_R is not at all present in our equations anymore. This phase can thus be chosen arbitrarily if one is only interested in the vanishing of the quadratic divergences up to 2-loop order!

Our equations thus do still not fix $|S_L|$, $|S_R|$ and V_{LR}. There are also no reasonable assumptions anymore that would us enable to fix all these 3 constants. One could assume $|S_L| = |S_R|$. This would mean that $V_{LLLL} = V_{RRRR}$ which is not so bad. However because of this assumption equations (202) and (203) become the same, so that we still have 2 degrees of freedom and 1 equation, thus 1 degree of freedom to much.

Figure 4: The area given by the inequalities (204) and (205). The cross indicates the position of the supersymmetric coupling constants.



However equation (202) does give a certain condition $|S_L|$ and $|S_R|$ should satisfy. In order that there exists a solution V_{LR} to this equation at all the discriminant should be larger than 0 which gives:

$$-7\frac{1}{2}e^4 - 3|S_L|^4 + 16\frac{1}{2}e^2|S_L|^2 + |S_R|^4 + 1\frac{1}{2}e^2|S_R|^2 \geq 0. \tag{204}$$

The same holds for equation (203), this gives the inequality:

$$-7\frac{1}{2}e^4 - 3|S_R|^4 + 16\frac{1}{2}e^2|S_R|^2 + |S_L|^4 + 1\frac{1}{2}e^2|S_L|^2 \geq 0. \tag{205}$$

So as long as we choose our $|S_L|$ and $|S_R|$ to be inside the region specified by (204) and (205) we get no quadratic divergences up to 2-loop order. The area these 2 inequalities give can be seen in figure (4). The area enclosed by the 4 curves is the area that satisfies our inequalities. The cross in figure (4) indicates the position of the supersymmetric coupling constants $\frac{|S_L|^2}{e^2}$ and $\frac{|S_R|^2}{e^2}$. Notice it is very close to the border, but it is not on it! Because this cross is inside the area given by (204) and (205) we have indeed verified that in minimal softly broken SQED there occur no quadratic divergences.

When we have chosen $|S_L|$ and $|S_R|$ the constants V_{LR}, V_L and V_R follow immediately. Also $|P_L|$ and $|P_R|$ are then of course known. The complex phases of S_L and S_R can still be chosen freely, there is absolutely no constraint on them (at least up to 2-loop order). When the phases of S_L and S_R are chosen the ones for P_L and P_R follow immediately. So actually, despite of all the constraints we have come up with, there is still a wide variety of S_L's and S_R's we can choose. Once those are fixed the rest of the constants are also fixed. This means that if one wants a theory free of quadratic divergences up to 2-loop order one has a lot of choice. Minimal softly broken SQED is certainly not the only option then.

4 Summary

In the first section of this thesis we derived minimal softly broken SQED in the conventional way. We saw what its particle content was and which interactions occurred.

Only in the second section we started with the main topic of this thesis. There we presented an alternative derivation of minimal softly broken SQED. We started by postulating the particle content and the identity of all the particles. Then we started introducing vertices. The 3-vertices we introduced all at once, but it should have become clear that we really need all of them. In this summary we will therefore describe where the need of all the different vertices clearly emerges.

We started with the electron photon interaction, which is of course an empirical fact. If we have only this vertex we saw that the photon propagator at 1-loop order has a quadratical divergence in its longitudinal part. We insisted that these quadratically divergent parts vanish in the complete propagators. Therefore we need the left selectron photon interaction. The form of this vertex is completely fixed by the Ward Takahashi identity. We also saw that when we have this new 3-vertex we also need a left selectron photon 4-vertex, on grounds of unitarity. After these introductions we can again look at the photon propagator. It appears then that only one half of the quadratical divergence in the longitudinal part is canceled. This forces us to introduce also the right selectron photon 3-vertex and with it the right selectron photon 4-vertex. Then the photon propagator at 1-loop order is indeed completely free of quadratic divergences.

After the photon propagator one can consider the left selectron propagator at 1-loop order. With the vertices introduced up to now this propagator has a quadratical divergence. This forces us to introduce the left selectron electron photino interaction. With the demand that no quadratic or worse divergences occur anymore after this introduction we can completely fix the form of the vertex expression and we get an equation for the 2 complex coupling constants occurring in this vertex expression. If we consider the right selectron propagator we are similarly forced to introduce the right selectron electron photino coupling and we can also fix the form of the vertex expression completely.

After these propagators we can consider the process where a left selectron goes to a right one at 1-loop order. This gives us another equation for the coupling constants present in the left selectron electron photino and right selectron electron photino vertices, but it doesn't force us to introduce a new vertex. Then we can still look at the electron and photino propagator but these appear to have no quadratical divergences at all.

After all these 1-loop propagators we can consider the same propagators again, but at 2-loop order. When one considers the left selectron propagator again, now at 2-loop order, one finds an equation which is inconsistent with what we found at 1-loop order. Again we are forced to introduce new vertices. To resolve the problem we introduce 3 4-vertices where only selectrons coincide. With these extra vertices our problem is resolved and the left selectron propagator at 2-loop order gives another equation for our coupling constants. The same goes for the right selectron propagator and the process where a left selectron goes to a right one at 2-loop order. The electron, photino and photon propagators at 2-loop order are automatically free of quadratic divergences, not giving us more constraints.

At the end of this path we have 4 complex and 3 real coupling constants which we have to fix. We only have 2 complex and 4 real equations to fix them. Indeed our equations turn out to be insufficient to fix everything uniquely. We tried to come up with some reasonable demands by hand to fix some more degrees of freedom but still we cannot fix all coupling constants completely.

Thus it appears that when one wants to build a theory with the particle content of SQED without quadratical divergences one does need all the vertices present in minimal softly broken SQED. Also the form of all these vertices is completely fixed. The coupling constants aren't fixed when one only demands that all quadratical divergences vanish up to 2-loop order, like we did. There still is some freedom in choosing them. When one also demands by hand that there is a chiral symmetry in our theory, like we did, the freedom consists of 2 complex parameters. The phases of these parameters can be chosen completely freely, whereas the magnitudes have to be in some given area in order that all quadratical divergences vanish.

5 Appendix 1

In this appendix we first want to calculate integrals of the form:

$$I_n = \int d^4l \frac{1}{(l^2 - m^2 + i\varepsilon)^n}, \tag{206}$$

where the integration is over the area where

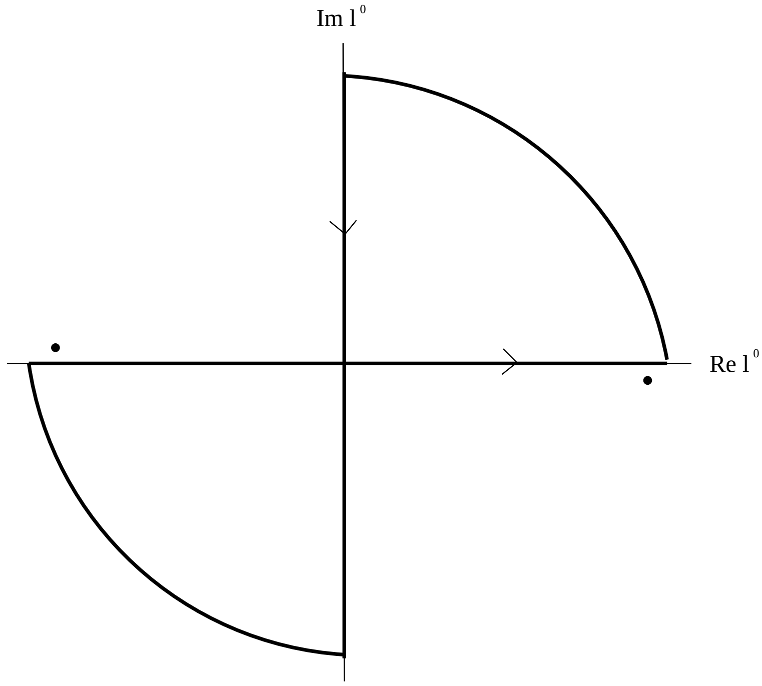
$$l_E \leq \Lambda. \tag{207}$$

To perform this integral we should first write the integral (206) over the momentum l in Minkowski space to an integral over the corresponding momentum l_E in Euclidean space. This is done by a Wick rotation. This is a rotation of the l^0 component of l. The poles in the integrand are located at

$$l^0 = \pm\sqrt{|\vec{l}|^2 + m^2} \mp \frac{i\varepsilon}{2\sqrt{|\vec{l}|^2 + m^2}}. \tag{208}$$

This is drawn in figure (5). From this picture we can also see it is correct to replace the integration

Figure 5: The position of the poles in the complex l^0 plane



over l^0 from $-\infty$ to $+\infty$ by an integration over l^0 from $-i\infty$ to $+i\infty$. These integrations are equal since the integration over the complete contour is zero, since no poles are enclosed, and the integrations over the two arcs are zero because of the form of the integrand. In our case we do not immediately let Λ approach infinity, so that the arcs give a contribution $\mathcal{O}(\frac{1}{\Lambda})$, which can also be neglected. After this rotation we introduce the new variable $l^4 = -il^0$ and substitute it for l^0. We then have an integral over the Euclidean momentum l_E with a normal (real) area to be integrated over. So we get:

$$I_n = i\int d^4l_E \frac{1}{(-(l^4)^2 - |\vec{l}|^2 - m^2 + i\varepsilon)^n} = i(-1)^n \int d^4l_E \frac{1}{(|l_E|^2 + m^2)^n}. \tag{209}$$

We have already taken the limit $\varepsilon \to 0$ here. This can be done since there are no more real poles present in the integrand. The only function of ε was to keep the poles in the original integrand away from the contour to be integrated over. Now we can perform the integral (209) by using spherical coordinates in 4-dimensional Euclidean space. The integrand doesn't contain any angles so these can be integrated out. We use:

$$\begin{aligned}\int d^4 l_E &= \int_0^{2\pi} d\theta_1 \int_0^{\pi} \sin\theta_2 d\theta_2 \int_0^{\pi} \sin^2\theta_3 d\theta_3 \int_0^{\Lambda} |l_E|^3 d|l_E| = \\ & 2\pi^2 \int_0^{\Lambda} d|l_E||l_E|^3. \end{aligned} \tag{210}$$

So I_n becomes:

$$\begin{aligned} I_n &= 2i\pi^2(-1)^n \int_0^{\Lambda} d|l_E| \frac{|l_E|^3}{(|l_E|^2 + m^2)^n} = \\ & i\pi^2(-1)^n \int_0^{\Lambda^2} dt \frac{t}{(t+m^2)^n}. \end{aligned} \tag{211}$$

Before we can use partial integration to remove the t in the numerator we have to distinct two cases, $n = 1$ and $n > 1$. First consider $n = 1$:

$$\begin{aligned} I_1 &= -i\pi^2 \int_0^{\Lambda^2} dt \frac{t}{t+m^2} = \\ & -i\pi^2[t \ln(t+m^2)]_0^{\Lambda^2} + i\pi^2 \int_0^{\Lambda^2} dt \ln(t+m^2) = \\ & -i\pi^2[t \ln(t+m^2)]_0^{\Lambda^2} + i\pi^2[(t+m^2)\ln(t+m^2) - (t+m^2)]_0^{\Lambda^2} = \\ & i\pi^2[m^2 \ln(t+m^2) - (t+m^2)]_0^{\Lambda^2} = \\ & i\pi^2(m^2 \ln(\Lambda^2+m^2) - \Lambda^2 - m^2 \ln(m^2)). \end{aligned} \tag{212}$$

The function $\ln(\Lambda^2 + m^2)$ can be expanded around $m^2 = 0$. Finally we get:

$$I_1 = -i\pi^2\Lambda^2 + 2i\pi^2 m^2 \ln\Lambda - i\pi^2 m^2 \ln(m^2) + \mathcal{O}\big(\frac{1}{\Lambda^2}\big). \tag{213}$$

Now consider the case $n > 1$.

$$\begin{aligned} I_n &= i\pi^2(-1)^n \int_0^{\Lambda^2} dt \frac{t}{(t+m^2)^n} = \\ & i\pi^2(-1)^n[t\frac{-1}{n-1}\frac{1}{(t+m^2)^{n-1}}]_0^{\Lambda^2} - i\pi^2(-1)^n \int_0^{\Lambda^2} dt \frac{-1}{n-1}\frac{1}{(t+m^2)^{n-1}} \end{aligned} \tag{214}$$

Also here we have to distinct to case again, $n = 2$ and $n > 2$. When $n = 2$ we get:

$$\begin{aligned} I_2 &= i\pi^2[-t\frac{1}{t+m^2}]_0^{\Lambda^2} + i\pi^2 \int_0^{\Lambda^2} dt \frac{1}{t+m^2} \\ & i\pi^2[-t\frac{1}{t+m^2}]_0^{\Lambda^2} + i\pi^2[\ln(t+m^2)]_0^{\Lambda^2} = \\ & -i\pi^2 \frac{\Lambda^2}{\Lambda^2+m^2} + i\pi^2\ln(\Lambda^2+m^2) - i\pi^2\ln(m^2) = \\ & 2i\pi^2 \ln\Lambda - i\pi^2\ln(m^2) - i\pi^2 + \mathcal{O}\big(\frac{1}{\Lambda^2}\big). \end{aligned} \tag{215}$$

When $n > 2$ we get:

$$\begin{aligned}
I_n &= i\pi^2(-1)^n[t\frac{-1}{n-1}\frac{1}{(t+m^2)^{n-1}}]_0^{\Lambda^2} - \\
& i\pi^2(-1)^n[\frac{1}{(n-1)(n-2)}\frac{1}{(t+m^2)^{n-2}}]_0^{\Lambda^2} = \\
& i\pi^2\frac{(-1)^{n+1}}{n-1}\frac{\Lambda^2}{(\Lambda^2+m^2)^{n-1}} - \\
& i\pi^2(-1)^n\frac{1}{(n-2)(n-1)}\frac{1}{(\Lambda^2+m^2)^{n-2}} + \\
& i\pi^2(-1)^n\frac{1}{(n-2)(n-1)}\frac{1}{(m^2)^{n-2}} = \\
& i\pi^2(-1)^n\frac{1}{(n-2)(n-1)}\frac{1}{(m^2)^{n-2}} + \mathcal{O}(\frac{1}{\Lambda^2}).
\end{aligned} \tag{216}$$

Now we have calculated all integrals of the form (206). We see that for these cases naive power counting indeed works. For $n = 1$ we get a quadratic divergence, a logarithmic divergence and finite terms. For $n = 2$ we only get a logarithmic divergence and finite terms. For $n > 2$ we only get finite terms.

The form (206) is not the most general form we will encounter in our calculations. We might also encounter integrals in which the factors in the denominator do not all have the same mass. For example we might encounter

$$I = \int d^4l\frac{1}{(l^2-m_1^2+i\varepsilon)(l^2-m_2^2+i\varepsilon)}. \tag{217}$$

With the following trick however we can always write them to an integral of the form (206). This trick is:

$$\begin{aligned}
\frac{1}{A^mB^n} &= \frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)}\int_0^1 dx_1\int_0^1 dx_2\delta(x_1+x_2-1)\frac{x_1^{m-1}x_2^{n-1}}{(x_1A+x_2B)^{m+n}} = \\
& \frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)}\int_0^1 dx_1\frac{x_1^{m-1}(1-x_1)^{n-1}}{(x_1A+(1-x_1)B)^{m+n}}.
\end{aligned} \tag{218}$$

This trick is easily proven for $m = n = 1$:

$$\begin{aligned}
\int_0^1 dx_1\frac{1}{(x_1A+(1-x_1)B)^2} &= \int_0^1 dx_1\frac{1}{((A-B)x_1+B)^2} = \\
& \frac{-1}{A-B}[\frac{1}{(A-B)x_1+B}]_0^1 = \\
& \frac{1}{B-A}\Big(\frac{1}{A}-\frac{1}{B}\Big) = \frac{1}{AB}.
\end{aligned} \tag{219}$$

By differentiating $m-1$ times to A and $n-1$ times to B we can then also prove the form (218).

Now look at the integral (217) again. With the trick just outlined this integral can be written to:

$$I = \int_0^1 dx_1\int d^4l\frac{1}{(l^2-x_1m_1^2-(1-x_1)m_2^2+i\varepsilon)^2}. \tag{220}$$

This is again an integral of the form (206), so we can also use the previous calculations for integrals of the type (217). The only problem in an actual calculation might of course be the integral over x_1. However to see how the integral diverges this method is very suited. We see that also the integral (217) only diverges logarithmically. Also this divergent term is exactly the same as for the integral where the two masses would have been the same, in other words the worst divergent term doesn't depend on the masses present in the integrand. Only the finite part is, as expected, different from the finite part of the integral with two equal masses.

6 Appendix 2

Below is printed the FORM program used in the calculation of diagram P_3 of the left selectron propagator.

```
FORM by J.Vermaseren,version 3.0(Jan 31 2001) Run at: Thu Jan 29 15:21:12 2004
    Off Statistics;

    D 4;

    I alpha, beta, mu1, nu1, mu2, nu2, delta1, gamma1, delta2, gamma2;

    S S, SC, P, PC, v1, v1C, v2, v2C, a1, a1C, a2, a2C, t1, t1C, t2, t2C,
      t3, t3C, me, mfino;

    V V, VC, A, AC, p, l;

    CT T, TC;

    L B = S*gi_(1) + V(mu1)*g_(1, mu1) +
          T(mu1, nu1)*i_*(g_(1, mu1)*g_(1, nu1) - g_(1, nu1)*g_(1, mu1))/2 +
          A(mu1)*g5_(1)*g_(1, mu1) + P*g5_(1);

    L BC = SC*gi_(1) + VC(mu2)*g_(1, mu2) +
           TC(mu2, nu2)*i_*(g_(1, mu2)*g_(1, nu2) - g_(1, nu2)*g_(1, mu2))/2 +
           AC(mu2)*g5_(1)*g_(1, mu2) - PC*g5_(1);

    L Tr = B*(g_(1, alpha)*(p(alpha)-l(alpha))+me)*
           BC*(g_(1, beta)*(-l(beta))+mfino);

    sum mu1, nu1, mu2, nu2, alpha, beta;

    trace4, 1;

    print B, BC, Tr;

    .sort

  B =
     4*S;

  BC =
     4*SC;

  Tr =
     4*S*SC*me*mfino - 4*P*PC*me*mfino + 4*V.VC*p.l - 4*V.VC*l.l + 4*V.VC*me*
     mfino - 4*V.p*VC.l + 4*V.p*SC*mfino - 4*V.l*VC.p + 8*V.l*VC.l - 4*V.l*SC
     *me - 4*V.l*SC*mfino + 4*VC.p*S*mfino - 4*VC.l*S*me - 4*VC.l*S*mfino + 4
     *A.AC*p.l - 4*A.AC*l.l - 4*A.AC*me*mfino - 4*A.p*AC.l - 4*A.p*PC*mfino
      - 4*A.l*AC.p + 8*A.l*AC.l - 4*A.l*PC*me + 4*A.l*PC*mfino - 4*AC.p*P*
     mfino - 4*AC.l*P*me + 4*AC.l*P*mfino - 4*p.l*S*SC - 4*p.l*P*PC + 4*l.l*S
     *SC + 4*l.l*P*PC + 4*e_(V,AC,p,l) - 4*e_(VC,A,p,l) + 4*T(VC,p)*i_*mfino
      + 4*T(VC,l)*i_*me - 4*T(VC,l)*i_*mfino - 4*T(p,VC)*i_*mfino + 4*T(p,l)*
     i_*SC + 4*T(p,N1_?)*TC(l,N1_?) - 4*T(p,N1_?)*TC(N1_?,l) - 4*T(l,VC)*i_*
     me + 4*T(l,VC)*i_*mfino - 4*T(l,p)*i_*SC + 4*T(l,N1_?)*TC(p,N1_?) - 8*T(
```

```
l,N1_?)*TC(l,N1_?) - 4*T(l,N1_?)*TC(N1_?,p) + 8*T(l,N1_?)*TC(N1_?,l) - 4
*T(N1_?,p)*TC(l,N1_?) + 4*T(N1_?,p)*TC(N1_?,l) - 4*T(N1_?,l)*TC(p,N1_?)
 + 8*T(N1_?,l)*TC(l,N1_?) + 4*T(N1_?,l)*TC(N1_?,p) - 8*T(N1_?,l)*TC(N1_?
,l) + 4*T(N1_?,N2_?)*e_(AC,p,N1_?,N2_?)*i_*mfino - 4*T(N1_?,N2_?)*e_(AC,
l,N1_?,N2_?)*i_*me - 4*T(N1_?,N2_?)*e_(AC,l,N1_?,N2_?)*i_*mfino - 4*T(
N1_?,N2_?)*e_(p,l,N1_?,N2_?)*i_*PC + 4*T(N1_?,N2_?)*TC(N1_?,N2_?)*me*
mfino - 4*T(N1_?,N2_?)*TC(N1_?,N2_?)*p.l + 4*T(N1_?,N2_?)*TC(N1_?,N2_?)*
l.l - 4*T(N1_?,N2_?)*TC(N2_?,N1_?)*me*mfino + 4*T(N1_?,N2_?)*TC(N2_?,
N1_?)*p.l - 4*T(N1_?,N2_?)*TC(N2_?,N1_?)*l.l - 4*TC(V,p)*i_*mfino - 4*
TC(V,l)*i_*me + 4*TC(V,l)*i_*mfino + 4*TC(p,V)*i_*mfino - 4*TC(p,l)*i_*S
 + 4*TC(l,V)*i_*me - 4*TC(l,V)*i_*mfino + 4*TC(l,p)*i_*S + 4*TC(N1_?,
N2_?)*e_(A,p,N1_?,N2_?)*i_*mfino - 4*TC(N1_?,N2_?)*e_(A,l,N1_?,N2_?)*i_*
me - 4*TC(N1_?,N2_?)*e_(A,l,N1_?,N2_?)*i_*mfino - 4*TC(N1_?,N2_?)*e_(p,l
,N1_?,N2_?)*i_*P;


id V = v1*(p-l) + v2*l;
id A = a1*(p-l) + a2*l;
id T(alpha?, beta?) = t1*(p(alpha)-l(alpha))*l(beta) +
       t2*(p(beta)-l(beta))*l(alpha) +
                    t3*e_(alpha, beta, delta1, gamma1)*
       (p(delta1)-l(delta1))*l(gamma1);
id VC = v1C*(p-l) + v2C*l;
id AC = a1C*(p-l) + a2C*l;
id TC(alpha?, beta?) = t1C*(p(alpha)-l(alpha))*l(beta) +
         t2C*(p(beta)-l(beta))*l(alpha) +
                     t3C*e_(alpha, beta, delta2, gamma2)*
                (p(delta2)-l(delta2))*l(gamma2);

sum delta1, gamma1, delta2, gamma2;

id e_(p, l, alpha?, beta?)*e_(p, l, alpha?, beta?) = 2*(p.l^2 - p.p*l.l);

bracket l.l, p.l, p.p;

print Tr;

.end

Tr =
   + p.p*p.l*l.l * ( 4*t1*t1C - 4*t1*t2C - 4*t1C*t2 + 4*t2*t2C + 16*t3*t3C
      )

   + p.p*p.l * (  - 4*v1*v1C - 4*a1*a1C )

   + p.p*l.l * (  - 4*i_*S*t1C + 4*i_*S*t2C + 4*i_*SC*t1 - 4*i_*SC*t2 + 8*
     i_*P*t3C + 8*i_*PC*t3 - 4*i_*v1*t1C*me + 4*i_*v1*t2C*me + 4*i_*v1C*t1
     *me - 4*i_*v1C*t2*me + 4*i_*v2*t1C*mfino - 4*i_*v2*t2C*mfino - 4*i_*
     v2C*t1*mfino + 4*i_*v2C*t2*mfino + 8*i_*a1*t3C*me + 8*i_*a1C*t3*me +
     8*i_*a2*t3C*mfino + 8*i_*a2C*t3*mfino + 4*v1*v1C - 4*v1*v2C - 4*v1C*
     v2 + 4*a1*a1C - 4*a1*a2C - 4*a1C*a2 + 4*t1*t1C*me*mfino - 4*t1*t2C*me
     *mfino - 4*t1C*t2*me*mfino + 4*t2*t2C*me*mfino - 16*t3*t3C*me*mfino )

   + p.p*l.l^2 * (  - 4*t1*t1C + 4*t1*t2C + 4*t1C*t2 - 4*t2*t2C - 16*t3*
```

```
  t3C )

+ p.p * ( 4*S*v1C*mfino + 4*SC*v1*mfino - 4*P*a1C*mfino - 4*PC*a1*mfino
   + 4*v1*v1C*me*mfino - 4*a1*a1C*me*mfino )

+ p.l*l.l * (  - 12*v1*v1C + 8*v1*v2C + 8*v1C*v2 - 4*v2*v2C - 12*a1*a1C
   + 8*a1*a2C + 8*a1C*a2 - 4*a2*a2C )

+ p.l * (  - 4*S*SC - 4*S*v1C*me - 8*S*v1C*mfino + 4*S*v2C*mfino - 4*SC
  *v1*me - 8*SC*v1*mfino + 4*SC*v2*mfino - 4*P*PC - 4*P*a1C*me + 8*P*
  a1C*mfino - 4*P*a2C*mfino - 4*PC*a1*me + 8*PC*a1*mfino - 4*PC*a2*
  mfino - 8*v1*v1C*me*mfino + 4*v1*v2C*me*mfino + 4*v1C*v2*me*mfino + 8
  *a1*a1C*me*mfino - 4*a1*a2C*me*mfino - 4*a1C*a2*me*mfino )

+ p.l^2*l.l * ( 4*t1*t1C - 4*t1*t2C - 4*t1C*t2 + 4*t2*t2C + 16*t3*t3C )

+ p.l^2 * ( 4*i_*S*t1C - 4*i_*S*t2C - 4*i_*SC*t1 + 4*i_*SC*t2 - 8*i_*P*
  t3C - 8*i_*PC*t3 + 4*i_*v1*t1C*me - 4*i_*v1*t2C*me - 4*i_*v1C*t1*me
   + 4*i_*v1C*t2*me - 4*i_*v2*t1C*mfino + 4*i_*v2*t2C*mfino + 4*i_*v2C*
  t1*mfino - 4*i_*v2C*t2*mfino - 8*i_*a1*t3C*me - 8*i_*a1C*t3*me - 8*i_
  *a2*t3C*mfino - 8*i_*a2C*t3*mfino + 8*v1*v1C + 8*a1*a1C - 4*t1*t1C*me
  *mfino + 4*t1*t2C*me*mfino + 4*t1C*t2*me*mfino - 4*t2*t2C*me*mfino +
  16*t3*t3C*me*mfino )

+ p.l^3 * (  - 4*t1*t1C + 4*t1*t2C + 4*t1C*t2 - 4*t2*t2C - 16*t3*t3C )

+ l.l * ( 4*S*SC + 4*S*v1C*me + 4*S*v1C*mfino - 4*S*v2C*me - 4*S*v2C*
  mfino + 4*SC*v1*me + 4*SC*v1*mfino - 4*SC*v2*me - 4*SC*v2*mfino + 4*P
  *PC + 4*P*a1C*me - 4*P*a1C*mfino - 4*P*a2C*me + 4*P*a2C*mfino + 4*PC*
  a1*me - 4*PC*a1*mfino - 4*PC*a2*me + 4*PC*a2*mfino + 4*v1*v1C*me*
  mfino - 4*v1*v2C*me*mfino - 4*v1C*v2*me*mfino + 4*v2*v2C*me*mfino - 4
  *a1*a1C*me*mfino + 4*a1*a2C*me*mfino + 4*a1C*a2*me*mfino - 4*a2*a2C*
  me*mfino )

+ l.l^2 * ( 4*v1*v1C - 4*v1*v2C - 4*v1C*v2 + 4*v2*v2C + 4*a1*a1C - 4*a1
  *a2C - 4*a1C*a2 + 4*a2*a2C )

+ 4*S*SC*me*mfino - 4*P*PC*me*mfino;
```

7 Appendix 3

In this appendix we focus on 2-loop integrals, integrals with two loop momenta to be integrated over. These we need in our 2-loop calculations of all the propagators in our theory. As we will see in a moment 2-loop integrals are much more difficult to handle than 1-loop integrals which we treated in appendix 1.

In all 2-loop calculations we already eliminated all integration variables from the numerator. So in this appendix we only have to calculate 2-loop integrals with numerator 1. We will start with the following integral I:

$$I = \int d^4l_1 d^4l_2 \frac{1}{l_1^2 - m_1^2 + i\varepsilon} \frac{1}{(l_1 - l_2)^2 - m_2^2 + i\varepsilon} \frac{1}{l_2^2 - m_3^2 + i\varepsilon}. \tag{221}$$

As in the case with the 1 loop integral we first have to regularize this integral. This immediately gives problems.

We might introduce a cut-off momentum Λ like we did in appendix 1. When doing this we should realize however that we cannot just shift the integration variables l_1 and l_2. Such a shift would make the boundary of the integral very difficult to handle. The original boundary was a 4-dimensional sphere with radius Λ, the new boundary would still be a sphere with radius Λ, but not with its center at $l_1 = 0, l_2 = 0$. Also this shift cannot be neglected, as is sometimes done when one has a term $(l - p)^2$ in the denominator where p is an external momentum. In such a case the shift is only finite and unimportant for the highest divergence. In our case however the shift can be infinite and is integrated over!

So introducing a cutoff to regularize (221) is a bad idea. We might instead consider dimensional regularization. This type of regularization is mostly used when one is renormalizing the theory. And indeed the integral (221) can be calculated when regularized in this way. However we are interested in the quadratically divergent part. This part is very hard to identify when using dimensional regularization! At 1-loop order one knows that a quadratic divergence gives a term with a pole at dimension $n = 4$ and a pole at dimension $n = 2$. So at 1-loop order picking out the quadratic divergences can be done. It is however not clear how a quadratic divergence manifests itself at 2-loop order. One might expect that also there a quadratic divergence gives a term with a pole for $n = 2$. But this is not true! When calculated with dimensional regularization the integral (221) does not have a pole for $n = 2$! But, as we will see below, this integral is quadratically divergent, as can also be expected from power counting. It appears the lowest dimension for which (221) has a pole is 3. So also dimensional regularization is not going to help us further.

A nice way to handle the integral (221) appears to be using an exponential to regularize it. Such a nonlocal regularization is described by Kleppe and Woodard in [6] and by Evens, Moffat, Kleppe and Woodard in [7]. This regularization means that one makes the following replacement:

$$\frac{1}{l_E^2 + m^2} \rightarrow \int_{\Lambda^{-2}}^{\infty} d\alpha e^{-\alpha(l_E^2 + m^2)}. \tag{222}$$

Such a regularization is called nonlocal because when it is introduced in the Lagrangian density it gives an exponential with a momentum operator in its argument. This clearly is a nonlocal Lagrangian density. The right hand side of (222) is equal to:

$$\frac{1}{l_E^2 + m^2} e^{-\Lambda^{-2}(l_E^2 + m^2)}. \tag{223}$$

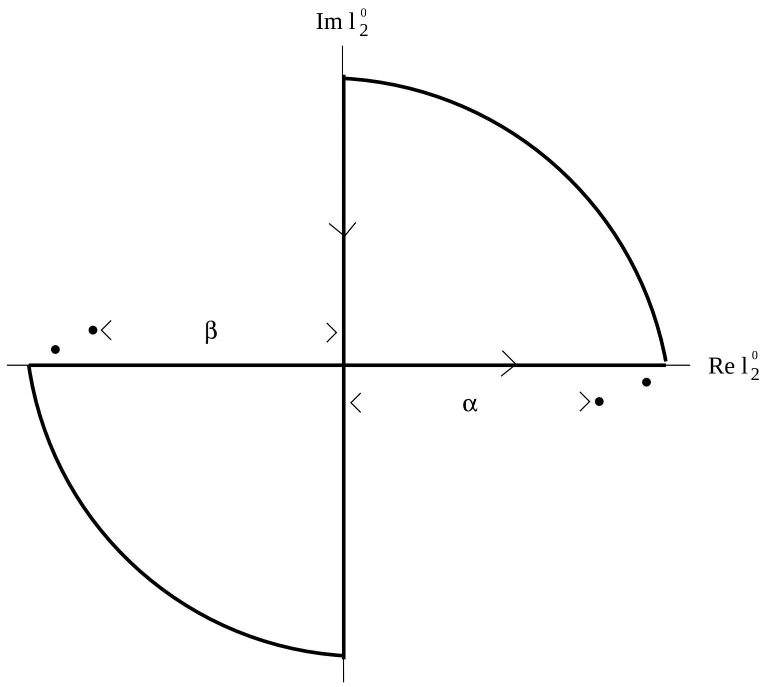
So we see that when Λ grows large we indeed get back the original expression on the left hand side of (222) as should be the case for a regularization scheme. Also we see that Λ can be interpreted as a cut-off momentum. For large Λ the exponential has dropped to e^{-1} for $|l_E| = \Lambda$. Thus our Λ is a smooth cut-off, but the important thing is that it can be interpreted as a cut-off. In this way we can clearly identify quadratic divergences and one doesn't have the problem we saw when considering dimensional regularization.

Before we can use this regularization scheme however we should first write the integral (221) to an integral over Euclidean momenta. At first sight this seems hard to do because of the position of the poles in the second denominator of (221). It is possible however, as we will see below. First rewrite (221) as follows.

$$
\begin{aligned}
I &= \int d^4l_1 d^4l_2 \frac{1}{l_1^2 - m_1^2 + i\varepsilon} \frac{1}{(l_1-l_2)^2 - m_2^2 + i\varepsilon} \frac{1}{l_2^2 - m_3^2 + i\varepsilon} \\
&= \int d^4l_1 \frac{1}{l_1^2 - m_1^2 + i\varepsilon} \\
&\quad \int d^3l_2 dl_2^0 \frac{1}{\left(l_2^0 - \sqrt{|\vec{l}_2|^2 + m_3^2} + \frac{i\varepsilon}{2\sqrt{|\vec{l}_2|^2+m_3^2}}\right)\left(l_2^0 + \sqrt{|\vec{l}_2|^2 + m_3^2} - \frac{i\varepsilon}{2\sqrt{|\vec{l}_2|^2+m_3^2}}\right)} \\
&\quad \frac{1}{\left(l_1^0 - l_2^0 - \sqrt{|\vec{l}_1 - \vec{l}_2|^2 + m_2^2} + \frac{i\varepsilon}{2\sqrt{|\vec{l}_1-\vec{l}_2|^2+m_2^2}}\right)\left(l_1^0 - l_2^0 + \sqrt{|\vec{l}_1 - \vec{l}_2|^2 + m_2^2} - \frac{i\varepsilon}{2\sqrt{|\vec{l}_1-\vec{l}_2|^2+m_2^2}}\right)}
\end{aligned}
\tag{224}
$$

From this expression we can clearly read off the poles of the l_2^0 integrand. These are situated as indicated in figure (6). In figure (6) the distances α and β are given by:

Figure 6: The position of the poles in the complex l_2^0 plane



$$
\begin{aligned}
\alpha &= l_1^0 + \sqrt{|\vec{l}_1 - \vec{l}_2|^2 + m_2^2} \\
\beta &= -l_1^0 + \sqrt{|\vec{l}_1 - \vec{l}_2|^2 + m_2^2}.
\end{aligned}
\tag{225}
$$

This means these distances can also be negative and the poles might be in the way of the Wick rotation we want to perform. The first fraction in (224) gives no problematic poles. The second fraction does, and the poles are situated wrongly in the cases:

$$\begin{aligned} l_1^0 &< -\sqrt{|\vec{l}_1 - \vec{l}_2|^2 + m_2^2} \\ l_1^0 &> \sqrt{|\vec{l}_1 - \vec{l}_2|^2 + m_2^2}. \end{aligned} \tag{226}$$

In the first case α becomes negative and the pole of the second fraction in (224) which is beneath the real axis goes to the left of the imaginary axis. This pole is then situated inside the contour indicated in figure (6). In the second case β becomes negative and the pole above the real axis goes to the right of the imaginary axis, into the area inside the contour. A Wick rotation thus cannot be done as usual, in this case we have to include corrections because the poles can be inside the contour. We split the l_1^0 integral into 3 parts, perform the Wick rotation and for the cases indicated in (226) we have to make corrections for the poles, which are inside the contour.

$$\begin{aligned} I &= i\int d^4l_1 \frac{1}{l_1^2 - m_1^2 + i\varepsilon} \int d^4l_{2E} \frac{1}{(l_1^0 - il_{2E}^4)^2 - |\vec{l}_1 - \vec{l}_{2E}|^2 - m_2^2 + i\varepsilon} \frac{1}{-l_{2E}^2 - m_3^2 + i\varepsilon} + \\ & 2\pi i \int d^3l_1 \int_{l_1^0 > \sqrt{|\vec{l}_1 - \vec{l}_2|^2 + m_2^2}} dl_1^0 \frac{1}{l_1^2 - m_1^2 + i\varepsilon} \int d^3l_2 \frac{1}{2\sqrt{|\vec{l}_1 - \vec{l}_2|^2 + m_2^2} - \frac{i\varepsilon}{\sqrt{|\vec{l}_1 - \vec{l}_2|^2 + m_2^2}}} \\ & \frac{1}{\left(l_1^0 - \sqrt{|\vec{l}_1 - \vec{l}_2|^2 + m_2^2} + \frac{i\varepsilon}{2\sqrt{|\vec{l}_1 - \vec{l}_2|^2 + m_2^2}}\right)^2 - |\vec{l}_2|^2 - m_3^2 + i\varepsilon} + \\ & 2\pi i \int d^3l_1 \int_{l_1^0 < -\sqrt{|\vec{l}_1 - \vec{l}_2|^2 + m_2^2}} dl_1^0 \frac{1}{l_1^2 - m_1^2 + i\varepsilon} \int d^3l_2 \frac{1}{-2\sqrt{|\vec{l}_1 - \vec{l}_2|^2 + m_2^2} + \frac{i\varepsilon}{\sqrt{|\vec{l}_1 - \vec{l}_2|^2 + m_2^2}}} \\ & \frac{1}{\left(l_1^0 + \sqrt{|\vec{l}_1 - \vec{l}_2|^2 + m_2^2} - \frac{i\varepsilon}{2\sqrt{|\vec{l}_1 - \vec{l}_2|^2 + m_2^2}}\right)^2 - |\vec{l}_2|^2 - m_3^2 + i\varepsilon} \end{aligned} \tag{227}$$

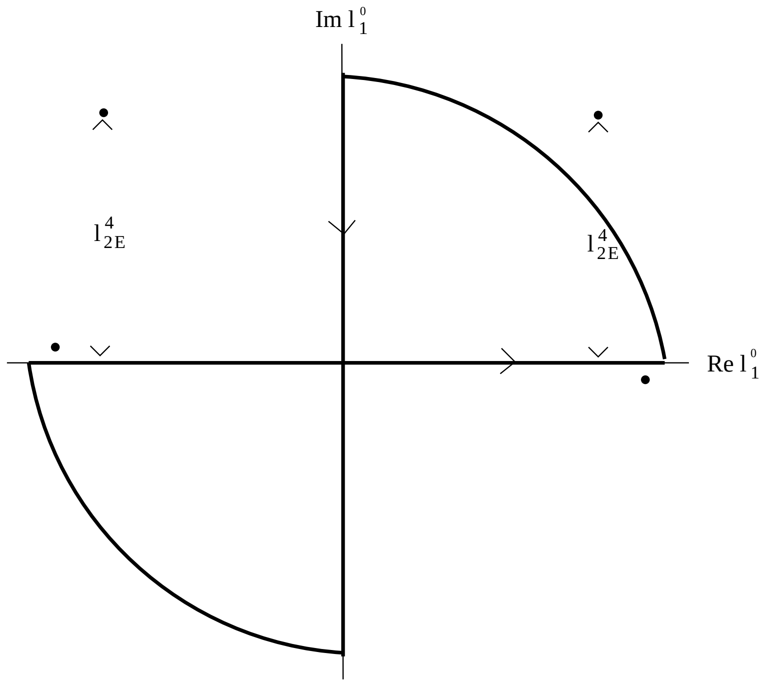
Now have a look at the residue terms. When one performs the variable substitution $l_1^0 \to -l_1^0$ in the last term one sees that this last term becomes exactly minus the second term! So the correction terms cancel each other and we are left with:

$$\begin{aligned} I &= -i\int d^4l_1 \frac{1}{l_1^2 - m_1^2 + i\varepsilon} \int d^4l_{2E} \frac{1}{(l_1^0 - il_{2E}^4)^2 - |\vec{l}_1 - \vec{l}_{2E}|^2 - m_2^2 + i\varepsilon} \frac{1}{l_{2E}^2 + m_3^2} = \\ & -i\int d^4l_{2E} \frac{1}{l_{2E}^2 + m_3^2} \\ & \int d^3l_1 dl_1^0 \frac{1}{\left(l_1^0 - \sqrt{|\vec{l}_1|^2 + m_1^2} + \frac{i\varepsilon}{2\sqrt{|\vec{l}_1|^2 + m_1^2}}\right)\left(l_1^0 + \sqrt{|\vec{l}_1|^2 + m_1^2} - \frac{i\varepsilon}{2\sqrt{|\vec{l}_1|^2 + m_1^2}}\right)} \\ & \frac{1}{\left(l_1^0 - il_{2E}^4 - \sqrt{|\vec{l}_1 - \vec{l}_{2E}|^2 + m_2^2} + \frac{i\varepsilon}{2\sqrt{|\vec{l}_1 - \vec{l}_{2E}|^2 + m_2^2}}\right)\left(l_1^0 - il_{2E}^4 + \sqrt{|\vec{l}_1 - \vec{l}_{2E}|^2 + m_2^2} - \frac{i\varepsilon}{2\sqrt{|\vec{l}_1 - \vec{l}_{2E}|^2 + m_2^2}}\right)}. \end{aligned} \tag{228}$$

Again we can read off the poles in the integrand of this expression. They are drawn in figure (7). Also here the poles of the first fraction in the l_1^0 integral give no problems. The poles of the second fraction do. If l_{2E}^4 is positive the pole on the right side of the imaginary axis is situated badly. On the other hand if l_{2E}^4 is negative the pole on the left side of the imaginary axis is inside the contour. Again we can perform the Wick rotation, but we must include terms for the pole inside the contour.

$$I = \int d^4l_{1E} d^4l_{2E} \frac{1}{l_{1E}^2 + m_1^2} \frac{1}{(l_{1E} - l_{2E})^2 + m_2^2} \frac{1}{l_{2E}^2 + m_3^2} +$$

Figure 7: The position of the poles in the complex l_1^0 plane



$$2\pi\int d^3l_{2E}\int_{l_{2E}^4>0} dl_{2E}^4\frac{1}{l_{2E}^2+m_3^2}\int d^3l_1\frac{1}{-2il_{2E}^4+2\sqrt{|\vec{l}_1-\vec{l}_{2E}|^2+m_2^2}-\frac{i\varepsilon}{\sqrt{|\vec{l}_1-\vec{l}_{2E}|^2+m_2^2}}}$$
$$\frac{1}{\big(-2il_{2E}^4+\sqrt{|\vec{l}_1-\vec{l}_{2E}|^2+m_2^2}-\frac{i\varepsilon}{2\sqrt{|\vec{l}_1-\vec{l}_{2E}|^2+m_2^2}}\big)^2-|\vec{l}_1|^2-m_1^2+i\varepsilon}+$$
$$2\pi\int d^3l_{2E}\int_{l_{2E}^4<0} dl_{2E}^4\frac{1}{l_{2E}^2+m_3^2}\int d^3l_1\frac{1}{-2il_{2E}^4-2\sqrt{|\vec{l}_1-\vec{l}_{2E}|^2+m_2^2}+\frac{i\varepsilon}{\sqrt{|\vec{l}_1-\vec{l}_{2E}|^2+m_2^2}}}$$
$$\frac{1}{\big(-2il_{2E}^4-\sqrt{|\vec{l}_1-\vec{l}_{2E}|^2+m_2^2}+\frac{i\varepsilon}{2\sqrt{|\vec{l}_1-\vec{l}_{2E}|^2+m_2^2}}\big)^2-|\vec{l}_1|^2-m_1^2+i\varepsilon} \tag{229}$$

Again we see that, when we perform the variable substitution $l_{2E}^4 \to -l_{2E}^4$ in the last term, the two correction terms cancel. So we are left with:

$$I=\int d^4l_{1E}d^4l_{2E}\frac{1}{l_{1E}^2+m_1^2}\frac{1}{(l_{1E}-l_{2E})^2+m_2^2}\frac{1}{l_{2E}^2+m_3^2}. \tag{230}$$

Now we can finally use the regularization scheme (222). The integral (230) then becomes:

$$\begin{aligned} I \;=\; & \int_{\Lambda^{-2}}^{\infty} d\alpha_1 d\alpha_2 d\alpha_3 \\ & \int d^4l_{1E}d^4l_{2E}\exp(-\alpha_1(l_{1E}^2+m_1^2)-\alpha_2((l_{1E}-l_{2E})^2+m_2^2)-\alpha_3(l_{2E}^2+m_3^2)) = \\ & \int_{\Lambda^{-2}}^{\infty} d\alpha_1 d\alpha_2 d\alpha_3 e^{-\alpha_1 m_1^2-\alpha_2 m_2^2-\alpha_3 m_3^2} \\ & \int d^4l_{1E}d^4l_{2E}\exp\Big(-(\alpha_1+\alpha_2)\big(l_{1E}-\frac{\alpha_2}{\alpha_1+\alpha_2}l_{2E}\big)^2-\big(\alpha_2+\alpha_3-\frac{\alpha_2^2}{\alpha_1+\alpha_2}\big)l_{2E}^2\Big) \end{aligned} \tag{231}$$

Now we can perform a variable shift in l_{1E} without any problems. Our boundaries of the l_{1E} and l_{2E} integrals are at infinity, so these boundaries do not change and we don't get the problems we would have gotten when we used a normal cut-off to regularize our integrals. So we make the variable substitutions:

$$\begin{aligned} l_1' &= \sqrt{\alpha_1+\alpha_2}\big(l_{1E} - \frac{\alpha_2}{\alpha_1+\alpha_2} l_{2E}\big) \\ l_2' &= \sqrt{\alpha_2+\alpha_3 - \frac{\alpha_2^2}{\alpha_1+\alpha_2}} l_{2E}. \end{aligned} \tag{232}$$

Our integral then becomes:

$$\begin{aligned} I &= \int_{\Lambda^{-2}}^{\infty} d\alpha_1 d\alpha_2 d\alpha_3 \frac{1}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)^2} e^{-\alpha_1 m_1^2 - \alpha_2 m_2^2 - \alpha_3 m_3^2} \\ & \int d^4 l_1' d^4 l_2' e^{-(l_1')^2} e^{-(l_2')^2}. \end{aligned} \tag{233}$$

Now use

$$\int d^4 l e^{-l^2} = \pi^2 \tag{234}$$

and our integral becomes

$$I = \pi^4 \int_{\Lambda^{-2}}^{\infty} d\alpha_1 d\alpha_2 d\alpha_3 \frac{1}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)^2} e^{-\alpha_1 m_1^2 - \alpha_2 m_2^2 - \alpha_3 m_3^2}. \tag{235}$$

Now we should wonder how to calculate the α integrals. We should remember that we are only interested in the quadratically divergent part of (235). This part comes from the area where all α's are small. There the exponential is very close to 1 and can be left out. This is nice, since this exponential prevents us form calculating the integral (235) exactly. Also, as can be seen below, the exponential is not needed to make the integral finite for large α's. It appears that the fraction already converges enough to make this happen. So, if we are only interested in the quadratically divergent part, our integral is:

$$I = \pi^4 \int_{\Lambda^{-2}}^{\infty} d\alpha_1 d\alpha_2 d\alpha_3 \frac{1}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)^2}. \tag{236}$$

One might wonder whether or not it is correct to leave out the exponential in (235). The calculation can also be done in a mathematically more elegant way. One could replace the integral (235) by an integral where the upper integration boundaries are some finite quantities instead of infinity. The parts we then leave out will never give quadratic divergences. In the integral, which now has finite boundaries, we can then expand the exponential around the point where all masses are zero. From this expansion only the first term, which is just 1, will give quadratic divergences. After discarding the higher order terms the integral can then also be calculated exactly. For the quadratically divergent part one then finds exactly the same as we will find below.

The integral (236) can now be solved if we introduce the variables β:

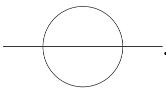
$$\beta_1 = \frac{1}{\alpha_1}, \quad \beta_2 = \frac{1}{\alpha_2}, \quad \beta_3 = \frac{1}{\alpha_3}. \tag{237}$$

With this substitution I becomes:

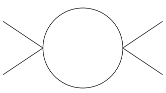
$$\begin{aligned} I &= \pi^4 \int_0^{\Lambda^2} d\beta_1 d\beta_2 d\beta_3 \frac{1}{(\beta_1+\beta_2+\beta_3)^2} = \\ & \pi^4 \int_0^{\Lambda^2} d\beta_1 d\beta_2 [-\frac{1}{\beta_1+\beta_2+\beta_3}]_0^{\Lambda^2} = \end{aligned}$$

$$\pi^4 \int_0^{\Lambda^2} d\beta_1 d\beta_2 \frac{1}{\beta_1+\beta_2} - \pi^4 \int_0^{\Lambda^2} d\beta_1 d\beta_2 \frac{1}{\beta_1+\beta_2+\Lambda^2} =$$
$$\pi^4 \int_0^{\Lambda^2} d\beta_1 [\ln(\beta_1+\beta_2)]_0^{\Lambda^2} - \pi^4 \int_0^{\Lambda^2} d\beta_1 [\ln(\beta_1+\beta_2+\Lambda^2)]_0^{\Lambda^2} =$$
$$\pi^4 \int_0^{\Lambda^2} d\beta_1 \ln(\beta_1+\Lambda^2) - \pi^4 \int_0^{\Lambda^2} d\beta_1 \ln(\beta_1) -$$
$$\pi^4 \int_0^{\Lambda^2} d\beta_1 \ln(\beta_1+2\Lambda^2) + \pi^4 \int_0^{\Lambda^2} d\beta_1 \ln(\beta_1+\Lambda^2) =$$
$$\pi^4(2\Lambda^2\ln(2\Lambda^2) - 2\Lambda^2 - \Lambda^2\ln(\Lambda^2) + \Lambda^2) -$$
$$\pi^4(\Lambda^2\ln(\Lambda^2) - \Lambda^2 + 0) -$$
$$\pi^4(3\Lambda^2\ln(3\Lambda^2) - 3\Lambda^2 - 2\Lambda^2\ln(2\Lambda^2) + 2\Lambda^2) +$$
$$\pi^4(2\Lambda^2\ln(2\Lambda^2) - 2\Lambda^2 - \Lambda^2\ln(\Lambda^2) + \Lambda^2) =$$
$$\pi^4(2\ln 2 - 3\ln 3 + 2\ln 2 + 2\ln 2)\Lambda^2 + \pi^4(2-1-1-3+2+2-1)\Lambda^2\ln(\Lambda^2) =$$
$$\pi^4 \ln\big(2\frac{10}{27}\big)\Lambda^2. \tag{238}$$

We see that the coefficients in front of terms with a $\Lambda^2\ln(\Lambda^2)$ exactly cancel. This is quite surprising since such a term is typically a term from a subdivergence. For example in φ^4 theory the integral we just calculated belongs to the diagram

.

Such a diagram has 3 subdiagrams, one for each internal line we can leave out. All these subdiagrams are logarithmically divergent. With the internal line attached again the worst divergence of the diagram is then expected to be proportional to $\Lambda^2\ln\Lambda$. As we saw above however this is not the case. In our expectation a very important thing is forgotten. The 3 subdiagrams, which look like:



also give a term $\ln p^2$, where p is an external momentum that is integrated over when one places the subdiagram in the larger diagram. Such a term must of course also be present on dimensional grounds if a term $\ln\Lambda^2$ is present. One would be inclined to discard this term because one is only looking for the most divergent term at 2-loop order. This does not mean however that one can throw away all terms which are finite in the subdiagram! The integral over p still makes a badly divergent term from the $\ln p^2$ term! It appears that this forgotten term exactly cancels the term proportional to $\Lambda^2\ln\Lambda$ which we only expected. Also the $\ln p^2$ term of the subdiagram gives the term proportional to Λ^2 which we indeed got in our calculations.

That a term proportional to $\Lambda^2\ln\Lambda$ cannot occur in (235) can also be seen in the following way. If such a term was present, also a term proportional to $\Lambda^2\ln m$, where m is a mass, should be present on dimensional grounds. Such a term would blow up if we take $m=0$. So then also our original integral (235) should blow up if we set $m=0$. This, as we saw above, is not the case, indeed confirming that the terms proportional to $\Lambda^2\ln\Lambda$ should vanish.

Now we calculated the quadratically divergent part of (221). There are however more integrals we will run into at 2-loop order. Some of these are independent, like for example:

$$I = \int d^4l_1 d^4l_2 \frac{1}{l_1^2 - m_1^2 + i\varepsilon}\frac{1}{(l_2^2 - m_2^2 + i\varepsilon)^2}. \tag{239}$$

With help of appendix 1 we can then easily see how this integral diverges. We will also run into the following type of integrals:

$$I = \int d^4l_1 d^4l_2 \frac{1}{(l_1^2 - m_1^2 + i\varepsilon)^a} \frac{1}{((l_1 - l_2)^2 - m_2^2 + i\varepsilon)^b}. \tag{240}$$

where a and b both are larger than 0. With the regularization scheme we have chosen also these integrals can be dealt with. First on should again do a Wick rotation where, as in the case of the integral (221), one again encounters badly situated poles. Again the correction terms will cancel and one is left with the Euclidean form of (240). Here one then uses our regularization scheme (222) again. Then one can do a variable substitution so that finally (240) reduces to 2 independent integrals.

In this way we will be able to handle all integrals which we shall encounter at 2-loop order.

8 Appendix 4

Below is printed the FORM program LOOPS.frm.

```
#define order "4"

S x, y, z, e, SL, SLC, PL, PLC, SR, SRC, PR, PRC, VLLLL, VRRRR, VLRLR, ieps, n;

#do i = 'order', 0, -1
        I mu'i', nu'i', m'i', n'i';
        V k'i';
#enddo

#do i = 1, 'order'
        V l'i';
#enddo

S ffdum1, ffdum2;

I mudum, nudum, mdum, ndum, mu, nu, a, b;

V q, p, kdum, ldum;

CF DD, gamma, gamma5;

F D;

L M = D('order', x, x, p, x, x, x, x, x, q, x, x, x);

* Below we start using the Schwinger Dyson equations for the expression D above. This we do until only D's at order 0 are left.

#do i = 'order', 1, -1
repeat;
* Using the Schwinger Dyson equations for the first photino line.
        if (match(D('i', x, ?a, x, ?b, x, ?c, x, ?d, x, kdum?, mdum?, ffdum1?, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j)));
                id D('i', x, ?a, x, ?b, x, ?c, x, ?d, x, kdum?, mdum?, ffdum1?, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) =
                        DD('i', x, ?a, x, ?b, x, ?c, x, ?d, x, kdum, mdum, ffdum1, y, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                        D('i', x, ?a, x, ?b, x, ?c, x, ?d, x, kdum, mdum, ffdum1, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j);
                id D('i', x, ?a, x, ?b, x, ?c, x, ?d, x, kdum?, mdum?, -1, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) =
                        i_*gamma(-kdum, mdum, m'i')/(kdum.kdum+ieps)*(
                        (SL*d_(m'i', n'i')+PL*gamma5(m'i', n'i'))*
                        D({'i'-1}, x, ?a, x, kdum-k'i', ?b, x, ?c, x, ?d, x, ?e, x, k'i', n'i', -1, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                        (SR*d_(m'i', n'i')+PR*gamma5(m'i', n'i'))*
                        D({'i'-1}, x, ?a, x, ?b, x, kdum-k'i', ?c, x, ?d, x, ?e, x, k'i', n'i', -1, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                        (-SLC*d_(m'i', n'i')+PLC*gamma5(m'i', n'i'))*
                        D({'i'-1}, x, k'i', n'i', -1, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, kdum-k'i', ?g, x, ?h, x, ?i, x, ?j) +
                        (-SRC*d_(m'i', n'i')+PRC*gamma5(m'i', n'i'))*
                        D({'i'-1}, x, k'i', n'i', -1, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, kdum-k'i', ?h, x, ?i, x, ?j));
                id D('i', x, ?a, x, ?b, x, ?c, x, ?d, x, kdum?, mdum?, 1, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) =
                        i_*gamma(kdum, m'i', mdum)/(kdum.kdum+ieps)*(
                        (SL*d_(n'i', m'i')+PL*gamma5(n'i', m'i'))*
                        D({'i'-1}, x, ?a, x, kdum-k'i', ?b, x, ?c, x, ?d, x, ?e, x, k'i', n'i', 1, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                        (SR*d_(n'i', m'i')+PR*gamma5(n'i', m'i'))*
                        D({'i'-1}, x, ?a, x, ?b, x, kdum-k'i', ?c, x, ?d, x, ?e, x, k'i', n'i', 1, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                        (-SLC*d_(n'i', m'i')+PLC*gamma5(n'i', m'i'))*
                        D({'i'-1}, x, k'i', n'i', 1, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, kdum-k'i', ?g, x, ?h, x, ?i, x, ?j) +
                        (-SRC*d_(n'i', m'i')+PRC*gamma5(n'i', m'i'))*
                        D({'i'-1}, x, k'i', n'i', 1, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, kdum-k'i', ?h, x, ?i, x, ?j));
                repeat;
                        id DD('i', x, ?a, x, ?b, x, ?c, x, ?d, x, kdum?, mdum?, ffdum1?, ?e1, y, ldum?, ndum?, ffdum2?, ?e2, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) =
                                D(0, x, x, x, x, x, kdum, mdum, ffdum1, ldum, ndum, ffdum2, x, x, x, x, x)*
                                D('i', x, ?a, x, ?b, x, ?c, x, ?d, x, ?e1, ?e2, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                                DD('i', x, ?a, x, ?b, x, ?c, x, ?d, x, kdum, mdum, ffdum1, ?e1, ldum, ndum, ffdum2, y, ?e2, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j);
                endrepeat;
                id DD('i', x, ?a, x, ?b, x, ?c, x, ?d, x, ?e, y, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) = 0;
        endif;
* Using the Schwinger Dyson equations for the first electron line.
        if (match(D('i', x, kdum?, mdum?, ffdum1?, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j)));
                id D('i', x, kdum?, mdum?, ffdum1?, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) =
                        DD('i', x, kdum, mdum, ffdum1, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, y, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                        D('i', x, kdum, mdum, ffdum1, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j);
                id D('i', x, kdum?, mdum?, -1, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) =
                        i_*gamma(-kdum, mdum, m'i')/(kdum.kdum+ieps)*(
                        i_*e*gamma(mu'i', m'i', n'i')*
                        D({'i'-1}, x, k'i', n'i', -1, ?a, x, ?b, x, ?c, x, kdum-k'i', mu'i', ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                        (SL*d_(m'i', n'i')+PL*gamma5(m'i', n'i'))*
                        D({'i'-1}, x, ?a, x, kdum-k'i', ?b, x, ?c, x, ?d, x, k'i', n'i', -1, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                        (SR*d_(m'i', n'i')+PR*gamma5(m'i', n'i'))*
                        D({'i'-1}, x, ?a, x, ?b, x, kdum-k'i', ?c, x, ?d, x, k'i', n'i', -1, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j));
                id D('i', x, kdum?, mdum?, 1, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) =
                        i_*gamma(kdum, m'i', mdum)/(kdum.kdum+ieps)*(
                        -i_*e*gamma(mu'i', n'i', m'i')*
                        D({'i'-1}, x, k'i', n'i', 1, ?a, x, ?b, x, ?c, x, kdum-k'i', mu'i', ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                        (SL*d_(n'i', m'i')+PL*gamma5(n'i', m'i'))*
                        D({'i'-1}, x, ?a, x, kdum-k'i', ?b, x, ?c, x, ?d, x, k'i', n'i', 1, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                        (SR*d_(n'i', m'i')+PR*gamma5(n'i', m'i'))*
                        D({'i'-1}, x, ?a, x, ?b, x, kdum-k'i', ?c, x, ?d, x, k'i', n'i', 1, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j));
                repeat;
                        id DD('i', x, kdum?, mdum?, ffdum1?, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f1, y, ldum?, ndum?, ffdum2?, ?f2, x, ?g, x, ?h, x, ?i, x, ?j) =
                                D(0, x, kdum, mdum, ffdum1, x, x, x, x, x, ldum, ndum, ffdum2, x, x, x, x)*
                                D('i', x, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f1, ?f2, x, ?g, x, ?h, x, ?i, x, ?j) +
                                DD('i', x, kdum, mdum, ffdum1, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f1, ldum, ndum, ffdum2, y, ?f2, x, ?g, x, ?h, x, ?i, x, ?j);
                endrepeat;
                id DD('i', x, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f, y, x, ?g, x, ?h, x, ?i, x, ?j) = 0;
        endif;
* Using the Schwinger Dyson equations for the first anti-electron line.
        if (match(D('i', x, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, kdum?, mdum?, ffdum1?, ?f, x, ?g, x, ?h, x, ?i, x, ?j)));
                id D('i', x, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, kdum?, mdum?, ffdum1, ?f, x, ?g, x, ?h, x, ?i, x, ?j) =
                        DD('i', x, y, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, kdum, mdum, ffdum1, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                        D('i', x, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, kdum, mdum, ffdum1, ?f, x, ?g, x, ?h, x, ?i, x, ?j);
                id D('i', x, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, kdum?, mdum?, -1, ?f, x, ?g, x, ?h, x, ?i, x, ?j) =
                        i_*gamma(kdum, mdum, m'i')/(kdum.kdum+ieps)*(
                        -i_*e*gamma(mu'i', m'i', n'i')*
                        D({'i'-1}, x, ?a, x, ?b, x, ?c, x, kdum-k'i', mu'i', ?d, x, ?e, x, k'i', n'i', -1, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                        (-SLC*d_(m'i', n'i')+PLC*gamma5(m'i', n'i'))*
                        D({'i'-1}, x, ?a, x, ?b, x, ?c, x, ?d, x, k'i', n'i', -1, ?e, x, ?f, x, kdum-k'i', ?g, x, ?h, x, ?i, x, ?j) +
```

```
                        (-SRC*d_(m‘i’, n‘i’)+PRC*gamma5(m‘i’, n‘i’))*
                        D({‘i’-1}, x, ?a, x, ?b, x, ?c, x, ?d, x, k‘i’, n‘i’, -1, ?e, x, ?f, x, ?g, x, kdum-k‘i’, ?h, x, ?i, x, ?j));
                id D(‘i’, x, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, kdum?, mdum?, 1, ?f, x, ?g, x, ?h, x, ?i, x, ?j) =
                        i_*gamma(-kdum, m‘i’, mdum)/(kdum.kdum+ieps)*(
                        i_*e*gamma(mu‘i’, n‘i’, m‘i’)*
                        D({‘i’-1}, x, ?a, x, ?b, x, ?c, x, kdum-k‘i’, mu‘i’, ?d, x, ?e, x, k‘i’, n‘i’, 1, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                        (-SLC*d_(n‘i’, m‘i’)+PLC*gamma5(n‘i’, m‘i’))*
                        D({‘i’-1}, x, ?a, x, ?b, x, ?c, x, ?d, x, k‘i’, n‘i’, 1, ?e, x, ?f, x, kdum-k‘i’, ?g, x, ?h, x, ?i, x, ?j) +
                        (-SRC*d_(n‘i’, m‘i’)+PRC*gamma5(n‘i’, m‘i’))*
                        D({‘i’-1}, x, ?a, x, ?b, x, ?c, x, ?d, x, k‘i’, n‘i’, 1, ?e, x, ?f, x, ?g, x, kdum-k‘i’, ?h, x, ?i, x, ?j));
                repeat;
                        id DD(‘i’, x, ?a1, y, ldum?, ndum?, ffdum2?, ?a2, x, ?b, x, ?c, x, ?d, x, ?e, x, kdum?, mdum?, ffdum1?, ?f, x, ?g, x, ?h, x, ?i, x, ?j) =
                                D(0, x, ldum, ndum, ffdum2, x, x, x, x, x, kdum, mdum, ffdum1, x, x, x, x)*
                                D(‘i’, x, ?a1, ?a2, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                                DD(‘i’, x, ?a1, ldum, ndum, ffdum2, y, ?a2, x, ?b, x, ?c, x, ?d, x, ?e, x, kdum, mdum, ffdum1, ?f, x, ?g, x, ?h, x, ?i, x, ?j);
                endrepeat;
                id DD(‘i’, x, ?a, y, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) = 0;
        endif;
* Using the Schwinger Dyson equations for the first photon line.
        id D(‘i’, x, ?a, x, ?b, x, ?c, x, kdum?, mudum?, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) =
                DD(‘i’, x, ?a, x, ?b, x, ?c, x, kdum, mudum, y, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                D(‘i’, x, ?a, x, ?b, x, ?c, x, kdum, mudum, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j);
        id D(‘i’, x, ?a, x, ?b, x, ?c, x, kdum?, mudum?, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) =
                -i_/(kdum.kdum+ieps)*(
                -i_*e*(2*k‘i’(mudum)-kdum(mudum))*
                D({‘i’-1}, x, ?a, x, k‘i’, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, kdum-k‘i’, ?g, x, ?h, x, ?i, x, ?j) +
                -i_*e*(2*k‘i’(mudum)-kdum(mudum))*
                D({‘i’-1}, x, ?a, x, ?b, x, k‘i’, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, kdum-k‘i’, ?h, x, ?i, x, ?j) +
                -i_*e*gamma(mudum, m‘i’, n‘i’)*
                D({‘i’-1}, x, k‘i’, m‘i’, 1, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, kdum-k‘i’, n‘i’, -1, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                2*i_*e^2*d_(mudum, mu‘i’)*
                D({‘i’-2}, x, ?a, x, k‘i’, ?b, x, ?c, x, k{‘i’-1}, mu‘i’, ?d, x, ?e, x, ?f, x, kdum-k‘i’-k{‘i’-1}, ?g, x, ?h, x, ?i, x, ?j) +
                2*i_*e^2*d_(mudum, mu‘i’)*
                D({‘i’-2}, x, ?a, x, ?b, x, k‘i’, ?c, x, k{‘i’-1}, mu‘i’, ?d, x, ?e, x, ?f, x, ?g, x, kdum-k‘i’-k{‘i’-1}, ?h, x, ?i, x, ?j));
        repeat;
                id DD(‘i’, x, ?a, x, ?b, x, ?c, x, kdum?, mudum?, ?d1, y, ldum?, nudum?, ?d2, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) =
                        D(0, x, x, x, x, kdum, mudum, ldum, nudum, x, x, x, x, x, x)*
                        D(‘i’, x, ?a, x, ?b, x, ?c, x, ?d1, ?d2, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                        DD(‘i’, x, ?a, x, ?b, x, ?c, x, kdum, mudum, ?d1, ldum, nudum, y, ?d2, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j);
        endrepeat;
        id DD(‘i’, x, ?a, x, ?b, x, ?c, x, ?d, y, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) = 0;
* Using the Schwinger Dyson equations for the first left selectron line.
        id D(‘i’, x, ?a, x, kdum?, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) =
                DD(‘i’, x, ?a, x, kdum, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, y, ?g, x, ?h, x, ?i, x, ?j) +
                D(‘i’, x, ?a, x, kdum, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j);
        id D(‘i’, x, ?a, x, kdum?, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) =
                i_/(kdum.kdum+ieps)*(
                -i_*e*(2*kdum(mu‘i’)-k‘i’(mu‘i’))*
                D({‘i’-1}, x, ?a, x, kdum-k‘i’, ?b, x, ?c, x, k‘i’, mu‘i’, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                (-SLC*d_(m‘i’, n‘i’)+PLC*gamma5(m‘i’, n‘i’))*
                D({‘i’-1}, x, k‘i’, m‘i’, 1, ?a, x, ?b, x, ?c, x, ?d, x, kdum-k‘i’, n‘i’, -1, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                i_*e^2*d_(mu‘i’, nu‘i’)*
                D({‘i’-2}, x, ?a, x, kdum-k‘i’-k{‘i’-1}, ?b, x, ?c, x, k‘i’, mu‘i’, k{‘i’-1}, nu‘i’, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                1/2*VLLLL*
                D({‘i’-2}, x, ?a, x, k‘i’, k{‘i’-1}, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, kdum-k‘i’-k{‘i’-1}, ?g, x, ?h, x, ?i, x, ?j) +
                VLRLR*
                D({‘i’-2}, x, ?a, x, k‘i’, ?b, x, k{‘i’-1}, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, kdum-k‘i’-k{‘i’-1}, ?h, x, ?i, x, ?j));
        repeat;
                id DD(‘i’, x, ?a, x, kdum?, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g1, y, ldum?, ?g2, x, ?h, x, ?i, x, ?j) =
                        D(0, x, x, kdum, x, x, x, x, x, ldum, x, x, x)*
                        D(‘i’, x, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g1, ?g2, x, ?h, x, ?i, x, ?j) +
                        DD(‘i’, x, ?a, x, kdum, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g1, ldum, y, ?g2, x, ?h, x, ?i, x, ?j);
        endrepeat;
        id DD(‘i’, x, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g, y, x, ?h, x, ?i, x, ?j) = 0;
* Using the Schwinger Dyson equations for the first right selectron line.
        id D(‘i’, x, ?a, x, ?b, x, kdum?, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) =
                DD(‘i’, x, ?a, x, ?b, x, kdum, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, y, ?h, x, ?i, x, ?j) +
                D(‘i’, x, ?a, x, ?b, x, kdum, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j);
        id D(‘i’, x, ?a, x, ?b, x, kdum?, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) =
                i_/(kdum.kdum+ieps)*(
                -i_*e*(2*kdum(mu‘i’)-k‘i’(mu‘i’))*
                D({‘i’-1}, x, ?a, x, ?b, x, kdum-k‘i’, ?c, x, k‘i’, mu‘i’, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                (-SRC*d_(m‘i’, n‘i’)+PRC*gamma5(m‘i’, n‘i’))*
                D({‘i’-1}, x, k‘i’, m‘i’, 1, ?a, x, ?b, x, ?c, x, ?d, x, kdum-k‘i’, n‘i’, -1, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                i_*e^2*d_(mu‘i’, nu‘i’)*
                D({‘i’-2}, x, ?a, x, ?b, x, kdum-k‘i’-k{‘i’-1}, ?c, x, k‘i’, mu‘i’, k{‘i’-1}, nu‘i’, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) +
                1/2*VRRRR*
                D({‘i’-2}, x, ?a, x, ?b, x, k‘i’, k{‘i’-1}, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, kdum-k‘i’-k{‘i’-1}, ?h, x, ?i, x, ?j) +
                VLRLR*
                D({‘i’-2}, x, ?a, x, k{‘i’-1}, ?b, x, k‘i’, ?c, x, ?d, x, ?e, x, ?f, x, kdum-k‘i’-k{‘i’-1}, ?g, x, ?h, x, ?i, x, ?j));
        repeat;
                id DD(‘i’, x, ?a, x, ?b, x, kdum?, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h1, y, ldum?, ?h2, x, ?i, x, ?j) =
                        D(0, x, x, x, kdum, x, x, x, x, x, ldum, x, x)*
                        D(‘i’, x, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h1, ?h2, x, ?i, x, ?j) +
                        DD(‘i’, x, ?a, x, ?b, x, kdum, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h1, ldum, y, ?h2, x, ?i, x, ?j);
        endrepeat;
        id DD(‘i’, x, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h, y, x, ?i, x, ?j) = 0;
endrepeat;
#enddo

id D(-1, ?a) = 0;
id D(n?{>0}, x, x, x, x, x, x, x, x, x, x) = 0;

.sort

V adum, bdum;

F phinoprops, eprops, phoprops, seLprops, seRprops;

* Below we will replace the D’s at order 0 by the functions phinoprops, eprops, phoprops, seLprops and seRprops. These functions will ultimately
* become the (lowest order) propagators of the particles.

id D(?a) = phinoprops*D(?a);
repeat;
        id phinoprops(?k)*D(0, x, ?a, x, ?b, x, ?c, x, ?d, x, adum?, mdum?, ffdum1?, ?e, bdum?, ndum?, ffdum2?, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) =
        phinoprops(?k, adum, mdum, ffdum1, bdum, ndum, ffdum2)*D(0, x, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j);
endrepeat;
```

```
id phinoprops = 1;

id D(?a) = eprops(x)*D(?a);
repeat;
        id eprops(?k, x, ?l)*D(0, x, adum?, mdum?, ffdum1?, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, bdum?, ndum?, ffdum2?, ?f, x, ?g, x, ?h, x, ?i, x, ?j) =
        eprops(?k, adum, mdum, ffdum1, x, ?l, bdum, ndum, ffdum2)*D(0, x, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j);
endrepeat;
id eprops(x) = 1;

id D(?a) = phoprops*D(?a);
repeat;
        id phoprops(?k)*D(0, x, ?a, x, ?b, x, ?c, x, adum?, mudum?, ?d, bdum?, nudum?, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) =
        phoprops(?k, adum, mudum, bdum, nudum)*D(0, x, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j);
endrepeat;
id phoprops = 1;

id D(?a) = seLprops(x)*D(?a);
repeat;
        id seLprops(?k, x, ?l)*D(0, x, ?a, x, adum?, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, bdum?, ?g, x, ?h, x, ?i, x, ?j) =
        seLprops(?k, adum, x, ?l, bdum)*D(0, x, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j);
endrepeat;
id seLprops(x) = 1;

id D(?a) = seRprops(x)*D(?a);
repeat;
        id seRprops(?k, x, ?l)*D(0, x, ?a, x, ?b, x, adum?, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, bdum?, ?h, x, ?i, x, ?j) =
        seRprops(?k, adum, x, ?l, bdum)*D(0, x, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j);
endrepeat;
id seRprops(x) = 1;

id D(0, x, adum?, mdum?, ffdum1?, ?a, x, ?b, x, ?c, x, ?d, x, ?e, x, x, ?g, x, ?h, x, ?i, x, ?j) = 0;
id D(0, x, x, ?b, x, ?c, x, ?d, x, ?e, x, bdum?, ndum?, ffdum2?, ?f, x, ?g, x, ?h, x, ?i, x, ?j) = 0;
id D(0, x, ?a, x, adum?, ?b, x, ?c, x, ?d, x, ?e, x, ?f, x, x, ?h, x, ?i, x, ?j) = 0;
id D(0, x, ?a, x, x, ?c, x, ?d, x, ?e, x, ?f, x, bdum?, ?g, x, ?h, x, ?i, x, ?j) = 0;
id D(0, x, ?a, x, ?b, x, adum?, ?c, x, ?d, x, ?e, x, ?f, x, ?g, x, x, ?i, x, ?j) = 0;
id D(0, x, ?a, x, ?b, x, x, ?d, x, ?e, x, ?f, x, ?g, x, bdum?, ?h, x, ?i, x, ?j) = 0;
id D(0, x, ?a, x, ?b, x, ?c, x, adum?, mudum?, x, ?e, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) = 0;
id D(0, x, ?a, x, ?b, x, ?c, x, ?d, x, adum?, mdum?, ffdum1?, x, ?f, x, ?g, x, ?h, x, ?i, x, ?j) = 0;
id D(0, x, x, x, x, x, x, x, x, x, x) = 1;

.sort

* Below we will symmetrize the propagator functions phoprops, seLprops and seRprops. This is needed when a propagator function has more than 2
* arguments, then the legs have to be tied in all different ways.

id seLprops(?a, x, ?b) = seLprops(?a, x, y, ?b);

repeat;
        id seLprops(?a, x, ?b, y, adum?, ?c) = seLprops(?a, x, z, adum, ?b, y, ?c);
        repeat;
                id seLprops(?a, x, ?b, z, adum?, bdum?, ?c, y, ?d) =
                seLprops(?a, x, ?b, adum, bdum, ?c, y, ?d) + seLprops(?a, x, ?b, bdum, z, adum, ?c, y, ?d);

                id seLprops(?a, x, ?b, z, adum?, y, ?c) = seLprops(?a, x, ?b, adum, y, ?c);
        endrepeat;
endrepeat;

id seLprops(?a, y) = seLprops(?a);

id seRprops(?a, x, ?b) = seRprops(?a, x, y, ?b);

repeat;
        id seRprops(?a, x, ?b, y, adum?, ?c) = seRprops(?a, x, z, adum, ?b, y, ?c);
        repeat;
                id seRprops(?a, x, ?b, z, adum?, bdum?, ?c, y, ?d) =
                seRprops(?a, x, ?b, adum, bdum, ?c, y, ?d) + seRprops(?a, x, ?b, bdum, z, adum, ?c, y, ?d);

                id seRprops(?a, x, ?b, z, adum?, y, ?c) = seRprops(?a, x, ?b, adum, y, ?c);
        endrepeat;
endrepeat;

id seRprops(?a, y) = seRprops(?a);

id phoprops(?a) = phoprops(y, ?a);

repeat;
        id phoprops(?a, y, adum?, mudum?, ?b) = 1/sqrt_(2)*phoprops(z, adum, mudum, ?a, y, ?b);
        repeat;
                id phoprops(?a, z, adum?, mudum?, bdum?, nudum?, ?b, y, ?c) =
                phoprops(?a, adum, mudum, bdum, nudum, ?b, y, ?c) + phoprops(?a, bdum, nudum, z, adum, mudum, ?b, y, ?c);

                id phoprops(?a, z, adum?, mudum?, y, ?b) = phoprops(?a, adum, mudum, y, ?b);
        endrepeat;
endrepeat;
id 1/sqrt_(2)/sqrt_(2) = 1/2;

id phoprops(?a, y) = phoprops(?a);

.sort

CF delta, inpr, mom;

Set l:
#do i = 1, `order'
        ,l`i'
#enddo
;

* Here we will replace all propagator functions by their proper expressions.

id adum?.bdum? = inpr(adum, bdum);
argument;
        id adum?.bdum? = inpr(adum, bdum);
endargument;

multiply mom();
```

```
while (match(eprops(adum?, mdum?, ffdum1?, ?a, x, bdum?, ndum?, ffdum2?, ?b)));
        if (match(eprops(adum?, mdum?, -1, ?a, x, bdum?, ndum?, 1, ?b)));
                id once eprops(adum?, mdum?, -1, ?a, x, bdum?, ndum?, 1, ?b) = i_*gamma(-adum, mdum, ndum)/
                        (inpr(adum, adum)+ieps)*delta(adum+bdum)*eprops(?a, x, ?b);
        else;
                id once eprops(adum?, mdum?, 1, ?a, x, bdum?, ndum?, -1, ?b) = i_*gamma(adum, ndum, mdum)/
                        (inpr(adum, adum)+ieps)*delta(adum+bdum)*eprops(?a, x, ?b);
        endif;
        splitfirstarg delta;
        $var1 = 0;
        id once delta(adum?$var1, bdum?$var2) = 1;
        id once delta(bdum?$var2) = 1;
        argument;
                id $var2 = -$var1;
                argument;
                        id $var2 = -$var1;
                endargument;
        endargument;
        id mom(?a) = mom(?a, $var2);
endwhile;

id eprops(x) = 1;

while (match(seLprops(adum?, ?a, x, bdum?, ?b)));
        id once seLprops(adum?, ?a, x, bdum?, ?b) = i_/(inpr(adum, adum)+ieps)*delta(adum+bdum)*seLprops(?a, x, ?b);
        splitfirstarg delta;
        $var1 = 0;
        id once delta(adum?$var1, bdum?$var2) = 1;
        id once delta(bdum?$var2) = 1;
        argument;
                id $var2 = -$var1;
                argument;
                        id $var2 = -$var1;
                endargument;
        endargument;
        id mom(?a) = mom(?a, $var2);
endwhile;

id seLprops(x) = 1;

while (match(seRprops(adum?, ?a, x, bdum?, ?b)));
        id once seRprops(adum?, ?a, x, bdum?, ?b) = i_/(inpr(adum, adum)+ieps)*delta(adum+bdum)*seRprops(?a, x, ?b);
        splitfirstarg delta;
        $var1 = 0;
        id once delta(adum?$var1, bdum?$var2) = 1;
        id once delta(bdum?$var2) = 1;
        argument;
                id $var2 = -$var1;
                argument;
                        id $var2 = -$var1;
                endargument;
        endargument;
        id mom(?a) = mom(?a, $var2);
endwhile;

id seRprops(x) = 1;

while (match(phoprops(adum?, mudum?, ?a, bdum?, nudum?)));
        id once phoprops(adum?, mudum?, ?a, bdum?, nudum?) = -i_*d_(mudum, nudum)/(inpr(adum, adum)+ieps)*delta(adum+bdum)*phoprops(?a);
        splitfirstarg delta;
        $var1 = 0;
        id once delta(adum?$var1, bdum?$var2) = 1;
        id once delta(bdum?$var2) = 1;
        argument;
                id $var2 = -$var1;
                argument;
                        id $var2 = -$var1;
                endargument;
        endargument;
        id mom(?a) = mom(?a, $var2);
endwhile;

id phoprops = 1;

while (match(phinoprops(adum?, mdum?, ffdum1?, ?a, bdum?, ndum?, ffdum2?)));
        if (match(phinoprops(adum?, mdum?, -1, ?a, bdum?, ndum?, 1)));
                id once phinoprops(adum?, mdum?, -1, ?a, bdum?, ndum?, 1) = i_*gamma(-adum, mdum, ndum)/
                        (inpr(adum, adum)+ieps)*delta(adum+bdum)*phinoprops(?a);
        else;
                id once phinoprops(adum?, mdum?, 1, ?a, bdum?, ndum?, -1) = i_*gamma(adum, ndum, mdum)/
                        (inpr(adum, adum)+ieps)*delta(adum+bdum)*phinoprops(?a);
        endif;
        splitfirstarg delta;
        $var1 = 0;
        id once delta(adum?$var1, bdum?$var2) = 1;
        id once delta(bdum?$var2) = 1;
        argument;
                id $var2 = -$var1;
                argument;
                        id $var2 = -$var1;
                endargument;
        endargument;
        id mom(?a) = mom(?a, $var2);
endwhile;

id phinoprops = 1;

.sort

$var = 1;

#do i = 1, `order'
        if ((match(mom(?a, k`i', ?b)) = 0) && (match(mom(?a, -k`i', ?b)) = 0));
                id k`i' = l[$var];
                argument;
                        id k`i' = l[$var];
                        argument;
```

```
                                id k`i' = l[$var];
                        endargument;
                endargument;
                $var = $var + 1;
        endif;
#enddo
id mom(?a) = 1;

id gamma(0, ?a) = 0;

id inpr(0, 0) = 0;
id inpr(adum?, 0) = 0;
id inpr(0, bdum?) = 0;
id inpr(adum?, bdum?) = adum.bdum;
argument;
id inpr(0, 0) = 0;
id inpr(adum?, 0) = 0;
id inpr(0, bdum?) = 0;
id inpr(adum?, bdum?) = adum.bdum;
endargument;

.sort

I odum, g5, sl;

F gam;

* Below all products of gamma matrices are worked out.

id gamma5(mdum?, ndum?) = gamma(g5, mdum, ndum);
repeat;
        id gamma(?a, mdum?, ndum?)*gamma(?b, ndum?, odum?) = gamma(?a, ?b, mdum, odum);
endrepeat;

#do i = 1, `order'
        id once gamma(?a, mdum?, mdum?) = gam(?a, `i');
#enddo

repeat;
        if (match(gam(g5, ?a, sl?)) = 0);
                id gam(mudum?, ?a, sl?) = g_(sl, mudum)*gam(?a, sl);
        else;
                id gam(g5, ?a, sl?) = g5_(sl)*gam(?a, sl);
        endif;
endrepeat;

* Here we take the trace of all sequences of gamma matrices. Also the fermion minus sign is taken into account here, with every trace corresponds
* a fermion loop and thus a fermion minus sign.

id gam(sl?) = -1;

#do i = 1, `order'
        trace4, `i';
#enddo

.sort

S powl1, powl2, powp, l1l2mix, o;

CF den;

* Below we start rewriting the integrands to identify the quadratically divergent parts. This part of this program is strictly for 2-loop calculations,
* and is not suited for any number of loops, like the previous part of this program. Also it is only suited to deal with processes with 2 external legs.

* First we expand all fractions with an external momentum p in the denominator around p = 0 and we keep only the first term in the expansion, since
* only this term will give quadratic divergencies. This amounts to just setting p equal to 0.

argument;
        id p = 0;
endargument;

* Then we remove the 1-particle reducible diagrams from our complete expression. We also remove the lowest order propagators of the two external legs.

id only ieps^-3 = 0;
id only ieps^-2 = -1;

* In the block below we identify the powers of loop momenta in each term with the variables powl1 and powl2. We also indicate integrals which are
* not independant with the variable l1l2mix.

id l1 = powl1*l1;
id l2 = powl2*l2;
id p = powp*p;
id p.p^-1 = powp^-2*p.p^-1;
id denom_(?a) = den(?a);
splitarg (l1.l1) den;
splitarg (l2.l2) den;
id den(n?, l1.l1) = powl1^-2*den(n+l1.l1);
id den(n?, l2.l2) = powl2^-2*den(n+l2.l2);
id den(n?, l2.l2, l1.l1) = powl1^-2*powl2^-2*l1l2mix*den(n+l1.l1+l2.l2);
id den(n?) = denom_(n);

* Below we discard integrals which really vanish because of their odd integrand. This also removes diagrams which have a tadpole as a subdiagram
* from our total expression. Only after we removed these we can set ieps to zero.

if (match(l1l2mix));
        if (match(powl1^n?even_)) id powl2^o?odd_ = 0;
        if (match(powl1^n?odd_)) id powl2^o?even_ = 0;
endif;
if (match(l1l2mix) = 0) id powl1^n?odd_ = 0;
if (match(l1l2mix) = 0) id powl2^n?odd_ = 0;

argument;
        id ieps = 0;
endargument;

.sort
```

```
S A, B;

* Below we will write all expressions to standard integrals. First we want to remove all loop momenta from the numerators.

id l1.p^2 = 1/4*p.p*l1.l1;
id l2.p^2 = 1/4*p.p*l2.l2;
id l1.p*l2.p = 1/4*p.p*l1.l2;

* Then we see to it that only factors (l1.l1-2*l1.l2+l2.l2), and no factors (l1.l1+2*l1.l2+l2.l2) occur in the denominators.

if (match(1/(l1.l1+2*l1.l2+l2.l2)));
        id l1 = -l1;
        argument;
                id l1 = -l1;
        endargument;
endif;

* Now all the p's still occuring in numerators will only give less than quadratically divergent terms, so we set these terms to zero.

id p = 0;

* Factors l1.l2 still occuring in the numerator are dealt with below.

repeat;
        id l1.l2/(l1.l1-2*l1.l2+l2.l2)*powl1^-1*powl2^-1*l1l2mix = -1/2 + 1/2*l1.l1/(l1.l1-2*l1.l2+l2.l2)*powl2^-2*l1l2mix +
                                                                          1/2*l2.l2/(l1.l1-2*l1.l2+l2.l2)*powl1^-2*l1l2mix;
endrepeat;

* Below we rewrite integrals which can be made independant to such integrals by a variable substitution.

if ((match(l1l2mix)) && (match(l1.l1^-1) = 0));
        id 1/(l1.l1-2*l1.l2+l2.l2) = 1/(l1.l1-2*l1.l2+l2.l2)*powl2^2/l1l2mix;
        id l1 = l1 + l2*powl1^-1*powl2;
        argument;
                id l1 = l1 + l2;
        endargument;
endif;
if ((match(l1l2mix)) && (match(l2.l2^-1) = 0));
        id 1/(l1.l1-2*l1.l2+l2.l2) = 1/(l1.l1-2*l1.l2+l2.l2)*powl1^2/l1l2mix;
        id l2 = l2 + l1*powl1*powl2^-1;
        argument;
                id l2 = l2 + l1;
        endargument;
endif;

* Then we throw away terms which are not quadratically divergent.

if (match(l1l2mix) = 0);
        id powl1^n?odd_ = 0;
        id powl2^n?odd_ = 0;
        if ((match(powl1^n?{>-3}) = 0) && (match(powl2^n?{>-3}) = 0)) discard;
endif;

if (match(l1l2mix));
        id powl1^n?{<-3}*powl2^o?{<-4} = 0;
        id powl1^n?{<-4}*powl2^o?{<-3} = 0;
endif;

* Of the quadratically divergent terms where the 2 integrals are independant we also discard the following group.

if (match(l1l2mix) = 0);
        id powl1^n?{<-4} = 0;
        id powl2^n?{<-4} = 0;
endif;

* Finally we define the standardintegrals A and B.

id only powl1^-4*powl2^-4*l1l2mix*l1.l1^-1*l2.l2^-1/(l1.l1-2*l1.l2+l2.l2) = A;
id only powl1^-2*powl2^-4*l1.l1^-1*l2.l2^-2 = B;
id only powl1^-4*powl2^-2*l1.l1^-2*l2.l2^-1 = B;

bracket A, B;

print M;

.sort

* By substituting the known coupling constants we can test whether our calculation was correct.

id SL = e/sqrt_(2);
id SLC = e/sqrt_(2);
id PL = e/sqrt_(2);
id PLC = e/sqrt_(2);
id SR = e/sqrt_(2);
id SRC = e/sqrt_(2);
id PR = -e/sqrt_(2);
id PRC = -e/sqrt_(2);
id VLLLL = -2*i_*e^2;
id VRRRR = -2*i_*e^2;
id VLRLR = i_*e^2;
id 1/sqrt_(2)/sqrt_(2) = 1/2;

bracket A, B;

print M;

.end
```

9 Appendix 5

Below is printed the output of LOOPS.frm in the case of $\tilde{e}_L \to \tilde{e}_L$. We replaced the program code by some dots in the output since it is rather lengthy and can already be seen in appendix 4.

```
FORM by J.Vermaseren,version 3.0(Jan 31 2001) Run at: Thu Feb  5 11:17:17 2004
    #define order "4"

    S x, y, z, e, SL, SLC, PL, PLC, SR, SRC, PR, PRC, VLLLL, VRRRR, VLRLR,
      ieps, n;

    #do i = `order', 0, -1
            I mu`i', nu`i', m`i', n`i';
     V k`i';
    #enddo

    #do i = 1, `order'
     V l`i';
    #enddo

    S ffdum1, ffdum2;

    I mudum, nudum, mdum, ndum, mu, nu, a, b;

    V q, p, kdum, ldum;

    CF DD, gamma, gamma5;

    F D;

    L M = D(`order', x, x, p, x, x, x, x, x, q, x, x, x);


    ..........


Time =    1044.01 sec    Generated terms =        174
                M        Terms in output =         37
                         Bytes used      =        820

   M =
       + A * (  - 2*i_*e^2*SL*SLC - 2*i_*e^2*PL*PLC + 11*i_*e^4 - 16*i_*SL*SLC
         *PL*PLC - 10*i_*SL*SLC*SR*SRC - 2*i_*SL*SLC*PR*PRC + 2*i_*SL*PLC*SR*
         PRC - 6*i_*SL*PLC*SRC*PR - 10*i_*SL^2*SLC^2 + 2*i_*SL^2*PLC^2 - 6*i_*
         SLC*PL*SR*PRC + 2*i_*SLC*PL*SRC*PR + 2*i_*SLC^2*PL^2 - 2*i_*PL*PLC*SR
         *SRC - 10*i_*PL*PLC*PR*PRC - 10*i_*PL^2*PLC^2 - 1/2*i_*VLLLL^2 - i_*
         VLRLR^2 + 2*e^2*VLLLL + 2*e^2*VLRLR - 2*SL*SLC*VLLLL - 2*PL*PLC*VLLLL
          - 2*SR*SRC*VLRLR - 2*PR*PRC*VLRLR )

       + B * ( 4*i_*e^2*SL*SLC + 4*i_*e^2*PL*PLC - 3*i_*e^4 - i_*VLLLL*VLRLR
          - i_*VLLLL^2 - i_*VRRRR*VLRLR - i_*VLRLR^2 - 2*e^2*VLLLL - 2*e^2*
         VLRLR + 4*SL*SLC*VLLLL + 4*PL*PLC*VLLLL + 4*SR*SRC*VLRLR + 4*PR*PRC*
         VLRLR );
```

```
* By substituting the known coupling constants we can test whether our
* calculation was correct.

id SL = e/sqrt_(2);
id SLC = e/sqrt_(2);
id PL = e/sqrt_(2);
id PLC = e/sqrt_(2);
id SR = e/sqrt_(2);
id SRC = e/sqrt_(2);
id PR = -e/sqrt_(2);
id PRC = -e/sqrt_(2);
id VLLLL = -2*i_*e^2;
id VRRRR = -2*i_*e^2;
id VLRLR = i_*e^2;
id 1/sqrt_(2)/sqrt_(2) = 1/2;

bracket A, B;

print M;

.end
```

```
Time =    1044.02 sec    Generated terms =          37
                M        Terms in output =           0
                         Bytes used      =           2

   M = 0;
```

10 Appendix 6

Below is printed the output of LOOPS.frm in the case of $\tilde{e}_R \to \tilde{e}_R$. Again the program code is left out.

```
FORM by J.Vermaseren,version 3.0(Jan 31 2001) Run at: Thu Feb  5 12:30:04 2004
    #define order "4"

    S x, y, z, e, SL, SLC, PL, PLC, SR, SRC, PR, PRC, VLLLL, VRRRR, VLRLR,
      ieps, n;

    #do i = 'order', 0, -1
            I mu'i', nu'i', m'i', n'i';
     V k'i';
    #enddo

    #do i = 1, 'order'
     V l'i';
    #enddo

    S ffdum1, ffdum2;

    I mudum, nudum, mdum, ndum, mu, nu, a, b;

    V q, p, kdum, ldum;

    CF DD, gamma, gamma5;

    F D;

    L M = D('order', x, x, x, p, x, x, x, x, x, q, x, x);


    ..........


Time =    1064.15 sec    Generated terms =        163
                M        Terms in output =         37
                         Bytes used      =        792

   M =
       + A * (  - 2*i_*e^2*SR*SRC - 2*i_*e^2*PR*PRC + 11*i_*e^4 - 10*i_*SL*SLC
         *SR*SRC - 2*i_*SL*SLC*PR*PRC + 2*i_*SL*PLC*SR*PRC - 6*i_*SL*PLC*SRC*
         PR - 6*i_*SLC*PL*SR*PRC + 2*i_*SLC*PL*SRC*PR - 2*i_*PL*PLC*SR*SRC -
         10*i_*PL*PLC*PR*PRC - 16*i_*SR*SRC*PR*PRC - 10*i_*SR^2*SRC^2 + 2*i_*
         SR^2*PRC^2 + 2*i_*SRC^2*PR^2 - 10*i_*PR^2*PRC^2 - 1/2*i_*VRRRR^2 - i_
         *VLRLR^2 + 2*e^2*VRRRR + 2*e^2*VLRLR - 2*SL*SLC*VLRLR - 2*PL*PLC*
         VLRLR - 2*SR*SRC*VRRRR - 2*PR*PRC*VRRRR )

       + B * ( 4*i_*e^2*SR*SRC + 4*i_*e^2*PR*PRC - 3*i_*e^4 - i_*VLLLL*VLRLR
          - i_*VRRRR*VLRLR - i_*VRRRR^2 - i_*VLRLR^2 - 2*e^2*VRRRR - 2*e^2*
         VLRLR + 4*SL*SLC*VLRLR + 4*PL*PLC*VLRLR + 4*SR*SRC*VRRRR + 4*PR*PRC*
         VRRRR );
```

```
* By substituting the known coupling constants we can test whether our
* calculation was correct.

id SL = e/sqrt_(2);
id SLC = e/sqrt_(2);
id PL = e/sqrt_(2);
id PLC = e/sqrt_(2);
id SR = e/sqrt_(2);
id SRC = e/sqrt_(2);
id PR = -e/sqrt_(2);
id PRC = -e/sqrt_(2);
id VLLLL = -2*i_*e^2;
id VRRRR = -2*i_*e^2;
id VLRLR = i_*e^2;
id 1/sqrt_(2)/sqrt_(2) = 1/2;

bracket A, B;

print M;

.end
```

```
Time =    1064.16 sec    Generated terms =         37
                M        Terms in output =          0
                         Bytes used      =          2

   M = 0;
```

11 Appendix 7

Below is printed the output of LOOPS.frm in the case of $\tilde{e}_L \to \tilde{e}_R$. Again the program code is left out.

```
FORM by J.Vermaseren,version 3.0(Jan 31 2001) Run at: Thu Feb  5 11:40:30 2004
    #define order "4"

    S x, y, z, e, SL, SLC, PL, PLC, SR, SRC, PR, PRC, VLLLL, VRRRR, VLRLR,
      ieps, n;

    #do i = ‘order’, 0, -1
            I mu‘i’, nu‘i’, m‘i’, n‘i’;
     V k‘i’;
    #enddo

    #do i = 1, ‘order’
     V l‘i’;
    #enddo

    S ffdum1, ffdum2;

    I mudum, nudum, mdum, ndum, mu, nu, a, b;

    V q, p, kdum, ldum;

    CF DD, gamma, gamma5;

    F D;

    L M = D(‘order’, x, x, p, x, x, x, x, x, x, q, x, x);


    ..........


Time =     1016.67 sec    Generated terms =         98
                 M        Terms in output =         20
                          Bytes used      =        510

   M =
       + A * (  - 2*i_*e^2*SLC*SR - 2*i_*e^2*PLC*PR - 8*i_*SL*SLC*PLC*PR - 10*
         i_*SL*SLC^2*SR + 2*i_*SL*PLC^2*SR - 8*i_*SLC*PL*PLC*SR - 8*i_*SLC*SR*
         PR*PRC - 10*i_*SLC*SR^2*SRC + 2*i_*SLC*SRC*PR^2 + 2*i_*SLC^2*PL*PR -
         10*i_*PL*PLC^2*PR - 8*i_*PLC*SR*SRC*PR + 2*i_*PLC*SR^2*PRC - 10*i_*
         PLC*PR^2*PRC - 2*SLC*SR*VLRLR - 2*PLC*PR*VLRLR )

       + B * ( 4*i_*e^2*SLC*SR + 4*i_*e^2*PLC*PR + 4*SLC*SR*VLRLR + 4*PLC*PR*
         VLRLR );


    * By substituting the known coupling constants we can test whether our
    * calculation was correct.

    id SL = e/sqrt_(2);
```

```
    id SLC = e/sqrt_(2);
    id PL = e/sqrt_(2);
    id PLC = e/sqrt_(2);
    id SR = e/sqrt_(2);
    id SRC = e/sqrt_(2);
    id PR = -e/sqrt_(2);
    id PRC = -e/sqrt_(2);
    id VLLLL = -2*i_*e^2;
    id VRRRR = -2*i_*e^2;
    id VLRLR = i_*e^2;
    id 1/sqrt_(2)/sqrt_(2) = 1/2;

    bracket A, B;

    print M;

    .end

Time =     1016.68 sec    Generated terms =         20
                 M        Terms in output =          0
                          Bytes used      =          2

   M = 0;
```

12 Appendix 8

Below is printed the output of LOOPS.frm in the case of $e \to e$. Again the program code is left out. The output is not the actual electron propagator, but proportional to the trace of $\not{p}$ times the actual electron propagator.

```
FORM by J.Vermaseren,version 3.0(Jan 31 2001) Run at: Thu Feb  5 12:56:38 2004
    #define order "4"

    S x, y, z, e, SL, SLC, PL, PLC, SR, SRC, PR, PRC, VLLLL, VRRRR, VLRLR,
      ieps, n;

    #do i = ‘order’, 0, -1
            I mu‘i’, nu‘i’, m‘i’, n‘i’;
     V k‘i’;
    #enddo

    #do i = 1, ‘order’
     V l‘i’;
    #enddo

    S ffdum1, ffdum2;

    I mudum, nudum, mdum, ndum, mu, nu, a, b;

    V q, p, kdum, ldum;

    CF DD, gamma, gamma5;

    F D;

    L M = 1/4*p.p^-2*gamma(p, a, b)*
          D(‘order’, x, p, a, 1, x, x, x, x, x, q, b, -1, x, x, x, x);


    ..........


Time =     1997.48 sec    Generated terms =          0
                 M        Terms in output =          0
                          Bytes used      =          2

   M = 0;


    * By substituting the known coupling constants we can test whether our
    * calculation was correct.

    id SL = e/sqrt_(2);
    id SLC = e/sqrt_(2);
    id PL = e/sqrt_(2);
    id PLC = e/sqrt_(2);
    id SR = e/sqrt_(2);
    id SRC = e/sqrt_(2);
    id PR = -e/sqrt_(2);
```

```
    id PRC = -e/sqrt_(2);
    id VLLLL = -2*i_*e^2;
    id VRRRR = -2*i_*e^2;
    id VLRLR = i_*e^2;
    id 1/sqrt_(2)/sqrt_(2) = 1/2;

    bracket A, B;

    print M;

    .end

Time =    1997.49 sec    Generated terms =          0
                M        Terms in output =          0
                         Bytes used      =          2

   M = 0;
```

13 Appendix 9

Below is printed the output of LOOPS.frm in the case of $\tilde{\gamma} \to \tilde{\gamma}$. Again the program code is left out. Also here the output is the trace of $\not{p}$ times the actual photino propagator.

```
FORM by J.Vermaseren,version 3.0(Jan 31 2001) Run at: Thu Feb  5 13:36:16 2004
    #define order "4"

    S x, y, z, e, SL, SLC, PL, PLC, SR, SRC, PR, PRC, VLLLL, VRRRR, VLRLR,
      ieps, n;

    #do i = ‘order’, 0, -1
            I mu‘i’, nu‘i’, m‘i’, n‘i’;
     V k‘i’;
    #enddo

    #do i = 1, ‘order’
     V l‘i’;
    #enddo

    S ffdum1, ffdum2;

    I mudum, nudum, mdum, ndum, mu, nu, a, b;

    V q, p, kdum, ldum;

    CF DD, gamma, gamma5;

    F D;

    L M = 1/4*p.p^-2*gamma(p, a, b)*
          D(‘order’, x, x, x, x, x, p, a, 1, q, b, -1, x, x, x, x, x);


    ..........


Time =     2839.85 sec    Generated terms =          0
                 M        Terms in output =          0
                          Bytes used      =          2

   M = 0;


    * By substituting the known coupling constants we can test whether our
    * calculation was correct.

    id SL = e/sqrt_(2);
    id SLC = e/sqrt_(2);
    id PL = e/sqrt_(2);
    id PLC = e/sqrt_(2);
    id SR = e/sqrt_(2);
    id SRC = e/sqrt_(2);
    id PR = -e/sqrt_(2);
    id PRC = -e/sqrt_(2);
```

```
    id VLLLL = -2*i_*e^2;
    id VRRRR = -2*i_*e^2;
    id VLRLR = i_*e^2;
    id 1/sqrt_(2)/sqrt_(2) = 1/2;

    bracket A, B;

    print M;

    .end

Time =    2839.85 sec    Generated terms =          0
                M        Terms in output =          0
                         Bytes used      =          2

   M = 0;
```

14 Appendix 10

Below is printed the output of `LOOPS.frm` in the case of $\gamma \to \gamma$. Again the program code is left out. The output gives the photon propagator contracted with $g^{\mu\nu}$.

```
FORM by J.Vermaseren,version 3.0(Jan 31 2001) Run at: Thu Feb  5 14:50:52 2004
    #define order "4"

    S x, y, z, e, SL, SLC, PL, PLC, SR, SRC, PR, PRC, VLLLL, VRRRR, VLRLR,
      ieps, n;

    #do i = `order', 0, -1
            I mu`i', nu`i', m`i', n`i';
     V k`i';
    #enddo

    #do i = 1, `order'
     V l`i';
    #enddo

    S ffdum1, ffdum2;

    I mudum, nudum, mdum, ndum, mu, nu, a, b;

    V q, p, kdum, ldum;

    CF DD, gamma, gamma5;

    F D;

    L M = D(`order', x, x, x, x, p, mu, q, mu, x, x, x, x, x, x);


    ..........


Time =    1185.31 sec    Generated terms =        124
                M        Terms in output =          0
                         Bytes used      =          2

   M = 0;


    * By substituting the known coupling constants we can test whether our
    * calculation was correct.

    id SL = e/sqrt_(2);
    id SLC = e/sqrt_(2);
    id PL = e/sqrt_(2);
    id PLC = e/sqrt_(2);
    id SR = e/sqrt_(2);
    id SRC = e/sqrt_(2);
    id PR = -e/sqrt_(2);
    id PRC = -e/sqrt_(2);
    id VLLLL = -2*i_*e^2;
```

```
id VRRRR = -2*i_*e^2;
id VLRLR = i_*e^2;
id 1/sqrt_(2)/sqrt_(2) = 1/2;

bracket A, B;

print M;

.end

Time =    1185.31 sec    Generated terms =          0
                M        Terms in output =          0
                         Bytes used      =          2

   M = 0;
```

15 Appendix 11

Below is printed the output of `LOOPS.frm` in the case of $\gamma \to \gamma$. Again the program code is left out. The output gives the photon propagator contracted with $\frac{p^\mu p^\nu}{p^2}$.

```
FORM by J.Vermaseren,version 3.0(Jan 31 2001) Run at: Thu Feb  5 15:22:07 2004
    #define order "4"

    S x, y, z, e, SL, SLC, PL, PLC, SR, SRC, PR, PRC, VLLLL, VRRRR, VLRLR,
      ieps, n;

    #do i = ‘order’, 0, -1
            I mu‘i’, nu‘i’, m‘i’, n‘i’;
     V k‘i’;
    #enddo

    #do i = 1, ‘order’
     V l‘i’;
    #enddo

    S ffdum1, ffdum2;

    I mudum, nudum, mdum, ndum, mu, nu, a, b;

    V q, p, kdum, ldum;

    CF DD, gamma, gamma5;

    F D;

    L M = 1/p.p*p(mu)*p(nu)*
          D(‘order’, x, x, x, x, p, mu, q, nu, x, x, x, x, x, x);


    ..........


Time =     1161.04 sec    Generated terms =        219
                 M        Terms in output =          0
                          Bytes used      =          2

   M = 0;


    * By substituting the known coupling constants we can test whether our
    * calculation was correct.

    id SL = e/sqrt_(2);
    id SLC = e/sqrt_(2);
    id PL = e/sqrt_(2);
    id PLC = e/sqrt_(2);
    id SR = e/sqrt_(2);
    id SRC = e/sqrt_(2);
    id PR = -e/sqrt_(2);
    id PRC = -e/sqrt_(2);
```

```
    id VLLLL = -2*i_*e^2;
    id VRRRR = -2*i_*e^2;
    id VLRLR = i_*e^2;
    id 1/sqrt_(2)/sqrt_(2) = 1/2;

    bracket A, B;

    print M;

    .end

Time =    1161.04 sec    Generated terms =          0
                M        Terms in output =          0
                         Bytes used      =          2

   M = 0;
```

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