Sleeping Beauty: Exploring the long quiescent period of 1H 1905+000

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Preface

Hereby, I would like to express my gratitude towards all who have helped me with the work on this Bachelor thesis. I would like to name the following people in particular:
First and foremost Jonas Sweep, for the succesful cooperation on our research project and for his continuous support, scrutiny and help during the writing of this thesis. Kess Marks, for putting up with me during the stressful moments. Gijs Nelemans and Wim Beenakker, for the fruitful discussions on the project and their guidance through the jungle of established scientific knowledge. Paul Groot and Peter Jonker for their insights on the matter and their willingness to answer our questions.

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Chapter 1

Introduction

Neutron stars are the most compact objects known in our universe. They are born in supernova explosions of massive stars (masses ranging from M $\sim$ 8 $M_\odot$ to $\sim$ 30 $M_\odot$, were $M_\odot$ is the solar mass), at a temperature of $\sim 10^{11}$ K. During a supernova explosion most of the mass of the supernova progenitor is blasted away, leaving a much lighter kernel: a neutron star. Neutron stars have a mass up to 2 $M_\odot$, while their radii are R $\sim$ 10 km. They most likely consist of a massive dense core surrounded by a thin crust. The density of the core ranges from $1.4 \cdot 10^{11}$ gr/cm^3 at the crust-core interface to as high as 10 ρ_0 to 20 ρ_0 in the center (with $\rho_0 = 2.8 \cdot 10^{14}$ gr/cm^3, which is the standard nuclear density).
The neutron star crust can be divided in an atmosphere, an outer crust and an inner crust, whereas the core can be divided in an outer and inner core. The figure displayed below gives a schematic overview of the structure of a neutron star.

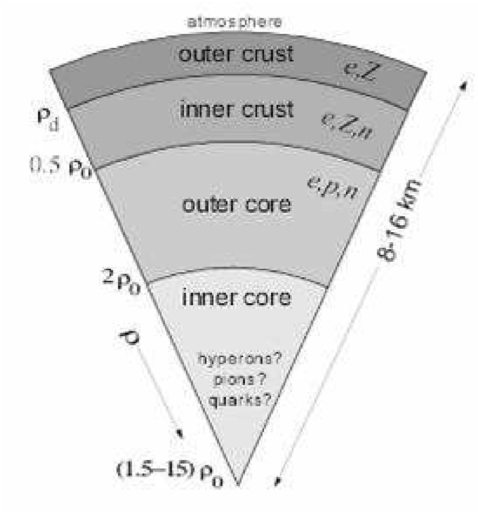

Figure 1.1: The global structure of a neutron star

- The atmosphere is a thin ($\sim$ 1 cm) plasma layer of iron in an isolated neutron star or hydrogen, helium and possibly (a little) heavier elements such as carbon if the neutron star is accreting matter. Accretion may take place if the neutron star is part of a binary system, in which it absorbs mass from the companion star.

- The outer crust (thickness ~ 0.4 km) consisting of neutron-rich nuclei and free electrons lies below the atmosphere and extends to the layer of density $\rho_d = 2.8 \cdot 10^{11}$ gr/cm^3. At this density, some neutrons can no longer be bound to a nucleus and form an independent neutron-fluid. This is done through a process whereby neutrons that are created through inverse β-decay are expelled from a nucleus because the only energy levels they can occupy exceed the binding energy.

- The inner crust lies below this layer and is made up from the same material as the outer crust with an admixture of neutrons because at densities of ρ_d and higher, more and more neutrons are expelled from their nuclei through the process described above.

- The outer core consists of highly degenerate matter. No nuclei can exist anymore because of the high density so matter consists of a Fermi gas of neutrons, protons, electrons and possibly muons (npeμ-matter). The composition of this gas is determined by beta equilibrium and global charge neutrality. This implies that matter in the neutron star is very neutron-rich. Hence the name neutron star. The density ranges from 0.5 ρ_0 to ~ 2 ρ_0.

- The inner core starts at a density of about 2 ρ_0 and is only present in massive neutron stars (M $\gtrsim$ M$_\odot$). The composition of the inner core and the equation of state (EOS) are very uncertain. There are several possibilities but most models assume a large fraction of npeμ-matter. Some models employ condensates of pions or kaons. Other models predict the appearance of hyperons such as Σ^- and Λ. More exotic models make use of a phase transition to deconfined quark matter.

After their birth, deprived of nuclear heat sources, neutron stars begin to cool down by thermal photon emission from the surface. However since the surface area of a neutron star is small, photon emission is not the main cooling mechanism, at least not in the first stages. A neutron star mainly cools by emitting neutrinos. Some 30 seconds after its birth, the star becomes transparent for neutrinos and it can lose a lot of thermal energy by emitting (lots of) neutrinos.
Most neutron stars are nicely explained by the cooling models developed so far. Recently however Jonker et al. (2006) suggested they had observed a neutron star binary system that was too cold to be explained by current theories. This particular binary system was a so-called soft X-ray transient (SXT. These systems experience very bright outburst during periods of accretion, interchanged with longer quiescent periods follows during which the star is much fainter.

The object we are discussing is 1H 1905+000, hereafter called Bob. It experienced an outburst of about eleven years with a luminosity of $4 \cdot 10^{36}$ ergs/s after which it went into quiescence more than ten years ago. During the quiescent phase the part of the sky where Bob stood during outburst was observed but a 300 ks Chandra survey did not detect anything, thus putting a stringent limit on Bob's quiescent photon luminosity of $L_{Bob} < 1.8 \cdot 10^{30}$ ergs/s. This limits effective surface temperature, measured at infinity, of Bob to $T_s^{\rm inf} < 3.5 \cdot 10^5$ K, assuming a canonical radius of 10 km. The effective surface temperature measured at infinity is lower than at the surface due to gravitational redshift, with a factor of $\sqrt{1 - R_s/R}$ where R_s is the Schwarzschild radius and R the neutron star radius.

No such faint SXT in quiescence has been observed so far. The faintness of Bob has prompted a debate between advocates of the ADAF theory and those who oppose it. Reason for this is that according to the ADAF theory, neutron star accretors should be significantly brighter than their black hole counterparts.

Opponents of this theory now claim that Bob, with its distinct faintness, clearly contradicts this theory as it is fainter than many observed black hole SXT's

In the following thesis an explanation will be given why Bob is as faint as he is. This explanation will have two distinct branches. One branch will involve the application of accretion models to Bob, thus explaining how hot Bob might have become after the outburst period or determining how long it takes to *not* significantly heat Bob with the amount of accreted matter given. The other branch will concentrate on Bob's interior and try to explain where all the heat Bob acquired went. The first branch will be covered by Frank Hemmes, the second by Jonas Sweep. Each branch will form a seperate Bachelor thesis but both theses together will give a complete explanation of Bob's behaviour.

Chapter 2

Background

As was mentioned in the introduction, 1H 1905+000 is a Soft X-ray Transient (SXT). SXT's are a subgroup of a certain class of binary systems, called Low Mass X-Ray Binaries (LMXB). LMXB's are binary systems where one of the stars is a compact object, i.e. a black hole or neutron star, accompanied by a low mass star with $M \leq M_\odot$ that donates mass to the compact object. A schematic overview of such a system can be seen in figure 2.1. The difference between a SXT and an ordinary LMXB is that the former accretes periodically, rather than continuously.
Transiently accreting stars have two distinct phases. The first phase is called an outburst and occurs when one star accretes a large amount of matter in a small time. This process of rapid accretion generates a lot of energy in two distinct ways. Firstly, the accreting matter moves ever deeper into the gravitational potential well, thus to states of lower gravitational energy. The excess energy is released in the form of radiation. Due to the high potential energy gradient close to the neutron star, most energy is emitted in the X-ray spectrum. Additionally, some stars emit indirectly in the optical part of the spectrum as well. Secondly, energy is released in the impact of matter on the surface of the neutron star. Part of this energy is used to heat the surface, and as a result is radiated away as thermal emission. In section 2.2 we will explore how part of the energy can also be stored within the star core and emitted at a later time.
After some time, usually a few weeks to months, the accretion disk has been depleted and the outburst stops. The SXT then enters its second phase, called quiescence. During quiescence, accretion onto the stellar surface practically halts and a new accretion disk is built up around the star. In this phase the luminosity is significantly lower than during outburst. This difference can range from two to six orders of magnitude. Typically, quiescent periods range from months to decades, depending on the outburst time and accretion rate of the system.
Even though the quiescent luminosity (L_q) of an SXT is much lower than during outburst, it might still be possible to observe it. This is certainly true in the case of neutron stars, which emit thermal radiation from their surface. For many neutron star SXT's, the observed L_q is higher than would be expected from their masses and radii. Although it was quickly realized that the quiescent luminosity is somehow connected to the accretion process, there are still competing theories as to what the responsible physical process should be. Two of these theories, one involving an Advection Dominated Accretion Flow (ADAF) and the other *deep crustal heating*, will be summarily introduced below.

2.1 The ADAF model

The ADAF theory, first proposed by Narayan & Yi in 1994, assumes that beyond a certain radius, the accretion disk becomes truncated because the matter evaporates. This evaporated matter then continues

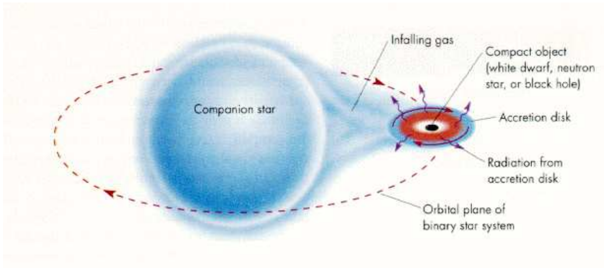

Figure 2.1: Schematic view of a binary system

to move towards the star in what is called an advection dominated accretion flow. The key idea is that the energy that is normally lost by radiation is now instead stored in the moving matter itself, making for an optically thick "cloud" of matter. As a result, the apparent luminosity of the system is lower than during an outburst, but the matter impacting on the star is still able to generate enough energy to sustain observed outburst luminosities, as it still contains the energy gained during its transport through the ADAF. The rate of mass transfer through the ADAF is also slower than if the disk itself would extend to the stellar surface. The result of this is that the disk remains stable for a longer period of time than it would without any form of continuous accretion. A disk is stable when it is either in quiescence or in outburst, but unstable when it is about to switch from one of these stages to another. The criteria for disk stability will be explained in detail later in this text. The prolonged stability makes this model useful for providing longer recurrence times for a given system, whilst still explaining the observed difference in luminosity between the outburst and quiescent state.

2.2 Deep crustal heating

Another model, put forward by Brown et al. (1998), considers *deep crustal heating* as the main process by which neutron star accretors acquire the energy needed to sustain their quiescent luminosities. It proposes that the matter deposited on the neutron star surface is the main source of energy. According to the model, the newly accreted matter itself loses most of its energy directly on impact by photon radiation from the stellar surface. However, the increase of mass at the surface of the neutron star increases pressure throughout the crust. This increase in pressure is sufficient to initiate pycnonuclear reactions deep in the stellar crust. Pycnonuclear reactions are nuclear fusion reactions that can occur at low temperatures if the pressure is sufficiently high. This opposed to thermonuclear reactions, which only occur above a certain critical temperature. Pycnonuclear reactions can, for instance, be induced by zero-point energy of arranged nuclei, which occur even at low temperatures. For high enough pressures, these zero-point energies can be high enough to traverse the Coulomb-barrier between nuclei, thus enabling fusion. This means that even in neutron stars where the temperature is too low to ignite thermonuclear reactions, fusion can still occur. Thus, if enough matter is deposited at the surface of the neutron star to significantly increase the pressure throughout the crust, pycnonuclear reactions occuring within the crust may provide an additional source of energy. The energy generated by these reactions is emitted in the form of neutrinos and photons. Due to the obliqueness of a neutron star for photons, it takes time for the energy to reach the surface and be emitted as photon radiation. Because of this delay, the energy is preserved within the crust, only to be emitted after the accretion process has stopped. In this way, the pycnonuclear reactions are responsible for the observed quiescent luminosity of a SXT. In this model, the quiescent luminosity is directly proportional to the rate of mass accretion, though it might be necessary to compensate for thermal relaxation of the crust if the outburst time is long. This is because it assumes a state of equilibrium between outburst and quiescent phases. According to Brown et al. an SXT is in steady state if the energy it gains during active accretion is equal to that emitted during quiescence. Effectively, the star is in a form of steady-state where the temperature does not notably change per cycle. Because the model assumes that energy is stored within the star for a period that is generally longer than the outburst time, it does not require continuous accretion to take that this model does not require continuous accretion during quiescence, as opposed to the ADAF model.

2.3 NS versus BH: Evidence for Event Horizons?

Recently, an interesting discussion has developed between both models described above, in which Bob plays an crucial part. This is because of a prediction made by the ADAF model, which states that black holes in quiescence should be significantly fainter than neutron stars. The reason for this is that the energy stored in the ADAF is released when matter is deposited on the surface of the accretor. However, in the case of a black hole, there is only an event horizon and no physical surface. At the moment the matter carried by the ADAF passes the event horizon, the energy contained within it is lost. This is different from a neutron star, where the matter radiates the stored energy when it impacts on the surface of the star.

The advocates of the ADAF model now claim the existence of such an advection flow would be evidence for black hole event horizons. Current observations show that SXT black holes have substantially smaller luminosities than their neutron star counterparts. They argue that within the ADAF model, this difference is explained by the existence of event horizons. However, this claim is still disputed, and one of the arguments against this theory is Bob. The quiescent luminosity of Bob is comparable to those of black holes, and even fainter than most of them. This seems to place Bob at odds with the interpretations of theADAF model, as it undermines the statement that black hole SXT's should be fainter than neutron stars. Proponents of the theory of deep crustal heating as such claim that Bob's behaviour is better explained by their model, while advocates of the ADAF theory refute this by stating that it is not only the quiescent luminosity that is important, but the ratio of L_q to the orbital period of the binary system. Assuming the ADAF theory holds, this would mean that black holes comparable to Bob in period would be even fainter during quiescence. During outburst, the difference in luminosity would be negated by the radiation from the accretion disc. Obviously, no such black holes have been observed so far, as their quiescent luminosity is too low for current methods of detection.

Chapter 3

Crustal Heating

When matter is accreted on a star, it provides it with energy in two different ways. First, the matter that impacts on the surface contains a lot of energy. This energy will mostly be radiated away immediately by thermal radiation. However, the process of adding mass to the primary also means the pressure of the underlying regions increases, because the mass that is added at the surface of the star now also presses on the lower regions of the crust. The accreted matter is then pushed into deeper layers of the star's surface as even more matter is piled onto it. When pressure and density continue to increase, the matter may be transformed by pycnonuclear reactions, neutron exchanges with the environment and β captures. All these processes generate energy, but contrary to the situation at the stellar surface, here it cannot radiate away instantaneously, because a neutron star is not transparant for photons. As a result, the interior of the neutron star is heated, especially the deeper layers of the crust. This energy can be lost in two different ways.

First there is thermal cooling. The energy released deep in the inner crust will increase its temperature. When this increase is significant enough and happens fairly quickly compared to the thermal relaxation timescale of the star, it will not be able to remain a thermal equilibrium between the inner crust and the core and outer crust. In normal SXT systems, where the recurrence times are low, the crust will not regain thermal equilibrium during the outburst. This means all the energy released will be stored in the crust during the outburst period. After the outburst, the neutron star will seek a new equilibrium and the heat will be transferred from the crust to the surface, where it can then be emitted as thermal radiation. Of course, a similar thing happens for SXT's with long outburst periods, but in such cases it is possible that the crust reaches thermal equilibrium before the outburst stops. This means that some of the energy generated inside can be transported to the surface and radiated away while accretion is still occuring. However, the luminosity from this process is several orders of magnitude smaller than the luminosities during accretion and thus not easily discernable.

A second process through which the star can loose energy from its interior is neutrino emission. Many of the processes that generate energy as a result of accretion also generate neutrinos. Even though neutron stars are the most dense objects in the universe, they are still transparent for neutrinos. Thus, every reaction that creates neutrinos results in a loss of energy for the neutron star, the amount of which depends on how much energy is given to the escaping neutrino, but is generaly of the order of kT.
A more detailed discussion on the various cooling mechanisms operating in neutron stars will be given by Jonas Sweep.

3.1 Deep Crustal Heating

Although the luminosity from the energy stored in the crust is much smaller than the luminosity from accretion, it becomes important during quiescence. The idea that the heat stored during accretion accounts for (most of the) observed luminosity of SXT's was first proposed by Brown et al in 1998. Since then, a number of other authors has also continued to develop the theory of *deep crustal heating*.
When applying the theory of deep crustal heating to Bob, there are two things to consider. First, there is the amount of energy gained during accretion. The calculation of this is relatively straightforward. When all aforementioned processes are combined, they yield about 1.2 - 1.6 MeV per accreted baryon. In this text we will adopt the formula recently proposed by Yakovlev et al. (2005) to calculate the amount of energy released for *deep crustal heating*.

$$L_{dh} = 1.45\text{MeV}\, \dot{M}/m_N \approx 8.74 \cdot 10^{33} \dot{M}/(10^{-10} M_{\odot}\text{yr}^{-1})\ \text{erg s}^{-1} \tag{3.1}$$

which gives the heating power deep in the crust. The average mass accretion rate during outburst can be found by dividing the $4 \cdot 10^{-9}$ $M_{\odot}$ by the 11 years of outburst and yields $\dot{M} \simeq 3.6 \cdot 10^{-10}$ erg s^{-1}. Using this and the outburst time t_o in equation (3.1), we can calcuate the total stored energy to be:

$$E_{dh} = 1.11 \cdot 10^{43}\text{erg} \tag{3.2}$$

From which the quiescent luminosity is easily calculated as:

$$L_q = \frac{E_{dh}}{t_r} \tag{3.3}$$

Here t_r is the "recurrence time" of the star, which is the time of one cycle of outburst and quiescence. If we would use the observed quiescent luminosity as input, we would gain a value for t_r of $1.77 \cdot 10^5$ yr. Here we took the recurrence and quiescent periods the same, since they differ only by 10 yr, which is neglectible. However, due to the abnormally long outburst time of Bob, matters are not as straigtforward as they seem.

3.2 Energy distribution

Although the previous calculations were quite simple, there is an additional issue that makes matters somewhat more complicated. The problem is that not all heat generated by deep crustal heating is actually used to heat up the core. It is only the energy stored there that will eventually be used to power the neutron star radiation during quiescence. However, a part of the heat can immediately flow outward and be radiated thermally from the surface and as such is lost on a much smaller timescale. Determining which fraction f of the energy released deep in the crust is eventually used to heat up the core is thus the main problem in estimating the quiescent luminosity of a star.
For short-period SXT's this still does not pose a real problem. During a short outburst, not enough heat is generated to significantly heat up the crust. As the temperature remains practically equal to a situation without *deep crustal heating*, the radiation emitted at the crust surface will also not deviate significantly. The additional energy generated by *deep crustal heating* cannot be radiated and remains to heat the core. Thus, it is safe to set $f \approx 1$ in these situations.
However, for longer outburst times, the crust can heat up more than 10%. Subsequently, the star will start to emit significantly more thermal radiation then during its quiescent period. The energy required to do so is deducted from the total amount of energy that is released through *deep crustal heating*, and we can set f to be (much) smaller than 1 as a result. Unfortunately, there is no clear-cut recipe to calculate f. It can be inferred from the neutron star EOS and the timescales for thermal diffusion and relaxation, but

that is beyond the scope of this text. What can be done, is using equation (3.3) to express L_i as a function of f when the recurrence time t_r is known. The recurrence time could then be inferred from the values calculated in chapter 4, i.e. from the allowed timescales on which the accretion disc can remain stable.

Chapter 4

Accretion

One of the aspects we need to take into consideration when viewing Bobs recurrence time is the process of accretion. In this chapter, I will first give a theoretical overview of the accretion process and the various theories that have been proposed to describe it. Then I will explain which theories I used, and how I applied them, to create a computer model of Bobs accretion disk. The latter was done to look at Bobs specific case and provide boundaries on the time Bobs disk could have remained stable given certain values for the mass transfer to the accretion disk. Lastly, there is a discussion of the results and viability of the model and other processes that could be of influence tot Bobs behaviour.

4.1 Theoretical background

One of the most distinct features of binary systems is accretion, the process whereby one of the stars in the system loses matter to its companion (hereafter called primary). This process can occur either by Roche-lobe overflow of the secondary star (hereafter secondary or donor) or by the capture of stellar wind matter from the secondary. Roche-lobe overflow occurs when the outer regions of the secondary spatially extend into the graviational potential well of the primary star. If matter traverses beyond the first Lagrange point of the combined potential landscape of both stars, it will feel a stronger gravitational attraction from the primary than from the donor star, and thus be detached from the secondary star to form a circular disc of matter around the primary.
For a long time it was unclear how matter captured by the primary would move inward and finally impact onto the stellar surface. Due to conservation of angular momentum, it would be expected that the accreted matter would form a toroidal ring around the primary, instead of forming a thin disc. Theories of viscous redistribution of angular momentum are used to explain the movement of matter from the outer part of the ring further inward, while a small portion of the mass carries the angular momentum outward. The two main theories are Modified Disc Instability Model (MDIM) and the magneto-rotational instability (MRI) theory. Both theories have evolved from a common precursor, the α-disc model

4.1.1 α-disc model

The α-disc model was the first attempt at a theory for viscous redistribution of angular momentum. It was proposed by Shakura and Sunyaev in 1973 and assumes differences in viscosity are brought about by different kinds of turbulence in the disc. This theory focuses on determining the viscosity parameter α in order to calculate the global stability of the disc. Basically, α describes the tendency of a unit of matter to exchange angular momentum with its environment. High α means angular momentum can be lost or gained at a high rate. When mass loses or gains angular momentum, it moves closer or further from the star respectively, as it has to occupy a Keplerian orbit that corresponds with its amount of angular momentum. There have been many studies aimed at determining α, from a variety of angles, since it was first introduced. Shakura and Sunyaev used α as a scaling parameter in the relation between the vertical change in viscous force and the pressure in a layer in the accretion disk:

$$\frac{dF_{viz}}{dz} = \frac{3}{2}\alpha\Omega_K P \tag{4.1}$$

with Ω_K the Keplerian frequency given by:

$$\Omega_K = \sqrt{\frac{G_N M_1}{R^3}} \tag{4.2}$$

and F_{viz} the viscous force, $\alpha \leqq 1$ and P the local pressure. By introducing α in this equation, the the transfer of angular momentum can become dependent on more than just the mechanical properties of the disk, since α can be related to a variety of other physical processes, such as magnetic fields, irradiation and gravitational tidal forces.
From then on, these possible contributions to α have been explored and cast into theory. Among these are the MRI and MDIM, but also theories focussing on tidal interactions, gravitational instabilities or hydrodynamically driven turbulence. However, in this text I will only review the MDIM and MRI, as these models are the most applicable to Bob and are also at the moment the subject of discussion. The MDIM, also called the "two-α" model directly evolved from the first attempts by Shakura and Sunyaev. It states that accretion is mainly initiated by the disc switching between being an ionized (hot) state or a recombined (cold) state as a result of changes in the disk structure and irradiation by the primary star. Each of these states has a different value of α assigned to it, hence the name "two-α" model.
The second theory is the magneto-rotational instability (MRI) theory. It explains the emergence of turbulent motion by interaction of the disc with the magnetic field of the primary. Although it was at first thought that such instabilities would occur only in high magnetic fields, it was shown by Balbus and Hawley in 1991 that even small magnetic fields had the capacity to initiate turbulent motion in accretion discs, thus making MRI a generally applicable theory.

In the case of Bob the main goal I set is to create a plausible model of accretion that can explain both the prolonged period of active accretion ($\sim$ 11 yr) and the extremely long recurrence time ($\sim$ 100 kyr) which results from our assumption that the system has already reached a state of equilibrium between the energy gained during accretion and that radiated away during quiescence. During quiescence, the disc has to remain stable long enough to accrete the amount of matter necessary to reach an outburst luminosity of $L_o = 4 \cdot 10^{36}$ erg s^{-1}. Following Jonker et al. (2006) I took a minimum of $4 \cdot 10^{-9}$ M$_\odot$ of accreted matter in order to sustain the observed luminosity. Assuming a state of equilibrium allows the use of the following formula from Brown et al (1998):

$$\frac{L_o t_o}{L_i t_r} \approx \frac{200}{f} \tag{4.3}$$

yielding a quiescence time of approximately $9.9 \cdot 10^4$ yr, using the observed values for L_0, t_o and L_i and assuming $f \approx 0.9$. It should be noted that I implicitly assume here that the energy acquired during accretion is all radiated as photons. Though the purpose of this thesis is to investigate the extent to which *enhanced* cooling processes may be needed, it is reasonable to assume the above calculation gives a practical estimate of t_r. The main argument here is that if other cooling mechanisms would be more than a few orders of magnitude stronger, Bob would cool excessively during its long recurrence time, which would violate our assumption that the system has reached steady-state.
Dividing the required mass by the recurrence time gives the average mass accretion rate $\dot{M}_{t_r} = 4.04 \cdot 10^{-14}$ M$_\odot$ yr$^-1$, which is presumably equal to the mass transfer rate from the donor. The question now is

whether it is possible to sustain a stable, passive accretion disk during quiescence with this mass accretion rate. Also, there is the question whether Bob continues to accrete, albeit at a lower rate, during quiescence. The latter question is important as continuous accretion yields considerably different disk dynamics than a passive disk. An additional problem with the extremely low mass transfer rate is that it is almost incompatible with current binary evolution theory, and that it would requires very special circumstances to have a system where such a rate can be structurally sustained for a long time. However, I will not go into this latter problem in this thesis.

There are three processes that determine the stability of our accretion disc. First, there is the interaction between the magnetic field of accretor and the accretion disc. Secondly we have the possibility of irradiation of the disc by the primary star, resulting in viscous instabilities. Lastly there is hydrodynamic or mechanical turbulence, caused by the viscous force in the disk as a result of the self-interaction of the accreted matter. All of these processes have to be sufficiently suppressed in order to be able to maintain an accretion disk for $9.9 \cdot 10^4$ yr. Though these constraints could be alleviated by assuming low-rate continuous accretion, I will show this scenario to be rather unlikely or of little effect.
Although most SXT's accrete hydrogen from a main-sequence companion star, Bob is different. Observations have not only failed to detect Bob itself, but also its companion. This excludes the possibility of the secondary being an ordinary main-sequence star. It is furthermore gathered from observations that the accretion disk of Bob extends to about 0.08 $R_\odot$, leading to the conclusion that Bob must be part of an ultracompact X-ray binary (UCXB). The companion is then either a white dwarf, or possibly a brown dwarf. The latter being a never fully developed protostar. Jonker et al (2007) conclude that the donor is most likely a white dwarf, and thus the accreted matter should consist mostly of Helium or Carbon/Oxygen. In this thesis, I will follow Jonker's assumption on the nature of the companion star.

4.1.2 The Modified Disk Instability Model

First I will review the impact of the MDIM on Bob's accretion disc. Following Lasota (2008) I describe the accretion disc as a helium dominated disc with a small radius. We are primarily concerned with the evolution of the disk during quiescence, and thus can use the evolutionary equations for non-irradiated dics, as the quiescent luminosity of Bob, at $L_q \leq 2 \cdot 10^{30}$ erg s^{-1}, is not enough to irradiate the disk except for the innermost regions. However, as the disk is almost completely depleted during outburst, virtually all mass will be newly accreted and deposited at the outer radii of the disk. I used the following two equations from Lasota (2008) to describe the critical surface density (Σ^-_{crit})[1] and critical mass accretion rate ($\dot{M}^-_{crit}$) above which a disk would become viscously unstable and will possibly start to accrete.

$$\Sigma^-_{crit}(R) = 1770\alpha_{0.1}^{-0.83} R_{10}^{1.20} M_1^{-0.40} \text{ g cm}^{-2} \tag{4.4}$$

$$\dot{M}^-_{crit}(R) = 5.00 \times 10^{-10} \alpha_{0.1}^{-0.01} R_{10}^{2.65} M_1^{-0.88} \text{ M}_\odot \text{ yr}^{-1} \tag{4.5}$$

Here $R_{10} = R \cdot 10^{-10}$ and R is used to calculate the density at different radii. $\alpha_{0.1} = 10\alpha$, whereas α is the viscosity parameter and M_1 is the mass of the primary in units of the solar mass.
The MDIM model explains the onset of active accretion (and thus an outburst) by assuming that hot and cold "fronts" move through the disk, initiating outburst or quiescence. Once a certain region has become unstable and moves from the cold to the hot branch, this will affect nearby regions, which are already a bit unstable themselves. This influence will cause these regions to move to the hot branch as well, and this process continues until the entire disc has switched from one branch to another, or the front encounters a stable region and is effectively quenched.
There are two kinds of these fronts, an "outside-in" and "inside-out" type. The "outside-in" type occurs when the outer regions of the disk move from one branch to another. Because the allowed values for Σ

[1]Where Σ should not be confused with the sigma particle

decrease with decreasing radius, the inner parts of the disc are easily affected by an instability moving inwards from the outer disc, and therefore "outside-in" type fronts propagate fast and affect the entire disc. "Inside-out" type instabilities encounter regions with increasing tolerance for values of Σ, which means that at some point they may reach a radius where the disc is too stable to be switched from one branch to another. When this happens the front stops. The disc now consists of two sections, each on a different branch. However, "inside-out" fronts usually occur at low radii. This means that the matter that is moved from the cold to the hot branch can actually accrete onto the primary. The density in the inner regions of disc then drops, and a new instability may form, resulting in a "inside-out" cooling front, thus stabilizing the disc again. However, as soon an "inside-out" front manages to reach the outer disc boundary, active accretion starts, only to halt when almost the entire disc has been depleted.

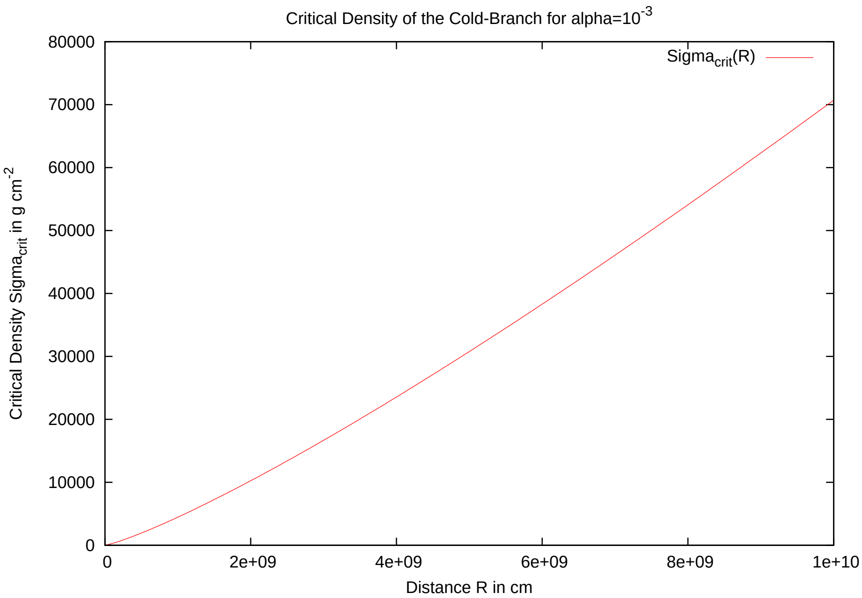

Figure 4.1: The lower critical surface density (in g cm^{-2}) as a function of the disk radius R (in cm). In this figure, $\alpha_c = 1 \cdot 10^{-3}$

We can apply this model to Bob, to estimate how long the system can remain stable, and thus how long a quiescent state can be maintained. If we calculate the critical mass accretion rate we find that it should not exceed $1 \cdot 10^{-12}$ M$_\odot$ yr^{-1} for the region between $1 \cdot 10^7$ m $\leq$ R $\leq 5.57 \cdot 10^7$ m, which is where ~96% of the disc is located (assuming a constant disc thickness). With our mass accretion rate of $4.04 \cdot 10^{-14} M_\odot$ yr^{-1}, this means the accretion rate itself never destabilizes the disc, provided it does not vary too much with the disc radius. Of course, higher transfer rates can occur when matter starts moving inward due to turbulent instabilities. However, at that point the disc is already unstable, so the exact value of $\dot{M}$ is unimportant.
The disc surface density is another story however. As we can clearly see from comparing figures 4.1 and 4.3 it is not possible to keep the entire ring stable for increasing surface densities. Even at the outermost radii, where the minimal surface density is highest, the maximum allowed value is only $\approx 3.4 \cdot 10^4$ g cm^{-2}. From figure 4.3, we see this density is reached *on average* already after about 40 kyr. Even when comparing the density with that in figure 4.2, which has the minimal value of α_c used in this thesis, it looks like about 100 kyr of quiescence is still not possible, because below a radius of $1 \cdot 10^7$ cm, the allowed density quickly drops below the $8 \cdot 10^4$ g cm^{-2}, which is the average density after 100 kyr in

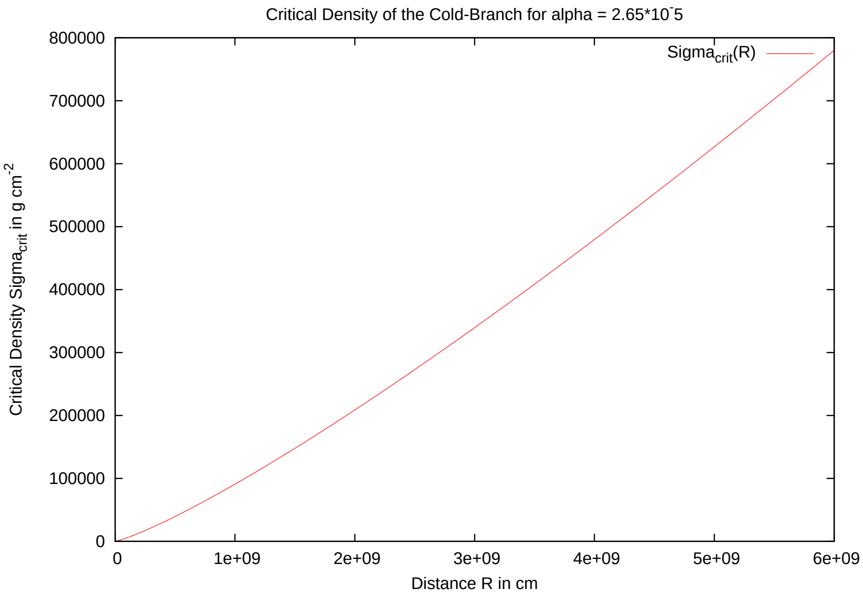

Figure 4.2: The lower critical surface density (in g cm^{-2}) as a function of the disk radius R (in cm). Here, $\alpha_c = 2.65 \cdot 10^{-5}$

figure 4.3. Figure 4.3 shows the average density of the accretion disc, assuming that matter is distributed homogenously over the entire disc, which is of course a very crude approximation. In reality mass will

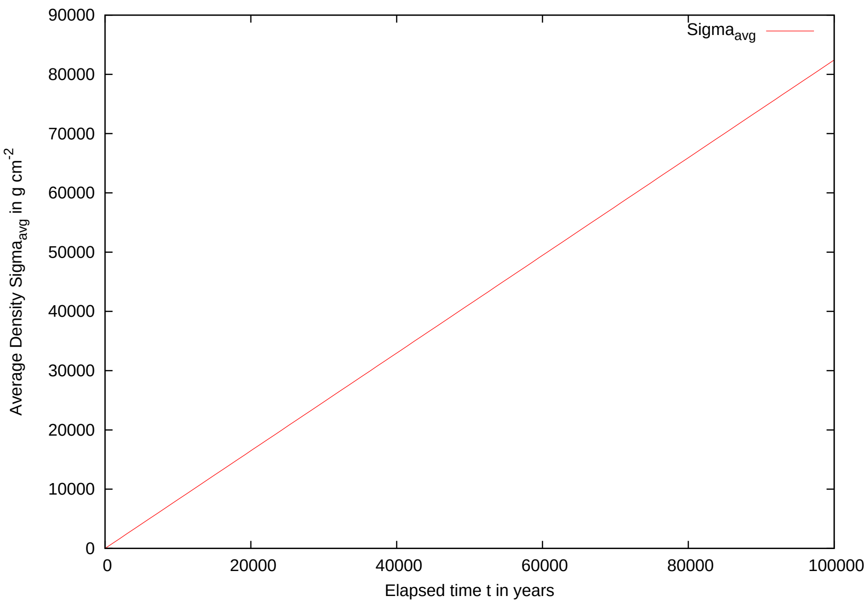

Figure 4.3: The average surface density (in g cm^{-2}) as a function of t (in years)

first concentrate at the higher radii, only moving inwards after having lost sufficient amounts of angular momentum. A non-homogenous distribution of matter through the disk could lead to a longer time until the disk becomes unstable, if most matter is contained at the outer radii where higher densities are allowed. This option will be explored further in this chapter.

4.1.3 Evaluating α with the MRI

As is clear from the figures included above, the stability of the disk and thus the time the system can remain quiescent, depends mostly on the value of α. Even though the physical parameters of Bob are unknown, neither the stellar radius nor its mass have a large range of possible values. For α on the other hand, there are a few orders of magnitude to consider. It is therefore important to devote some attention to the studies of α that have already been performed, in order to get a decent estimate on the parameters value in our particular system.

I will adopt the "two-α"-model from Dubus et al. (2001) and thus assume a different value for α during the cold phase compared to α during the hot phase. During the cold phase, there is little transport of matter throughout the disk, and we thus expect α_c to be low (I use the subscript "c" to denote the cold phase value of α). Furthermore, I assume that the value of α_c is determined by the interaction of a magnetic field with the accretion disc.
There have been numerous studies on the effect of a magnetic field on the accretion disk. In this thesis I draw from the conclusions of Balbus (2001) and Fromang et al.(2008), who investigated the dependence of α on the magnetic Reynolds and Prandtl number, Re_M and Pr_M. Both numbers describe the relation between the ohmic and viscous diffusion in the disk by the following equations

$$Re_M = \frac{c_s H}{\eta} \tag{4.6}$$

$$Pr_M = \frac{\nu}{\eta} \tag{4.7}$$

Where c_s is the speed of sound in the disk, H is the scale-height, ν the viscous diffusity and η the magnetic diffusity. For α, this results in three different equations:

$$\alpha_{Rey} = \frac{T^{Rey}_{r\phi}}{P_0} = \frac{1}{P_0}\langle \rho(v_x - \bar{v_x})(v_y - \bar{v_y})\rangle \tag{4.8}$$

due to Reynolds stresses and

$$\alpha_{Max} = \frac{T^{Max}_{r\phi}}{P_0} = \frac{1}{P_0}\langle -\frac{B_x B_y}{4\pi}\rangle \tag{4.9}$$

as a result of Maxwell stresses. Combined, these give

$$\alpha = \alpha_{Rey} + \alpha_{Max} \tag{4.10}$$

which is the α mentioned thusfar.

Using numerical simulations they found that for increasing Pr_M, or decreasing Re_M, α decreases.

The magnetic Reynolds number effectively describes the efficiency with which charged particles can be influenced by the magnetic field. A high collision rate of charged particles with other (neutral) particles and a low fraction of charged particles in the disk give a lower Reynolds number. We can physically understand this if we suppose that turbulence depends on the particle flux generated by the magnetic field. A low fraction of charge carrying particles and a short mean free path make for a small particle flux. Below a certain certain value of Re_M, α can become so small that turbulent motion is almost completely suppressed. For these situations, both Fromang and Balbus find values of α of the order of $1 \cdot 10^{-3}$ for lowly ionized disks in magnetic fields with zero net flux. The latter not meaning the complete absence of a magnetic field, but rather a spatially periodic magnetic field such that the averaged net flux is zero. (An example of such a field would for instance be $B_z = B_0 sin(2\pi x)$) An even lower value of α is obtained by Fleming et al.(2000), who reports finding $\alpha = 2.65 \cdot 10^{-5}$. We could even extend our model to values lower than $2.65 \cdot 10^{-5}$, as all previous models were based on H-dominated disks, while Bob's is He-dominated. The higher ionization energy of He would further reduce the number of charged particles, thus further decreasing Re_M and yielding an even lower value for α. This is especially true since we model our disc during quienscence, and the extremely low luminosity of Bob would not be able to ionize a substantial amount He-atoms. Thus, drawing from the above, I have used the values of $2.65 \cdot 10^{-5}$, $1 \cdot 10^{-4}$ and $1 \cdot 10^{-3}$ for α_c, while I kept α_h constant at a value of 0.2, which is a value commonly used in disk dynamics, throughout the simulations I performed.

4.2 Application

4.2.1 Model Structure

Because the stability of the disk depends quite strongly on the distribution of mass throughout the disk, it is very hard to make any statements on the stability of Bob using the average density alone. To get a more accurate estimate of the time it takes the disk to destabilize, I programmed a 1-dimensional model of the accretion disk[2]. In table 4.1 we have given a number of these results, which will be discussed in the next subsection. I have set the outer boundary of the accretion disk at $0.08R_\odot$, which is the estimate of Jonker et al. for the outer boundary of the accretion disc, and the inner boundary at the stellar surface. To more accurately predict the properties of our accretion disk, I modelled the disk as a collection of cells, each with two critical densities, Σ_c and Σ_h assigned to it. The model compares the surface density of each cell with both critical densities. Σ_c is given by (4.4), while Σ_h has the form[3]:

$$\Sigma_h = 227\alpha_{0.1h}^{-0.79} R_{10}^{0.96} M_1^{-0.25} \tag{4.11}$$

If the surface density is greater than Σ_c, the cell will transfer mass to the one adjacent to it. This process continues until the surface density becomes lower than Σ_h, when the mass outflow is halted until the surface density exceeds Σ_c again. The values for both critical densities were calculated from the equations obtained by Lasota (2008). Although his model is initially used to describe heating and cooling fronts moving through an already present disk, there are some reasons which validate their use in our situation:

Low α_c To be able to sustain the accretion disk for a long time, it is clear that at least a significantly lower value of α_c is needed than for SXT's with recurrence times of the order of days. A low value for α_c does not only result in a higher allowed density, but also decreases overall rate of angular momentum transfer between particles. This in turn means that the rate at which matter moves inward when not in a hot-branch "outburst" is very low. Such a inward movement of matter would amount to continuous filling of the "bins" I used to model the disk. I assumed here that the amount of matter transferred this way before the adjacent cell goes into a hot-branch outburst is negligible compared to the total matter deposited in the cell during outburst.

[2]Two versions of this model can be found in appendix A.

[3]The original formula contained an additional parameter C_3. However, following Lasota (2008) we have set this equal to 1

Long recurrence time The time provided by assuming equilibrium between outburst and quiescence is quite long. This means that if the disk is stable, mass transfer throughout the disk is slow and for most of the time it is more or less a thin ring, rather than a flat disk. The result of this is that the matter will be stored in the outer regions of the disk for most of the time, where the critical density is high. Only when matter gets very close to the surface of the star, a cascade-flow[4] will occur and active accretion will start.

Many stable regions As soon as a region becomes unstable, it will deplete itself until the density has reached Σ_h. The difference between Σ_c and Σ_h is very large for the outer regions of the disk, assuring that once a region has regained stability by transferring mass to adjacent cells, it will remain stable for a long time and cannot be easily excited again by instabilities propagating from lower radii. As a result, even "outside-in" fronts will for a long time not be able to push mass towards the star, as the mass gets sufficiently diluted to eventually quench the front.

As boundary condition I assumed a constant mass input at the first cell, which I placed at $0.08R_\odot$. To prevent mass from infinitely accumulating somewhere, I at first took the outflow rate of the last cell as $3.64 \cdot 10^{-10}$ $M_\odot$ yr^{-1}, which is the mass accretion rate that matches with the outburst luminosity of Bob, and then affinely increased the transfer rate between the first cell and the last. This way, the outflow rate of each cell is always greater than the input rate, and cells above their critical density will always lose more mass than they gain. However, modelling at this mass transfer rate led to recurrence times too small to accumulate enough matter to sustain such an outflow during an outburst of eleven years. Therefore, I also ran a few simulations with $\dot{M}_{out}$ varied, the results of which are displayed in tables 4.2 and 4.3.

As there is no compelling reason to assume the mass transfer rate has an affine form throughout the disc, aside from its simplicity, I also used a slightly different formula. This formula is based on the fraction of Σ_c and Σ_h, which gives a radial dependent relationship between both critical densities. It is not inconceivable that the mass transfer rate does depend on these densities, and this form has the added advantage of being dimensionless. This gives the folowing relation for the new $\dot{M}_{tr}$:

$$\dot{M}_{tr} = a + bR^{-0.24} \tag{4.12}$$

where the constants a and b can be deduced from boundary conditions. If we give R in meters, and use $\dot{M}_{tr}(R = R_{out}) = 9.95268 \cdot 10^{17}$g s^{-1} and $\dot{M}_{tr}(R = R_{in}) = 1.9905 \cdot 10^{21}$ g s^{-1} the equation resolves to[5] :

$$\dot{M}_{tr} = 2.07672 \cdot 10^{22} R^{-0.24} - 2.85593 \cdot 10^{20} \tag{4.13}$$

Where I obtained the boundary conditions by assuming that the inflow of mass at the outer radius of the disc is equal to the average mass transfer rate[6] and the outflow equals $\dot{M}_{acc}$. The behaviour of $\dot{M}_{tr}$ can also be seen in figure 4.4.

Ideally, the mass transfer function throughout the disc would be depending on R and α. If such a function were known, we could alter the model in such a way that mass would be flowing continuously, while the model would check where critical densities were reached. Certain conditions could be set for an event to be recognized as an outburst[7]. However, current knowledge about the accretion process is apparently not sufficient to provide generally usable expressions mapping $\dot{M}$ as a function of α and R, as I have not been able to find any such expression. Therefore, I have not been able to extend the model beyond its current "outburst"-like behaviour.

[4] I defined the onset of the cascade flow as the moment where the time it takes to fill the next cell becomes equal to one time step.

[5] These values are for $t_{gauge} = 2.2 \cdot 10^3$ yr

[6] which is practically equal to the quiescent mass transfer rate with our long recurrence time

[7] These conditions could include a minimal number of cells being in outburst, or a propagation of a cold front to a certain predefined radius

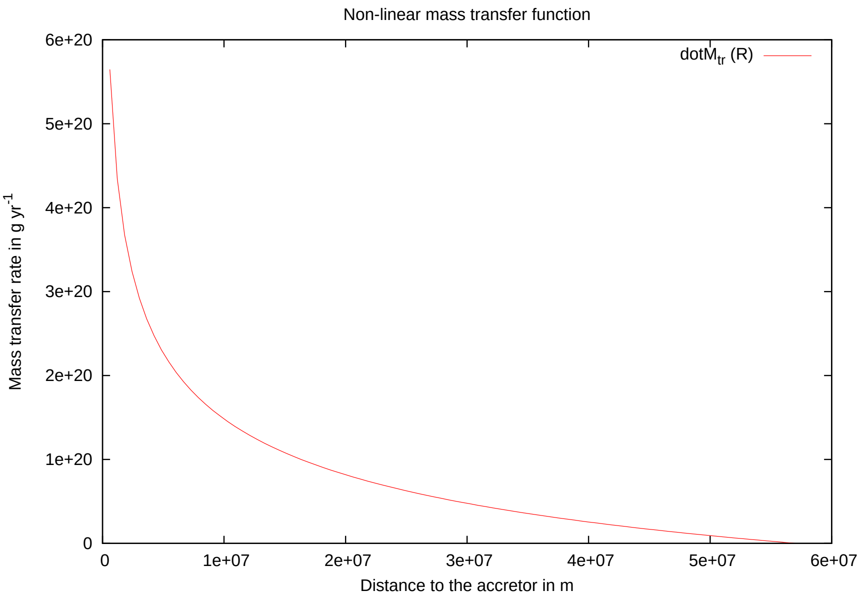

Figure 4.4: The mass transfer rate (in g yr^{-1} as a function of the radius R (in cm), given by (4.13)

4.2.2 Model Results

The results of the simulations are listed in the tables below. Table 4.1 contains the values obtained with the first version of the model. Here I tried to find a value for α_c for which the disc would remain stable for $9.9 \cdot 10^4$ year. If this were possible, it would be possible to get the observed luminosity from (4.3) with $f = 0.9$. However, as can be seen from the acquired results, α_c has to be chosen unrealistically low to be able to sustain the disc for the required amount of time. Even with $\dot{M}_{out}$ reduced to a value two orders of magnitude below its value during outburst the longest time for t_r is still too short.

Resolution (# cells)	$\dot{M}_{out}$ (M$_\odot$ yr^{-1})	α_c	t_{crit} (years)
$1 \cdot 10^5$	$3.64 \cdot 10^{-10}$	$1 \cdot 10^{-3}$	397
$1 \cdot 10^5$	$3.64 \cdot 10^{-10}$	$1 \cdot 10^{-4}$	2124
$1 \cdot 10^5$	$3.64 \cdot 10^{-10}$	$2.65 \cdot 10^{-5}$	$7.6 \cdot 10^3$
$1 \cdot 10^5$	$3.64 \cdot 10^{-12}$	$2.65 \cdot 10^{-5}$	$5.98 \cdot 10^4$
$1 \cdot 10^5$	$3.64 \cdot 10^{-12}$	$1 \cdot 10^{-4}$	$1.94 \cdot 10^4$
$1 \cdot 10^4$	$3.64 \cdot 10^{-12}$	$1 \cdot 10^{-4}$	$2.55 \cdot 10^4$
$1 \cdot 10^4$	$3.64 \cdot 10^{-10}$	$2.65 \cdot 10^{-5}$	$1.5 \cdot 10^3$

Table 4.1: Results for simulations with f=0.9, $L_i = 2 \cdot 10^{30}$ erg s^{-1}

Since a recurrence time as long as 100 kyr seemed unplausible, I did a second series of simulations, now with a target time of 22 kyr, the results of which can be found in table 4.2. This would still be possible with the observed luminosity if $f \lesssim 0.2$. However, the shorter recurrence time also means that $\dot{M}_{in}$ has to be higher. We now have $4 \cdot 10^{-9} M_\odot / 22 \cdot 10^3 yr = 1.81 \cdot 10^{-13} M_\odot yr^{-1}$. The results obtained with these simulations are somewhat more plausible, as the difference between the t_{crit} and t_r is less, both absolutely as relatively, than with the previous physics input. However, the increase in mass influx at the outer boundary of the disc now destabilizes the disc sooner than before and it stil seems impossible to reach the target time. The only exception here is the situation for $\dot{M} = 3.64 \cdot 10^{-12}$ M$_\odot$ yr^{-1} with $\alpha_c = 2.65 \cdot 10^{-5}$ where the outermost cell had not yet been reached when the model reached its target time T_{end} and terminated. Still, the chosen mass transfer rate here has been chosen specifically to obtain this result and does no longer correspond with the boundary conditions at the surface.

Resolution (# cells)	$\dot{M}_{out}$ (M$_\odot$ yr^{-1})	α_c	t_{crit} (years)
$1 \cdot 10^4$	$3.64 \cdot 10^{-10}$	$1 \cdot 10^{-4}$	406
$1 \cdot 10^4$	$3.64 \cdot 10^{-10}$	$2.65 \cdot 10^{-5}$	$1.2 \cdot 10^3$
$1 \cdot 10^4$	$3.64 \cdot 10^{-11}$	$2.65 \cdot 10^{-5}$	$7.3 \cdot 10^3$
$1 \cdot 10^4$	$3.64 \cdot 10^{-12}$	$1 \cdot 10^{-3}$	$1.7 \cdot 10^3$
$1 \cdot 10^4$	$3.64 \cdot 10^{-12}$	$1 \cdot 10^{-4}$	$1.11 \cdot 10^4$
$1 \cdot 10^4$	$3.64 \cdot 10^{-12}$	$2.65 \cdot 10^{-5}$	$3.34 \cdot 10^4$
$1 \cdot 10^5$	$3.64 \cdot 10^{-12}$	$1 \cdot 10^{-3}$	$1.8 \cdot 10^3$

Table 4.2: Results for simulations with f=0.2, $L_i = 2 \cdot 10^{30}$ erg s^{-1}

The last series of calculations were performed with a different, non-linear mass transfer function, obtained by comparing the radial dependancies of Σ_h and Σ_c. As can be seen from figure 4.4, the relation $\dot{M} \propto R^{-0.24}$ gives much lower transfer in the outer regions of the disc, and increases dramatically for smaller R. Unfortunately, I needed to compute a $\dot{M}_{in}$ to operate my model, even though the average mass transfer rate is inversely proportional to t_r, which I was trying to compute. To achieve this I defined t_{gauge} as the time for which I calculated $\dot{M}$. Ideally, t_r should be very close to t_{gauge}. I did this for both $t_{gauge} = 2.2 \cdot 10^3$ yr and $1 \cdot 10^5$ yr. The results from these simulations were far more positive than those obtained using the linear mass transfer rate. For $t_{gauge} = 2.2 \cdot 10^4$ the results are closer to the target time even for high values of $\dot{M}_{out}$.

Resolution (# cells)	t_{gauge} (years)	α_c	t_{crit} (years)
$1 \cdot 10^4$	$1.5 \cdot 10^4$	$2.65 \cdot 10^{-4}$	$1.1 \cdot 10^4$
$1 \cdot 10^5$	$2.2 \cdot 10^4$	$1 \cdot 10^{-3}$	780
$1 \cdot 10^5$	$2.2 \cdot 10^4$	$1 \cdot 10^{-4}$	$4.1 \cdot 10^3$
$1 \cdot 10^4$	$2.2 \cdot 10^4$	$2.65 \cdot 10^{-5}$	$1.2 \cdot 10^4$
$1 \cdot 10^4$	$1 \cdot 10^5$	$1 \cdot 10^{-4}$	$1.2 \cdot 10^4$
$1 \cdot 10^4$	$1 \cdot 10^5$	$2.65 \cdot 10^{-5}$	$3.6 \cdot 10^4$

Table 4.3: Results for simulations with the non-linear mass transfer function

4.2.3 Graphic Output

In February 2009, I made an attempt to add a function to the program that would enable me to display the outputted data in a graph. At certain benchmarks, both in the R and t domain, the program would write all arrays containing density values to a .data file. Using Gnuplot, I created a number of graphs that displayed the time-evolution of the system. Two of these grpahs are displayed below.
The most interesting feature that presented itself, was that the matter in the disk would not spread out over the disk. Instead, the mass would seperate itself into large parts that were around the critical

density level, with parts of the disk almost completely void of matter in between[8]. The result of this can be seen very clearly in the figures. Unfortunately, the program seemed to have an inexplicable error that inhibited me from completing a full cycle with the newly added output function. As such, only a limited part of the evolution is available. More attention to this will be given in the discussion below.

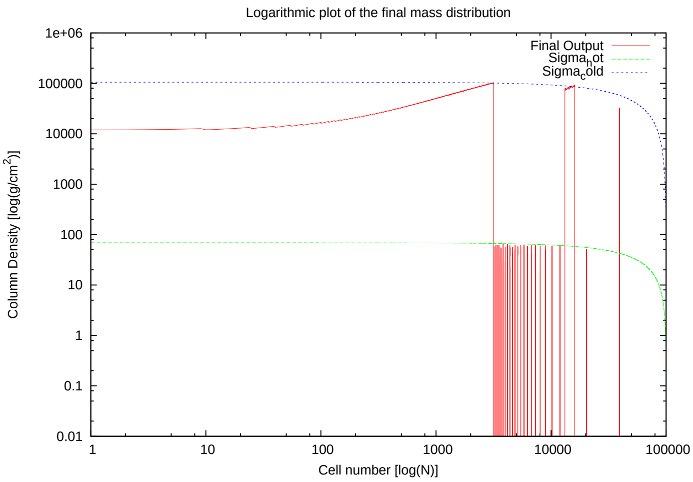

Figure 4.5: The configuration of the disk at T=1600000. This is the most progressed figure possible.

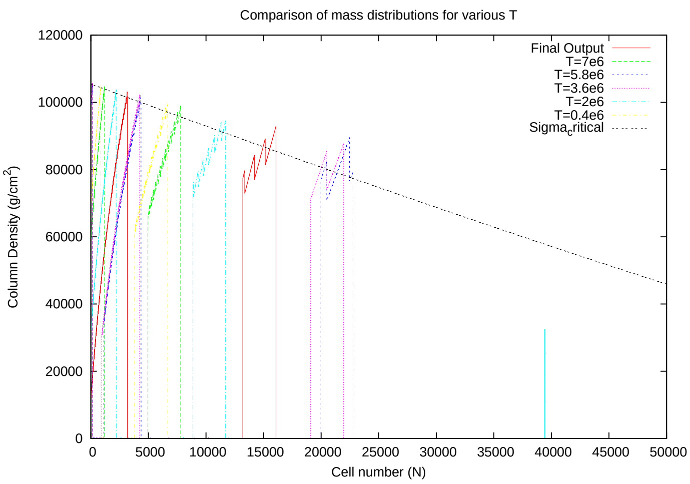

Figure 4.6: The disk configuration for several T, showing the time evolution of the disk.

[8]Some "residual" mass would remain there, but that might have been caused by the program rather than a physical effect.

4.3 Discussion

4.3.1 Results

Due to a lack of accurate observational data, it is hard to draw any hard conclusions, however, there are a few things that can be deduced from this chapter.
Obviously, it is quite hard to use standard disk dynamics to come up with a recurrence time of the order of 10^4 to 10^5 yr. This is particularly so as most of the current theories are geared towards explaining the behaviour of short-period SXT's. Still, I think it is possible to arrive at a recurrence time $t_r \sim 2{\cdot}10^4 - 4{\cdot}10^4$ yr, provided a few conditions are met. The most apparent of these conditions is that α_c should be low, at least of the order of $1 \cdot 10^{-4}$ to keep Σ_c at a large enough value. The mass transfer rate should also be sufficiently low[9], especially in the outer regions of the disc. As can be seen from the model results, this is achieved more easily when $\dot{M} \propto R^{-0.24}$ compared to $\dot{M} \propto R$. The results for $\alpha_c = 2.65 \cdot 10^{-5}$ seem to depend on t_{gauge} only slightly. Thus, one would expect that at a value of $t_{gauge} \approx 1.5 \cdot 10^4$ an equivalent t_r would be achieved. As there are still many unknowns, I will propose a rather broad range for t_r, ranging from $1 \cdot 10^4$ till $2 \cdot 10^4$.

4.3.2 Model

As stated before, the model employed is rather crude and does not do justice to the complex dynamics involved in disk accretion processes. Still, it does give an estimate on the period Bobs accretion disk will remain stable given a specific distribution of mass throughout the disk.
The main shortcoming of the model is that it does not simulate continuous transport of matter through the disk. In an actual system, matter would be moving inward even during the "cold" periods, albeit much slower than during the "hot periods. In turn, this would ensure that it would take longer for a critical density to be reached anywhere, but that once a front would form, it would meet less resistance. Such a model would be much less reliant on the simulated "nova-type outbursts" that currently ensure the transport of matter. Rather, the onset of outburst and front formation would be a sign of global instability of the disk. The stability criterium against which the model would check should then be the possibility of an inward type front being able to reach all to the stellar surface, which would mean the system entered a phase of active accretion.
A second issue not addressed is the two-dimensional character of an accretion disk. Even though it is no problem to assume the disk is circularly symmetric, significant differences in the height of the disk can exist. The density used was a column density, which negates any influence of the height of the column on the total picture. It could however be possible that differences in height would require a different approach, requiring a two dimensional model and a three dimensional density parameter.

A later revision of the computer model, performed in March 2009, indicated several more problems with than previously anticipated and discovered. When working with the datasets outputted by the program, it was found that the program allowed for "negative densities" to occur. This happened when the mass outflow in a cell was high enough to more than deplete the cell in a single iteration. The main reason for this was that there was a two orders of magnitude difference between the minimum density for the hot phase ($\mathrm{Sigma}^{hot}_{crit} \sim 10^1$g cm^{-2}), and the mass transfer rate per iteration ($\dot{\rho} \sim 10^3$g cm^{-2}). At such an instance, more matter would be drawn from the cell than was actually present.
Intuitively, this could be solved by further decreasing the time-intervals of each iteration. However, the resulting calculation time would have become exteremely long, probably requiring a few days on the computers available. As such, this was not an option. A more elegant solution was adopted, where a cell would simply check whether the amount of matter that would be drawn from it could actually be supplied. If this was not the case, the cell would transfer all mass present, but not more. One could say

[9] And this indeed corresponds to a low value of α_c

that the program was forbidden to "overdraw" the accounts of each cell.
Unfortunately, while this solved the initial problem, a new one arose. After the implementation of the aforementioned improvements, the program developed an unexpected fault where it became unable to progress beyond a particular cell in the radial range[10]. Because there was no apparent reason for this, nor any clear fault in the code of the program, I was unable to resolve this particular issue. Moreover, to my great regret, this casts some amount of doubt on all previous data outputted by the program.
Still, I think the behaviour displayed before the error occurs could be seen as a reasonable approximation of the accretion process, and one could extrapolate this to estimate the behaviour after the faulty cells. Using the program primarily served to increase the accuracy of our first very rough estimate done on the average density in the disk. This rough estimate still holds, regardless of the program. Also, the amount of "negative" mass generated by the program is negligible compared to the total mass moving through the disk. While its occurence means there is something wrong in the code, the influence on the physical result is small. We can thus still assume the data from the first few runs of the program has some validity, since the bulk of the matter is not affected by the occurence of negative masses.

4.3.3 An alternative: Continuous accretion

To make the constraints derived above less stringent, it could be an option to assume there is still accretion taking place, though at much a lower transfer rate. Continuous accretion would allow mass to flow from the accretion disk onto the surface of the star. Even a low outflow of mass could increase the stability of the disk by depleting the inner regions. The result would be a longer recurrence time by a similar mass transfer rate from the secondary star to the accretion disk. The luminosity as a function of mass transfer is given by[11]

$$L_q = \frac{M_1 \dot{M}_{tr} G_N}{R} \tag{4.14}$$

The upper limit on the luminosity of $2 \cdot 10^{30}$ erg s^{-1} yields an upper limit on the mass transfer rate in quiescence of $\dot{M}_{tr} = 1.68 \cdot 10^{-16}$ M$_\odot$ yr^{-1}. This is much smaller than the averaged accretion rate of $4.04 \cdot 10^{-14}$ M$_\odot$ yr^{-1} and it depends greatly on the mass transfer function throughout the disc whether this would be able to sufficiently stabilize the disc[12]. The way I modelled the disc, the mass transfer rate increases with decreasing radius. As the mass outflow would probably only start when the disc gets close enough to the star, the outflow rate would probably be negligible. Still, in a continuous mass transfer model where the transfer rate would decrease with decreasing radius, the effect could be significant if $\dot{M}_{disk} \sim 10^{-15} M_\odot yr^{-1}$. If this would be the transfer rate at the inner radii, continuous accretion could provide an outflow effect of $\sim 10\%$, significantly increasing the recurrence time. Transferring more mass from the accretion disc could be achieved by making use of an ADAF model. However, this in turn creates the problem of having to dissipate a lot of extra energy from the neutron star surface, something which would be equally hard to achieve. A last consideration on this point is that the processes governing accretion at very low rates are perhaps even less understood than accretion in general. Thus, it could be very well possible that other effects disperse a part of the accretion disc into space. Examples of such processes would be the propellor effect or jet-formation. Such dispersion would allow for a higher outflow rate than would be inferred from direct observation of the quiescent luminosity and could further stabilize the disc.

[10] Cell 39455 of 100000
[11] As most of the energy from accreted matter is released at the surface, it might be useful to stress this cannot be compensated for by enhanced cooling processes
[12] Calculated for $t_r \approx 100$ kyr, the value would be even higher for shorter recurrence times

4.3.4 Summary

Although the model has serious shortcomings, I believe it provides at least a crude estimate to the possible recurrence times of Bob. From table 4.3 we see that periods in the order of $\sim 10^4$ yr are possible for very low α_c. In an optimistic scenario, we could now set f=0.2 and assume $t_r = 1 \cdot 10^4 \rightarrow 2 \cdot 10^4$ yr, to get a quiescent luminosity of $L_q = 3.2 \cdot 10^{30} \rightarrow 7.0 \cdot 10^{30}$ erg s^{-1}. These luminosities are of the same order as the observed photon luminosity ($L_{Bob} < 1.8 \cdot 10^{30}$ erg s^{-1}. The difference in luminosity should be the result of enhanced cooling processes, which are of the same order of magnited as the thermal cooling. In the next section, I summarized the results of Jonas Sweep's research into various cooling mechanisms. Finally, we will match our results to see which cooling mechanisms are likely candidates to provide an enhanced cooling luminosity (L_{enh}) of the same order of magnitude as required by the accretion timescales.

Chapter 5

On the possibility of enhanced neutrino cooling

In his thesis, Jonas Sweep has explored the various non-thermal processes that could help cool Bob if photon radiation proves to be insufficient. In this chapter, I will give a summary of his work and his results. Combining our respective theses to formulate a single answer to the question posed in the introduction will be done in the conclusion.

5.1 Cooling processes

Despite its extremely high densities, a neutron star is still transparent to neutrinos. Thus, the main cooling mechanisms aside from photon emission from the stellar surface are all neutrino related. What happens is that although the star is in chemical equilibrium, reactions still occur provided the ratios of the particles involved do not change on average. This means that a variety of nuclear equilibrium reactions occur in the stellar core. The neutrinos created in these reactions escape the star, carrying off energy and thus cooling the stellar core. Because the neutrinos do not affect the chemical equilibrium, their emission has no influence on the ratios of the other particles. The most prominent processes that are candidates for neutrino powered neutron star cooling are listed below and will be explained further on in this section.

1. Direct URCA
2. Modified URCA
3. Pasta phase URCA
4. π-condensates
5. Pair-breaking
6. Hyperons

The first process is an equilibrium reaction between β decay and electron capture of protons. Although these reactions do not change the composition of the star, the reaction produces a large amount of neutrinos.

$$n \rightarrow p + e^- + \bar{\nu}_e \tag{5.1}$$

$$p + e \rightarrow n + \nu_e \tag{5.2}$$

Although the reaction is quite simple, it is not commonplace in most neutron stars. This is because under the conditions present in neutron stars, it is very hard to adhere to conservation of energy and momentum, in effect, this supresses URCA (almost) completely. The problem can be circumvented if a spectator nucleon is introduced, which can be used to absorb or supply the difference in momentum that would otherwise prevent the reaction from occuring. Schematically:

$$N + n \rightarrow N + p + e^- + \bar{\nu_e} \tag{5.3}$$

$$N + p + e^- \rightarrow N + n + \nu_e \tag{5.4}$$

Though this reaction, which is referred to as modified URCA, is generally allowed in most neutron stars, the necessity of a fourth particle makes these reactions less likely to occur, resulting in a much lower reaction rate. Thus, compared to direct URCA, it is much less efficient. Still, it is the modifed URCA process that is used mostly to explain neutron star cooling and luminosities.
A different solution is the use of hyperons, such as Σ particles, to create an direct URCA like process. The reactions governing this are very much like those stated above for ordinary direct URCA:

$$\Sigma^- \rightarrow n + l + \bar{\nu_l} \tag{5.5}$$

$$n + l \rightarrow \Sigma^- + \nu_l \tag{5.6}$$

Again, the use of Σ particles as mediator particles in a reaction can greatly enhance the rate at which it occurs and thus the efficiency of the cooling process. However, this does require a large amount of Σ particles to be present, something of which we cannot be certain.
Another way to solve the problem with direct URCA is to switch to a quasiparticle description by using a pion condensate. Under favourable circumstances, excitations that resemble π particles may be formed in the neutron star core. Because pions are bonons, they are able to condense in the same state. However, the nucleons are greatly affected by the condensate, to the extent that their states become mixed. Using the quasiparticle description, a direct URCA like process is now allowed:

$$\tilde{n} \rightarrow \tilde{p} + e^- + \bar{\nu_e} \tag{5.7}$$

$$\tilde{p} + e \rightarrow \tilde{n} + \nu_e \tag{5.8}$$

The problem here is that it is still uncertain whether pion condensates actually form, and under what conditions. The creation of a π-condensate depends on the equation of state (EOS) of the neutron star, which is still a great unknown in most cases. Nonetheless, pion condensates can make a significant contribution to neutrino emissivity if they are allowed.

There are two other processes that are a bit more elaborate than the above particle physics. The first is pair-breaking-formation (PBF). This process occurs when there is an attractive interaction between the nucleons and Cooper-pairs may be formed. If we again use a quasiparticle description, we can associate a quasiparticle with an *unpaired* real particle. If two particles form a pair, the two corresponding quasiparticles are annihilated and this process creates neutrinos. When a pair is again broken, two quasiparticles are formed. The quasiparticle description of the neutrino creating reaction is given below:

$$\tilde{B} + \tilde{B} \rightarrow \nu + \bar{\nu} \tag{5.9}$$

There is however a slight problem with PBF. Cooper-pairing of the nucleons results in superfluidity and makes particles unavailable to the other neutrino generating processes, because those processes require *single* particles to participate. Thus, though the presence of superfluidity can enhance the PBF process, it suppresses all other processes.

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Cooling Process	Integrated Luminosity (erg s^{-1})
Thermal photon emission	$< 1.8 \cdot 10^{30}$
Direct URCA	$1 \cdot 10^{32}$
Modified URCA	$3 \cdot 10^{23}$
Pion condensate	$2.7 \cdot 10^{31}$
Hyperonic URCA	$5 \cdot 10^{30}$
PBF-processes	$2.6 \cdot 10^{22}$
Pasta phase processes	$3 \cdot 10^{25}$

Table 5.1: An overview of order of magnitude estimates on various enhanced cooling processes

The last source of neutrinos are so-called "pasta phase processes" (P^3), which is based on non-spherical shapes of nuclei in the mantle. These shapes may resemble certain Italian types of food, hence the name[1]. The deformed shape of the nuclei may force them to adopt a kind of lattice structure, resulting in a different potential than is the case for free moving particles. Because of this, the relations for momentum and energy are altered, and the direct URCA process may become available. However, one has to keep in mind that the region in the mantle where this can occur is quite small and thus the process is not as efficient as normal direct URCA. Still, in the absence of the latter, P^3 may become important.

In table 5.1 an overview is given of the efficiency of the various cooling mechanisms. As is evident from the table, the most likely candidates for cooling mechanisms are a pion condensate and hyperonic URCA. Direct URCA is excluded as it cannot take place in a "standard" neutron star. However, the estimates provided in table 5.1 have not yet fully taken into account the effect of different equations of state (EOS's). The EOS of a neutron star strongly influences the likelihood of occurence of certain processes, such as the presence of a pion condensate or hyperons. Therefore, some more thought on the EOS will be spend on the next section.

5.2 Relevant Interior parameters

The effectiveness of the various processes described above depend highly on the structure of the neutron star interior. Unfortunately, our knowledge of neutron star cores is limited, but various models have been suggested and their effects on the neutrino emissions have been calculated. The description of these would deserve an entire text by itself, however, I will here summarize the most important factors for determining the magnitudes of the different cooling processes.

First, there is the density throughout the star, which is related to its mass. Only for very high densities is it possible to use the direct URCA process. However, with a canonical mass of $M \sim 1.4\, M_\odot$ Bob is not massive enough to initiate direct URCA, so that process can be discarded as a significant contribution to the cooling.
For a given density function, there can be different relations between density and pressure in the resulting equation of state. If the pressure is relatively high for a given density, the EOS is said to be stiff, for low pressures it is said to be soft. The specific form of the EOS depends on the model used for the nucleon-nucleon interaction. Generally, a stiff EOS tends to suppress neutrino emission. The last physical quantity to be taken into account is the temperature. This is especially the case since Bob is quite cold for a neutron star, which means it might behave differently from a standard SXT. If we look at

[1] Thus another example of the peculiar sense of humour among physicists

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Scenario	Total Luminosity
No neutron superfluidity, soft EOS	$3.2 \cdot 10^{31}$
No neutron superfluidity, stiff EOS	$4.9 \cdot 10^{30}$
Neutron superfluidity	$2.5 \cdot 10^{30}$
Neutron superfluidity, no pions	$5 \cdot 10^{29}$
Neutron superfluidity, no hyperons	$2 \cdot 10^{30}$
Neutron superfluidity, no fast processes	Photon cooling ($\leq 1.8 \cdot 10^{30}$)
No fast processes	Photon cooling ($\leq 1.8 \cdot 10^{30}$)

Table 5.2: An overview of the total luminosity as a result of different EOS and superfluidity scenarios

modified URCA, we find it has a T^8 dependence. This means that it might be heavily supressed at lower temperatures. The same is true for PBF.
Finally, the existence of superfluidity also needs to be considered. As stated before, superfluidity in itself can be a source of cooling. However, it also reduces the effectiveness of all other processes by making particles unavailable for interactions. Because the PBF-processes only contribute a neglectable fraction to the total luminosity, their presence as caused by the superfluidity cannot compensate for the loss in luminosity of other processes. In total, superfluidity will thus reduce the magnitude of the enhanced cooling luminosity.
In table 5.2, estimates of the total cooling luminosity have been made with regards to various EOS's and the possible presence of superfluidity. As is evident from the table, the best case scenario is no neutron superfluidity and a soft EOS. The resulting cooling luminosity is then just the sum of the contributions of the pion condensate and hyperonic URCA from table 5.1. However, even less efficient models are still viable, with $L_{enh} \sim 1^{30}$ erg s^{-1}. Only the scenario in which no pions are present poses a problem, with $L_{enh} \sim 10^{29} < L_{Bob} = 1.8 \cdot 10^{30}$. A more detailed application of these results will be done in the conclusion.

Chapter 6

Conclusions

Let us briefly recall what we set out to do. In the introduction, we said that "an explanation will be given why Bob is as faint as he is.". Now it is time to see whether we have succeeded in doing so.
First, we tried to make a reasonable estimate on the maximum recurrence time of Bobs accretion by modelling the accretion processes and checking their stability. The result of this is an estimate of $t_r \approx 1 \cdot 10^4 \rightarrow 2 \cdot 10^4$ yr. Although these values were obtained using a crude model, we think they are nonetheless reasonable as no out of the ordinary physics had to be invoked to arrive at them. Also, one has to keep in mind that the processes involved are not yet fully understood. Combined with the lack of observational data on Bob, this makes more accurate estimates difficult.
All this assumes an accretion equilibrium is reached: the energy gained during accretion is radiated away during quiescence. This assumption only holds if Bob is an old cooling neutron star, because it takes a number of cycles before such an equilibrium is reached. Since Bob's accretion cycles are extraordinarily long, the process of reaching such an equilibrium might have taken in the order of 100 kyr. Also, judging from its temperature and assuming such an equilibrium exists, Bob should be at least 1 million years old according to neutron star evolution models. At a certain point, after those at least a million years of cooling, Bob started to accrete matter, most likely reheating himself. After several phases of accretion an equilibrium was reached between the heat deposited during and the heat radiated away after accretion.
Now, we return to equation 3.3. Acquiring the incandescent luminosity is simply dividing the amount of energy stored by the time available to reemit it. Or, as an equation[1]:

$$L_i = \frac{f E_{tot}}{t_r} \text{ ergs}^{-1} \tag{6.1}$$

This is now basically a relation between the storage fraction parameter f, mentioned in the summary of Frank Hemmes' work, and the quiescent luminosity. From here, we can discern two extreme situations. We can either make things difficult for ourselves, or fairly easy. The difficult approach would be using the lowest value for t_r and setting $f = 0.9$, in other words: almost all heat generated in the star stays there. If this is done, it yields an quiescent luminosity of $L_i = 3.16 \cdot 10^{31}$ ergs/s, which is more than ten times the luminosity observed in the x-ray spectrum. We can account for this excess of energy by incorporating mildly enhanced neutrino emission in Bobs inner core, ie. hyperonic processes and processes involving a pion condensate. A way to arrive at this result is to forbid neutron superfluidity and use a NN-interaction model which implies a medium to soft EOS.
To be able to employ neutrino processes we have to assume there is at least some part of the star that is not superfluid. This would require a central density of at least 3.5 ρ_0 this would imply a mass greater than 1.1 $M_\odot$.

[1] This equation yields a slightly different luminosity than equation 4.3. This is because in equation 4.3 a different amount of energy gained per nucleon is assumed in their approximation

The most optimistic result we get by setting $f = 0.1$, which is the minimal value assumed by Jonker et al. (2006) and using a long recurrence time. If we do so, the luminosity becomes $L_i = 1.77 \cdot 10^{30}$ erg s^{-1}. This is still a little below the observed luminosity, and so it would not require additional cooling mechanisms.

However, should future measurements constrain the luminosity to below this value, neutrino processes may still be at work in the core but there is a remark to make: Due to the low temperature in the core accompanying such a low thermal luminosity, neutrino processes are operating at a much slower pace because of their great temperature dependence. The only possibility left in that case is stating that accretion equilibrium is not yet reached, and that the Bobs temperature is rising because of periodic accretion, if we still employ the 'deep crustal heating' model. An extreme case of stating that equilibrium is not yet reached is supposing Bob only accreted matter one single time, but us witnessing that event seems unlikely.

Even though the severe constrains imposed, it is still possible to explain the behaviour of Bob without making use of the ADAF model. However, if the value for α_c would for some physical argument be higher, or if it is highly implausible to assume a non-superfluid kernel in the star, it might be necesary to invoke continuous accretion processes and possibly an ADAF. Still, it might be quite difficult to employ this succesfully given the constraints on the quiescent luminosity, unless other mechanisms (i.e. jet formation) are assumed. Then again, such mechanisms could also be invoked to make the *deep crustal heating* model more plausible. We did not do this as it was our aim to explain Bob while using the least amount of fancy physics possible. Thus, we think that with current knowledge, Bob still remains a strong argument in favour of the *deep crustal heating* theory.

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