The ‘vanishing’ of SXT 1H 1905+000 explained from within

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Preface

I would like to express my sincere thanks to everyone who has made this Bachelor thesis a success. Especially the following people:
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Jonas Sweep
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Contents

Chapter 1

Introduction

Neutron stars are among the most compact objects known in our universe. They are born in supernova explosions of massive stars (masses ranging from M $\sim$ 8 $M_\odot$ to $\sim$ 30 $M_\odot$, where $M_\odot$ is the solar mass), at a temperature of $\sim 10^{11}$ K. During a supernova explosion most of the mass of the supernova progenitor is blasted away, leaving a much lighter kernel: a neutron star. Neutron stars have a mass up to 2 $M_\odot$ while their radii are R $\sim$ 10 km. They most likely consist of a massive dense core surrounded by a thin crust. The density of the core ranges from $1.4{\cdot}10^{11}$ gr/cm^3 at the crust-core interface to as high as $10\rho_0$ to $20\rho_0$ in the center, where $\rho_0 = 2.8{\cdot}\ 10^{14}$ gr/cm^3, which is the density of nucleons inside nuclei.).
The neutron star crust can be divided in an atmosphere, an outer crust and an inner crust. Whereas the neutron star core can be devided in an outer and inner core. The figure below nicely summarizes the global structure of a neutron star.

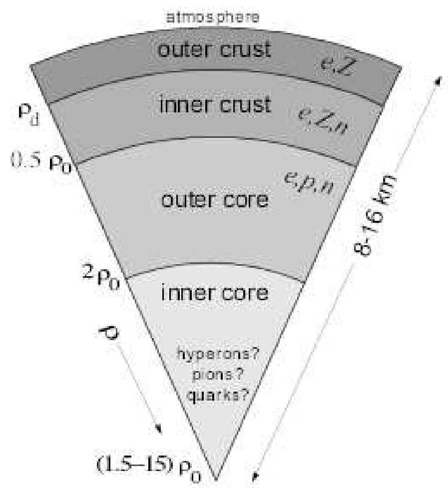

Figure 1.1: The global structure of a neutron star.

The atmosphere is a thin ($\sim$ 1 cm) plasma layer of iron in an isolated neutron star, or hydrogen, helium and possibly a small amount of heavier elements such as carbon if the neutron star is accreting matter. Accretion may take place if the neutron star is part of a binary system, in which it absorbs mass from the companion star. A detailed description of the atmosphere will be given in subsection 3.1.1.

The outer crust (thickness a few hundred metres), consisting of neutron-rich nuclei and free electrons, lies below the atmosphere and extends to the layer of density $\rho_d = 4{\cdot}10^{11}$ gr/cm^3. The outer crust is described more elaborate in subsection 3.1.2

The inner crust lies below this layer. It is made up from the same material as the outer crust with an admixture of neutrons, because at the neutron drip density ρ_d neutrons start dripping from the nuclei. A more specific approach to the inner crust is given in subsection 3.1.3.

The outer core consists of highly degenerate matter. No nuclei can exist anymore because of the high density, so matter consists of a Fermi gas of neutrons, protons, electrons and possibly muons (npeμ-matter). The composition of this gas is determined by beta equilibrium and global charge neutrality. This implies that matter in the neutron star is very neutron-rich. Hence the name neutron star. An explanation of the composition of the outer core will be given in subsection 3.1.4. The density ranges from 0.5 ρ_0 to roughly 2 ρ_0.

The inner core, as will be explained in subsection 3.1.5, starts at a density of about 2 ρ_0 and is only present in massive neutron stars (M $\gtrsim$ M$_\odot$). The composition of the inner core and the equation of state (EOS) are very uncertain. There are several possibilities but most models assume a large fraction of npeμ-matter. Some models employ condensates of pions or kaons. Other models predict the appearance of hyperons such as Σ^- and Λ. More exotic models make use of a phase transition to deconfined quark matter.

After their birth, deprived of nuclear heat sources, neutron stars begin to cool down by thermal photon emission from the surface. However since the surface area of a neutron star is small, photon emission is not the main cooling mechanism, at least not in the first stages. A neutron star mainly cools by emitting neutrinos as will be explained in section 3.2. Some 30 seconds after its birth, the star becomes transparent for neutrinos and it can lose a lot of thermal energy by emitting (lots of) neutrinos.
Most neutron stars are nicely explained by the cooling models developed so far. Recently however Jonker et al. (2006) suggested that a neutron star in a binary system was too cold to be explained by current theories. The binary system under attention is a so-called soft X-ray transient (SXT), a system that experiences bright outbursts caused by accretion followed by longer periods of quiescence in which the system is very much fainter.
The object we are talking about is 1H 1905+000, hereafter called Bob. It experienced an outburst of about eleven years with a luminosity of $4{\cdot}10^{36}$ ergs/s after which it went into quiescence, more than ten years ago. During the quiescent phase the part of the sky where Bob stood during outburst was observed but a 300 ks Chandra survey did not detect anything, thus putting a stringent limit on Bob's quiescent photon luminosity of $L_{Bob} < 1.8{\cdot}10^{30}$ ergs/s. Thus limiting the effective surface temperature, measured at infinity, of Bob to $T_s^\infty < 3.5{\cdot}10^5$ K, assuming a canonical 10 km radius. The effective surface temperature measured at infinity is smaller than measured on the surface because of gravitational redshift. It is multiplied by a factor of $\sqrt{1 - R_S/R}$ in which R_S is the Schwarzschild radius and R is the neutron star radius. No such faint SXT in quiescence has been observed so far. The faintness of Bob has prompted a debate between advocates of the ADAF theory and those who oppose it. The ADAF theory and other models will be shortly summarized in chapter 2. Reason for this is that according to the ADAF theory, neutron star accretors should be significantly brighter than their black hole counterparts. Opponents of this theory now claim that Bob, with its distinct faintness, clearly contradicts this theory as it is fainter than many observed black hole SXT's

In the following thesis an explanation will be given why Bob is as faint as he is.
This explanation will have two distinct branches. One branch will involve the application of accretion models to Bob, thus explaining how hot Bob might have become after the outburst period or determining how long it takes to *not* significantly heat Bob with the amount of accreted matter given. The other branch will concentrate on Bob's interior and try to explain where all the heat Bob acquired went. The first branch will be covered by Frank Hemmes, the second by Jonas Sweep. Each branch will form a seperate Bachelor thesis but both theses together will give a complete explanation of Bob's behaviour.

Chapter 2

On the possibility of persistent accretion

In the following chapter I will summarize the work performed on Bob by Frank Hemmes. It will mainly concern accretion and deep crustal heating. Frank puts limits on the amount of heat deposited into Bob under certain conditions. I will review how he arrived at his results.

2.1 Accretion models

As stated in the introduction, Bob is (most likely) a transiently accreting neutron star recieving matter from a companion star, probably a white dwarf. There are a few ways to explain how matter actually reaches Bob. The normal explanation involves Roche lobe overflow, as opposed to stellar wind capturing. Roche lobe overflow occurs if the outer regions of the companion star extend into the gravitational potential well of the primary, i.e. beyond the first Lagrange point. In this way the outer material of the companion feels a stronger attraction towards the primary than to the companion. Now because of conservation of angular momentum the matter that has just traversed the Lagrange point has no reason to fall any further but forms a ring around the primary. The figure below is a humble schematic picture of Roche lobe overflow.

The explanation for the fact that accretion happens nonetheless is viscosity. If the matter inside the newly formed ring about the primary is viscous, angular momentum may be redistributed: some matter carries a large amount of angular momentum and flows outward, some (typically more) matter carries less angular momentum and is now able to fall towards the primary. The exact value of the viscosity parameter α is uncertain. A model was constructed to account for this α, the so-called Magneto-Rotational Instability Model (MRI).

As the name suggests instabilities occur in the accretion disc leading ultimately to matter falling on the surface of the neutron star. The effects of these instabilities are described in the Modified Disc Instability Model (MDIM). The essence of the MDIM is the occurrence of hot and cold fronts in the accretion disc. Hot fronts are ionized and have a large value for α, they tend to move inwards at a high pace. Recombined, cold fronts are stable and have a much smaller value for α. Hot fronts, which lead to instability and thus to fast paced accretion, can be achieved in basically three ways: irradiation, a high mass transfer rate to the disc, a high surface density of the disc. Bob must have an unstable disc, as it experiences outbursts, in which the disc is falling onto the surface of the star, and quiescent periods, in which a new accretion disc is prepared which has to remain stable for a long time.

Due to the low luminosity of Bob, Frank excluded irradiation as a source of heat leading to instability. The low luminosity is also a reason to assume a low mass transfer rate. Thus for Bob, reaching the

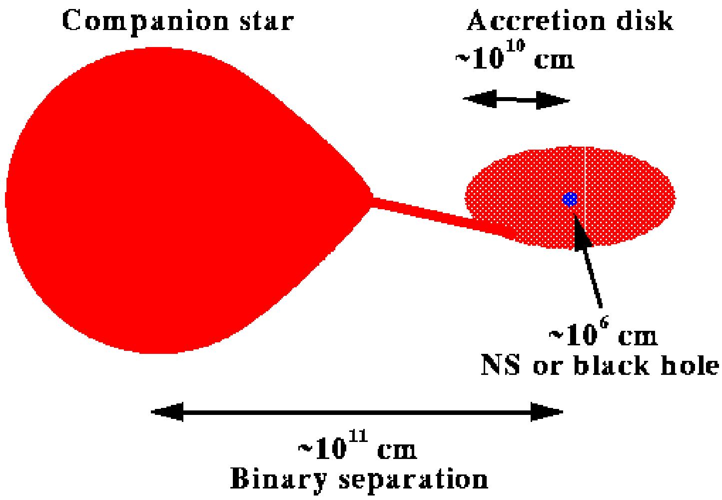

Figure 2.1: Roche lobe overflow in a binary system. The accretion disc around the primary is clearly visible.

critical surface density must be the reason for the instability.

2.2 Crustal heating

In 1998 Brown et al. assumed that the main source of the luminosity of quiescent SXT's is the heat acquired during accretion. In outburst, the main contribution to the luminosity ($4 \cdot 10^{36}$ ergs/s in Bobs case) is the hot inner part of the accretion disc. Only a small part of the heat generated during accretion reaches the star. A part of this is immediately radiated away from the stellar surface and another part can heat the star itself. The fraction of the heat deposited on the star that is actually used to heat the star is called f.
The infalling matter loses gravitational energy, about 200 MeV per nucleon, heating the inner part of the disc. This accounts for the luminosity we observe during outburst. However, accreting matter can also heat the neutron star in the following way. Due to the high pressure, pycnonuclear reactions can occur, caused by the newly infalling matter. These are fusion reactions ignited by zero point vibration of the arranged nuclei in the crust, which can even occur at T=0. This heating mechanism attributes about 200 times less energy per nucleon than is gained by the nucleons falling inwards heating the accretion disc. So the total energy deposited inside the neutron star becomes approximately:

$$E \approx f \frac{L_o t_o}{200} \tag{2.1}$$

Here L_o is the observed luminosity in outburst and t_o is the outburst time. Subsequently one assumes equilibrium between the heat generated inside the star during accretion and the heat radiated away during quiescence, which leads to the following relation:

$$f \frac{L_o t_o}{200} = L_i t_r \tag{2.2}$$

In the above equation L_i is the so-called incandescent luminosity, the amount of energy that is radiated away during quiescence by neutrino and photon emission and t_r is the recurrence time. The exact amount of heat obtained by pycnonuclear processes is given by the following formula:

$$L_{dh} = 1.45\text{MeV}\dot{M}/m_N \approx 8.74 \cdot 10^{33}\dot{M}/(10^{-10}M_{\odot}\text{yr}^{-1})\ \text{ergs/s} \tag{2.3}$$

Here m_N is the mass of a infalling nucleon. The heat generated in this way is released deep in the crust. The total amount of heat E_{tot} is calculated by multiplying this value with the accretion time. The quiescent luminosity can easily be calculated by dividing again by the quiescent or recurrence time:

$$L_q = f\frac{E_{tot}}{t_r} \tag{2.4}$$

If the star only experiences short periods of accretion the heat cannot escape to the surface, putting f equal to one. But when the accretion is persistent some heat might be lost during accretion, lowering f. The exact value of f as a function of accretion time is very uncertain, trying to keep things as general as possible Frank has calculated the quiescent timescale for high (f = 1) and low (f = 0.1) values of f.
So, there is a consistent model, *if* the disc can remain stable for the entire recurrence time following from the theory of Brown et al. Frank concluded that in the framework of the MDIM the critical mass transfer rate is never reached, as opposed to the critical density.
Frank used the MRI model to determine a value of α in the cold phase described with the MDIM. This value of α is then used to determine the time it takes for a region of the accretion disc to reach its maximum stable density. As soon as the density exceeds this critical density matter starts moving inwards. To arrive at recurrence times long enough to explain Bobs faintness the accretion disc has to remain stable for as long as possible, i.e. $2 \cdot 10^4$ yr. However, a too large recurrence time is also unfavorable since that would infer a mass transfer rate from the secondary star of less than 10^{-13} $M_{\odot}$/yr, which is unlikely considering the current binary evolution models.

Frank incorporated the mentioned density dependent inflow of matter into a computer model. The goal of modelling the accretion in this way was to determine a plausible timescale for Bobs quiescent period. This quiescent time was then used in the crustal heating model to arrive at a quiescent luminosity that explains Bobs faintness, taking into account all relevant processes of which I will come to speak.

2.3 Discussion

The quiescent luminosity is possibly partly determined by residual accretion during quiescence, to accomodate larger average mass transfer rates. Furthermore, 'crustal heating' is not the only model trying to explain the luminosity of transients. The ADAF-theory, an older alternative to the 'crustal heating' model was put forward by Lasota et al. in 1996. The ADAF theory states that during quiescence matter continues to accrete but evaporates at a certain point, thus truncating the accretion disc. This evaporated matter continues to flow towards the star in a so-called Advection Dominated Accretion Flow (ADAF). It forms an optically thick cloudlike band around the star, shielding it from observation. Because the accretion disc is truncated the accretion rate is smaller, making the disc more stable. Furthermore the ADAF contains a great amount of energy that we cannot observe, since the ADAF does not radiate as bright as an accretion disc extending to the stellar surface.
Advocates of the ADAF theory predict the quiescent luminosity of black holes to be much lower than that of neutron stars. After all, when the evaporated matter passes the event horizon it is lost from sight, not being able to generate energy by falling on some surface. Since Bobs quiescent luminosity is comparable to that of black holes, the ADAF-theory might be getting into trouble.

Chapter 3

Theoretical background

3.1 The structure of a neutron star

To fully understand the processes responsible for the cooling of a neutron star and to be able to apply them to Bob we need to understand the structure of a neutron star in a general manner.
As stated in the introduction a neutron star is composed of a crust and a core, which can be subdivided in an atmosphere, outer crust and inner crust, and an inner core and outer core respectively. The more we descend into the depths of a neutron star the more uncertain things such as the equation of state (EOS) become. Thus I will give an outline of the most important models existing today when uncertainties arise. Most differences between these models arise from the nucleon-nucleon (NN) interaction model used. An important difference between EOSs is their stiffness. An EOS is said to be stiff if the pressure at a given density is relatively high, it will be called soft if the pressure is relatively low. I will tell very little about the NN-interaction models because it is of limited importance. The behaviour of Bob can mostly be explained without it. Furthermore, the material about the NN-interaction is so elaborate[1] that it would only cloud this thesis, making it less readible and considerably longer.

3.1.1 Atmosphere

The atmosphere of a neutron star is a thin plasma layer, consisting of ionized hydrogen, helium and possibly a small amount of heavier elements such as carbon if its origin is accretion[2]. In the case of an isolated neutron star the atmosphere is made of ionized iron. Because of the accretion model used I have reason to believe that Bobs atmosphere is composed of ionized helium. The thickness of the atmosphere varies from about ten centimetres in a hot neutron star (with a surface temperature of $T_s \sim 3{\cdot}10^6$ K) to a few millimetres in a cold one ($T_s \sim 3{\cdot}10^5$ K). Its density varies from 10^{-3} gr/cm^3 at the top to 10^3 gr/cm^3 at the bottom. While the atmosphere determines much of the spectrum of a neutron star, the importance of it for our discussion is limited. The atmosphere influences the intensity as a function of energy per photon, but the total radiated power stays almost the same (Zavlin et al. 1996). Since the observations of Bob were carried out in different wavelengths (X-ray 0.2-10 keV and optical) we have a good idea of the total maximum luminosity. The atmosphere also influences the temperature gradient of the crust. This is of some importance to us, since we know something about the surface temperature and nothing about the temperature beneath the crust. We have to use a model of the atmosphere to

[1] Hans Bethe remarked in 1953 that in the preceding 25 years more labour was put in understanding the nucleon-nucleon interaction than in solving any other scientific problem. The amount of material has only increased since then and most likely not at a slower pace

[2] Hydrogen would originate from a main sequence star or a brown dwarf, helium or heavier elements could come from a white dwarf.

determine the temperature at the bottom of the crust which in turn determines the neutrino emissivity, which I will explain in the next section.

3.1.2 Outer crust

The outer crust of a neutron star, with a thickness of a few hundred metres, consists of free electrons and nuclei. The electrons form an ideal Fermi gas which becomes ultrarelativistic at $\rho \gg 10^6$ gr/cm^3. The nuclei become increasingly neutron-rich with increasing density because of beta equilibrium. At increasing density, because of the high pressure, the electrons present will be captured by the protons inside the nuclei to form neutrons (inverse beta decay). For a general nucleus X that means:

$$ {}^A_Z X + e^- \rightarrow {}^A_{Z-1} Y + \nu_e \tag{3.1}$$

where Z is the number of protons per nucleus and A the total number of nucleons per nucleus. Beta decay

$$ {}^A_Z X \rightarrow {}^A_{Z+1} Y + e^- + \bar{\nu_e} \tag{3.2}$$

will also occur, but since beta decay increases the number of particles the equilibrium is shifted towards more neutrons as the density increases. The density ranges from 10^3 gr/cm^3 to $\rho_d = 4{\cdot}10^{11}$ gr/cm^3, which is the neutron drip density. At this density neutrons start dripping from the nuclei. This occurs because at this density the energy gained by decreasing the number of neutrons in a nucleus is greater than the energy gained by capturing another electron.
Now we turn to a more quantitative treatment of nuclear matter in beta equilibrium, which is correctly described by the Harrison-Wheeler EOS (HW). We also want to include the possible formation of a Coulomb lattice of nucleons in a sea of electrons. To do this we will extend the HW treatment by including the lattice energy just as Baym, Pethick and Sutherland (hereafter BPS) have done in 1971. In other words, I will derive the BPS-EOS. Although classical in nature, the BPS-EOS describes matter in the outer crust reasonably well.
We wish to find the lowest energy state of our neutron star: a system of $\sim 10^{57}$ baryons in beta equilibrium with relativistic electrons. To do this we write the energy density of all the constituents of our system:

$$\epsilon = n_N M_N(A,Z) + \epsilon_e(n_e) + \epsilon_L \tag{3.3}$$

there n_N is the number density of nuclei and M_N(A,Z) is the energy per nucleus[3], so the first term is the total energy density of the nuclei. In the same way the second term is the energy density of the free electrons minus their rest mass. The third term denotes the lattice energy, should a crystal lattice form. Such a lattice is important for the composition of the outer crust but the effect on the pressure is very small. The energy per nucleus is adequately given by the semi-empirical mass formula (SEMF):

$$M_N(Z,A) = m_u c^2 [b_1 A + b_2 A^{\frac{2}{3}} - b_3 Z + b_4 A (\frac{1}{2} - \frac{Z}{A})^2 + b_5 \frac{Z^2}{A^{\frac{1}{3}}}] \tag{3.4}$$

where m_u is the atomic mass unit. The constants b_i are given by:

$$b_1 = 0.991749 \; , b_2 = 0.01911 \; , b_3 = 0.000840 \; , b_4 = 0.10175 \; , b_5 = 0.000763 \tag{3.5}$$

An explanation of the terms in the SEMF can be found in any nuclear physics textbook and is beyond the scope of this thesis.
The free electrons effectively constitute a T=0 Fermi gas because the temperature of the neutron star crust($< 10^8$K) is much smaller than the Fermi temperature of the electrons ($T_F \approx 10^9$ K at $\rho =$

[3]including the rest mass of the corresponding electrons

$10^6 gr/cm^3$). The energy density of the free electrons ϵ_e is given by the following expression, because the electron mass is already incorporated in the SEMF:

$$\epsilon_e = \frac{2}{h^3} \int_0^{p_F} (\sqrt{p^2c^2 + m_e^2 c^4} - m_e c^2) 4\pi p^2 dp \tag{3.6}$$

In the ultra-relativistic limit ϵ_e is given by:

$$\epsilon_e = \frac{c}{4\pi^2 \hbar^3} p_F^4 \tag{3.7}$$

For the Fermi momentum p_F we have by definition:

$$n_e = \frac{2}{h^3} \int_0^{p_F} 4\pi p^2 dp = \frac{p_F^3}{3\pi^2 \hbar^3} \tag{3.8}$$

Assuming a body centered cubic lattice structure[4], the lattice energy is given by

$$\epsilon_L = -1.444 Z^{\frac{2}{3}} e^2 n_e^{\frac{4}{3}} \tag{3.9}$$

In equilibrium the derivatives of the total energy density with respect to Z and A must vanish[5]. First we note that:

$$\frac{d\epsilon_e}{dn_e} = E_{F_e} - m_e c^2 \tag{3.10}$$

where the Fermi energy E_F is given by

$$E_F = \sqrt{p_F^2 c^2 + m^2 c^4} \tag{3.11}$$

The equilibrium composition for a fixed value of the nucleon density n, i.e. the density of protons and neutrons together, is now found by minimizing ϵ with respect to pairs (A,Z), putting the nucleon number density n_N equal to n/A and the electron number density n_e equal to nZ/A (in view of electrical neutrality). The pressure is subsequently calculated using the first law of thermodynamics, giving us ultimately an equation of state, which is what we set out to do.

$$P = n^2 \frac{\partial(\epsilon/n)}{\partial n}\bigg|_{A,Z} = n_e^2 \frac{\partial(\epsilon_e/n_e)}{\partial n_e} + n_e^2 \frac{\partial(\epsilon_L/n_e)}{\partial n_e} = P_e + P_L = P_e + \frac{1}{3}\epsilon_L \tag{3.12}$$

where P_e is the Fermi electron pressure and P_L is the lattice pressure. The electron pressure is equal to the pressure inside the nuclei because of mechanical equilibrium. If the electron pressure would be greater than the nuclear pressure electrons would flow into the nuclei until equilibrium is reached and vice versa.

3.1.3 Inner crust

The inner crust is about a kilometre thick and is composed of nuclei, electrons and neutrons. The inner crust starts at the neutron drip point and extends to the density ($\rho \approx 0.5\rho_0$) where no nuclei are present anymore and matter is completely composed of neutrons, protons electrons and possibly muons.
We use a compressible liquid drop model (CDLM) to describe it. Incorporating the compressibility of the nucleons is important because for instance at some high density the equilibrium state will contain non-spherical nuclei, which a non-compressible model would not allow us to explain. The CDLM allows us

[4] this is a reasonable approximation given the fact that detailed numerical calculations of the lattice energy with other, more realistic structures yield almost the same result

[5] Like Harrison and Wheeler we treat Z and A as continuous variables

to understand the physics at work best, arriving at the same results as complicated microscopic models. The foundation for the CDLM was laid by Baym, Bethe and Pethick in 1971, the model has of course been refined since then but the basic ideas are the same.
The formula for the total energy density is:

$$\epsilon(A, Z, n_N, n_n, V_N) = n_N(W_N + W_L) + (1 - V_N n_N)\epsilon_n(n_n) + \epsilon_e(n_e) \tag{3.13}$$

This expression differs from the one describing the outer core by the appearance of the energy density associated with the Fermi gas of the neutrons with number density n_n. This neutron Fermi gas is only present outside the nuclei, hence the factor (1-$V_N n_N$), in which V_N is the average volume occupied by a nucleus.
W_N is just the energy of a nucleus including its rest mass, whereas W_L is the lattice energy of the nuclei including the interaction energy of the electrons. As before n_N is the number density of the nuclei and ϵ_e is the energy density of the Fermi gas of ultra-relativistic electrons. The number density of electrons n_e satisfies the neutrality condition: n_e=n_NZ.

There are several contributions to W_N. The first contribution involves the rest mass M times c^2 of the nuclei.
The second contribution is the binding energy of the nucleons $W_B(n)$. A lot of work on this nucleon-nucleon (NN) interaction term has been performed since BBP introduced their EOS in 1971 and a lot of progress has been made, however the interaction between nucleons is still very hard to incorporate in a neutron star. I will come back to that in the next section when I will elaborate on the structure of the neutron star core, where the NN interaction is of more importance.Also the Coulomb interaction energy between the nucleons W_C is important as well as a surface energy term W_S. We thus get:

$$W_N = Mc^2 + W_B + W_C + W_S \tag{3.14}$$

The sum of the lattice and the Coulomb terms is approximately equal to:

$$W_C + W_L \approx \frac{3}{5}\frac{Z^2e^2}{r_N}\left(1 - \frac{r_N}{r_c}\right)^2\left(1 + \frac{r_N}{2r_c}\right) \tag{3.15}$$

Here r_N is the average radius of a nucleus and r_c is defined as the radius of a sphere containing exactly one nucleus:

$$\frac{4\pi}{3} n_N r_c^3 \equiv 1 \tag{3.16}$$

The energy density of the neutrons is composed of a rest mass term and a NN-interaction term which we take the same as for the nucleons inside the nuclei. Therefore:

$$\epsilon_n(n_n) = n_n(W_B(n_n) + m_n c^2) \tag{3.17}$$

We can approximate the energy density of the Fermi gas of electrons with the extreme relativistic formula given below, since the contribution to the energy due to their interaction is already incorporated in W_L:

$$\epsilon_e = \frac{\hbar c}{4\pi^2}(3\pi^2 n_e)^{\frac{4}{3}} \tag{3.18}$$

The surface energy W_S depends on the shape of the nuclei and on their curvature. Curvature corrections arise from the fact that nuclei can have different surface energies for different shapes with the same area. This is a small effect that we can neglect for simplicity. The surface energy becomes thus:

$$W_S = u\sigma S/V \tag{3.19}$$

Where σ is the energy per unit area, S is the area and V is the volume of the nucleus. The parameter u is the fraction of space filled with dense matter.

Because we assume thermodynamic equilibrium there are a four[6] conditions the material in the crust must fulfil to minimize the total energy.
First the nuclei must have a fixed number of nucleons A for fixed nucleon density n. This composition is determined by minimalisation of the energy per nucleon inside a nucleus as a function of A, so:

$$\frac{\partial}{\partial A}\frac{W_N + W_L}{A} = 0 \tag{3.20}$$

Second the nuclei should be in beta equilibrium with the electron gas. This means that the following relation holds between the chemical potentials of the protons and neutrons in the nuclei and the electrons:

$$\mu_e = \mu_n^N - \mu_p^N \tag{3.21}$$

Third the neutrons in the nuclei must be in chemical equilibrium with the neutron gas, ie. transferring a neutron from the gas to a nucleus or vice versa must cost no energy. So the chemical potentials of the neutron gas and the neutrons inside the nuclei should be equal.
Fourth there must be mechanical equilibrium between the nuclei and the neutron gas, i.e. the nuclear pressure P_N inside the nuclei should be the same as the pressure of the neutron gas outside of the nuclei. Now for the total pressure we again have:

$$P = n^2 \frac{\partial}{\partial n}\left(\frac{\epsilon}{n}\right) = P_{neutron\ gas} + P_e + P_L = P_N + P_e + P_L \tag{3.22}$$

where P_e and p_L are the partial pressures due to the electrons and the lattice. As I already mentioned, at some point the equilibrium geometry of the nuclei is non-spherical. This in fact depends on the NN interaction model used. I comment on two models that are applicable in the entire neutron star. The Friedman-Pandharipande-Skyrme (FPS) model, for instance, comes with non-spherical geometries, while the Skyrme-Lyon effective interaction (SLy) does not. The possibility of non-spherical nuclei was first discovered by Ravenhall, Pethick and Wilson in 1983. Due to the high contribution of the Coulomb lattice energy to the minimal energy and the relatively low contribution of the surface energy term and the nuclear Coulomb energy, the lowest amount of surface area does not minimize the total energy anymore at high enough densities. Therefore cylindrical, planar and tube- or bubble-like geometries can occur.
The different shapes of the nuclei are given in the figure below. In the case of spherical, cylindrical or

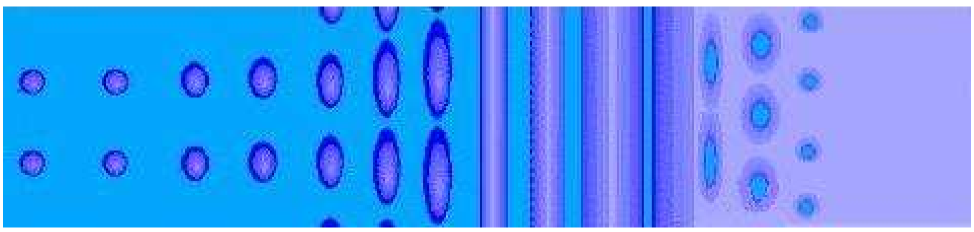

Figure 3.1: Different phases of neutron star matter in the mantle. From left to right: spherical nuclei, spaghetti, lasagna, swiss cheese and uniform dense matter.

planar nuclei W_S is given by

$$W_S = \frac{u\sigma d}{r} \tag{3.23}$$

[6]since ϵ depends on five variables

where r is the radius of the nucleus and d is the dimensionality of its geometrical boundary. So we have d=3 for spherical nuclei (3N), d=2 for rods (2N) which can be bounded by planes, just like a knive cutting spaghetti bounding a spaghetti thread from one side, and d=1 for planar nuclei (1N) that have one-dimensional boundaries..
Arranging the different phases according to boundaries that are not evident in a neutron star seems strange, but that is the way Ravenhall, Pethick and Wilson classified these phases. I use their description, although somewhat vague, to avoid possible discrepancies with other literature. Furthermore, at high enough densities these exotic nuclear shapes may be turned inside out to further minimize the Coulomb lattice energy. Holes (bubbles) are created inside the almost uniform nuclear matter, leading to tube-like (2B) and eventually to bubble-like (3B) phases. Note that planar bubbles (1B) and normal planar phases are the same. For obvious reasons the exotic phases 2N, 1N=1B and 3B have been given the following nicknames: spaghetti, lasagna and Swiss cheese respectively.

In the Wigner Seitz approximation the Coulomb energy of the lattice can be given by the following expression:

$$W_C = \frac{4\pi}{5}(n_{p,i}re)^2 f_d(u) \tag{3.24}$$

The Wigner Seitz cells are given by: 3B spheres, 2B coaxial cylinders and 1B1N parallel slabs. The radius of a nucleus, a rod, a tube or a bubble, depending on the dimensionality, is equal to r. The number density of the protons inside the nucleus or around the hole is $n_{p,i}$, e is the electron charge and f_d is a filling factor dependent dimensionality factor given by:

$$f_d(u) = \frac{5}{d+2}\left(\frac{1}{d-2}\left(1 - \frac{1}{2}du^{1-\frac{2}{d}}\right) + \frac{u}{2}\right) \tag{3.25}$$

where for d=2 one takes the limit d→2. For the correct description of bubble-like phases one should replace u by $1-u$.
The virial theorem for nuclei in equilibrium states:

$$W_S = 2W_C \tag{3.26}$$

All this leads to the phase diagram on the next page, showing which phases are present at which densities.

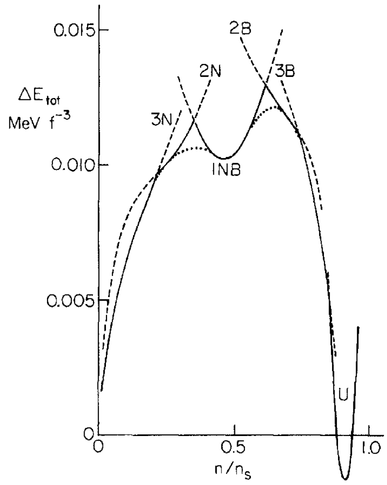

Figure 3.2: The different phases of the inner core as a function of the density given as a fraction of n_s, the standard nuclear number density. At a density of about 0.85 n_s the 3B phase turns into uniform matter which also makes up the core. Taken from Ravenhall and Pethick 1983.

3.1.4 Outer core

The outer core begins at the point where nuclei cannot occur in equilibrium. In the language of the CLDM it is hard to calculate this point exactly, we can however determine the lowest density at which uniform npe(μ)-matter is stable, up to this density translational invariance is violated. So looking from the inside out, at this point we expect nuclei to form, thus defining the crust-core interface at a density ρ_{cc}.
The outer core is a few kilometres thick and its density ranges from about 0.5 ρ_0 to about 2 ρ_0. The aforementioned npe-matter is a plasma of neutrons, protons and electrons in beta equilibrium. If the Fermi Energy of the electrons exceeds $m_\mu c^2 = 105.7$ MeV, muons begin to form. The description of core matter as an npeμ plasma is valid until a density of about $2\rho_0$. Above that hyperons may form or our EOS can be influenced by the appearance of meson condensates.
To find a suitable EOS for the outer core for a given number density, I will first derive what I can without a model for the NN-interaction. We consider a nearly ideal Fermi gas of electrons and possibly muons. The nucleons form a strongly interacting Fermi liquid, i.e. lattice or Coulomb corrections are not relevant this time. The energy density is given by:

$$\epsilon(n_n, n_p, n_e, n_\mu) = \epsilon_b(n_n, n_p) + \epsilon_e(n_e) + \epsilon_\mu(n_\mu) \tag{3.27}$$

where $\epsilon_{b,e,\mu}$ represent the energy densities of the baryons, electrons and muons respectively. We again expect beta equilibrium resulting in the now familiar condition

$$\mu_e + \mu_p = \mu_n \tag{3.28}$$

We also demand electric neutrality, so $n_e + n_\mu = n_p$. From this last condition we derive a relation between the chemical potentials of the muons and the electrons using a Lagrange multiplier λ. In order to minimize the energy density under the condition $n_e + n_\mu = n_p$ one should minimize the following expression with respect to n_e and n_μ:

$$\tilde{\epsilon} = \epsilon - \lambda(n_e + n_\mu - n_p) \tag{3.29}$$

$$\frac{\partial \tilde{\epsilon}}{\partial n_e} = \frac{\partial \epsilon}{\partial n_e} - \lambda = \mu_e - \lambda = 0 \quad \text{and} \quad \frac{\partial \tilde{\epsilon}}{\partial n_\mu} = \frac{\partial \epsilon}{\partial n_\mu} - \lambda = \mu_\mu - \lambda = 0 \tag{3.30}$$

Substracting these equations yields

$$\mu_e = \mu_\mu \tag{3.31}$$

From this we see that muon beta equilibrium is achieved as well. When muons are present, i.e. if μ_e >105.7 MeV, we can treat them on equal ground as the electrons, apart from their mass of course. The pressure can again be calculated using the first law of thermodynamics, where the derivatives have to be taken at equilibrium.

$$P = P_b + P_e + p_\mu = n_b^2 \frac{d(\epsilon_b/n_b)}{dn_b} + n_e^2 \frac{d(\epsilon_e/n_e)}{dn_e} + n_\mu^2 \frac{d(\epsilon_\mu/n_\mu)}{dn_\mu} \tag{3.32}$$

Where the subscript b refers to the baryons, i.e. the protons and neutrons. The electron pressure can be calculated using the first law of thermodynamics as well. We take the energy of the ultra-relativistic electrons to be the same as in equation 3.7, because now their mass is negligible. The muon energy can be calculated in the same way using T=0 Fermi theory, the muons are however mildly relativistic so their mass cannot be neglected:

$$\epsilon_\mu = \frac{2}{h^3} \int_0^{p_F} \sqrt{p^2c^2 + m_\mu^2 c^4} 4\pi p^2 dp \tag{3.33}$$

The pressure of the baryons (subscript b) can be written as follows, because of a possible particle fraction ($x_i \equiv n_i/n_b$) dependence of the energy per baryon:

$$P_b = n_b^2 \left(\frac{\partial \epsilon_b / n_b}{\partial n_b} \right) + n_b \sum_{i=p,e,\mu} \left(\frac{\partial \epsilon_b}{\partial x_i} \right) \left(\frac{\partial x_i}{\partial n_b} \right) \tag{3.34}$$

However the last term vanishes because of n_b=n_p+n_n, electrical neutrality, 3.28 and 3.31. The energy per baryon, depending on n_b, x_p, x_e and x_μ, will be calculated in the following part.

To calculate the energy per baryon we use a considerably simplified model. We split the total energy into a part attributable to symmetric nuclear matter and a symmetry energy. Analogous to the treatment of heavy nuclei we consider two independent Fermi gasses of protons and neutrons. Detailed many-body calculations performed by Lagaris et al. in 1981, Wiringa et al. in 1988 and Akmal et al. in 1998 show that we can approximate the total energy per baryon to a high precision, even when δ is almost equal to one, by stating:

$$E_n = E_0(n_b) + S(n_b)\delta^2 \tag{3.35}$$

Here E_0 is the energy of symmetric nuclear matter, involving equal numbers of protons and neutrons. S is the symmetry energy and $\delta = 1 - 2x_p$ is the neutron excess. So, the more asymmetric the matter, the higher the symmetry energy becomes. For a conceptual understanding of the outer core we consider the Free Fermi Gas FFG model of nuclear matter which states:

$$E_{FFG}(n_b, \delta) = \frac{3}{10} E_F(n_b) \left((1+\delta)^{\frac{5}{3}} + (1-\delta)^{\frac{5}{3}} \right) \tag{3.36}$$

Here

$$E_F = \frac{\hbar^2}{2m} \left(\frac{3}{2} \pi^2 n_b \right)^{\frac{2}{3}} \tag{3.37}$$

with $m = (m_n + m_p)/2$ the mean nucleon mass. In the approximation justified above the difference between the chemical potentials of the protons and neutrons would become:

$$\mu_n - \mu_p = 4(1 - 2x_p)S(n_b) \tag{3.38}$$

Without the appearance of muons we have $x_p = x_e$, which together with 3.28 implies:

$$x_p(n_b) = \frac{64S^3}{3\pi^2(\hbar c)^3 n_b} \approx 4.75 \cdot 10^{-2} \left(\frac{n_0}{n_b} \right) \left(\frac{S(n_b)}{S(n_0)} \right)^3 \tag{3.39}$$

where n_0 is the standard nuclear nucleon density and experiments have shown that $S(n_0) \approx 30$MeV This Fermi gas model with a symmetry energy lets us explain the important features of neutron star behaviour. To be more accurate one should use a realistic NN interaction model and even consider three body interactions (NNN). This can be done using a lot of computing power but is beyond the scope of this thesis. Realistic models are the FPS model and the SLy model, which can in principle be applied to the entire neutron star.

3.1.5 Inner core

The inner core starts at a density of about 2 ρ_0. It can be up to a few kilometres thick, however light neutron stars do not have an inner core. Due to the high densities in the inner core uncertainties arise about the composition of matter and its interaction.
To describe the inner core we use a minimal model which incorporates and explains the most important features and is easy to understand. Extra attention will be given to the formation of pion condensates, and the possible formation of hyperons.
The essence of a minimal model is assuming the same composition in the entire core: npeμ-matter, which lets us use the machinery of the previous section. Doing this we can say something important about the cooling of neutron stars in general.
The most important cooling mechanism for neutrino cooling is the direct URCA process, see section 3.2.1. The exact rate will also be calculated in the next section. There is however a severe condition which has to be met for the direct URCA process to be allowed. We can best understand this condition by employing some T=0 Fermi theory.
Should the temperature be absolute zero, due to the Pauli exclusion principle fermions will occupy all the states below a certain level, the Fermi surface. If the possible thermal excitations k_BT, have a much lower energy than the Fermi energy $E_F \equiv k_B$T we can approximate the situation with a T=0 Fermi gas. In a neutron star core we would get a T=0 Fermi gas for the electrons and a T=0 Fermi liquid for the protons and neutrons, because they experience strong interactions. Only in a thin spherical shell of relative thickness T/T_F around the Fermi surface thermal excitations occur. This means that some states in this small energy band are empty, so particles with energies close to the Fermi energy can participate in (neutrino producing) reactions, for instance the direct URCA process. Due to the small size of the participating energy band we can approximate the momenta of the interacting particles with their Fermi momenta p_F. The momentum of the produced neutrino is of order $k_BT/c \ll p_{Fn}$ because of the previous argument. So we have by momentum conservation in the (inverse) beta-decay reactions 3.43 and 3.44:

$$p_{Fn} < p_{Fp} + p_{Fe} \tag{3.40}$$

As can be seen from 3.39 the proton fraction increases with increasing density. The direct URCA process is allowed if the proton fraction x_p exceeds 1/9. The density at which this condition is satisfied depends on the symmetry energy S(n_b) and different models of the NN-interaction give different symmetry energies. Stiff EOS's require high densities for such a high proton fraction while soft EOS's can achieve a substantial proton fraction much easier. The corresponding mass of a neutron star with a high enough proton fraction to accomodate direct URCA would become 1.9 $M_\odot$ for a stiff EOS and as low as 1.4 $M_\odot$ for a soft model. Invoking the minimal model of a core consisting of npeμ-matter is a bold assumption, including hyperonization and the formation of meson condensates refines the model for the inner core.
The physical argument for π^- condensation is the same as the argument for the occurrence of muons. If the chemical potential of the electrons exceeds the rest energy of the negative pion it is favorable to create a pion. Since pions are bosons they can occupy the same state thus forming a condensate.
The exact creation mechanism follows from spontaneous chiral symmetry breaking as is discussed by Muto in 1993. There is an attractive interaction between nucleons in the L=1 state[7] and pions. Because of

[7]The nucleon L=1 orbital angular momentum state is often called the p-wave, since it corresponds to the L=1 term in partial wave analysis.

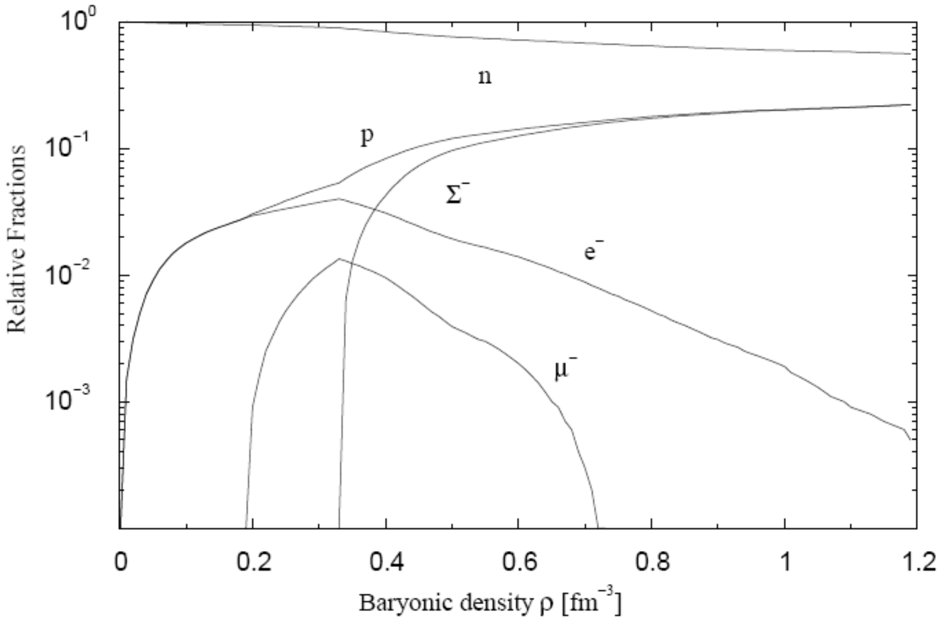

Figure 3.3: Particle fractions as a function of the baryon density obtained by Vidaña et al. The figure was obtained with model E of Rijken et al. for the NN-interaction, NNN-interaction, NH-interaction and HH-interaction.

this interaction excitations occur that resemble real pions in a finite momentum state. These excitations are referred to as the condensate.
The first hyperon to appear following the same argumentation, is the Σ^- particle (with quark composition dds). It occurs when

$$\mu_{\Sigma^-} = \mu_n + \mu_e \tag{3.41}$$

Although one would initially suspect neutral Λ hyperons to form because they are lighter, this is not the case because they have to take the place of neutrons which means:

$$\mu_\Lambda = \mu_n \tag{3.42}$$

This happens at a higher density. In the figure below, obtained by Vidaña et al. in 2000 by using Nijmegen model E (Rijken et al. 1999) for the NN, NNN and NH interactions, particle fractions as a function of the baryonic density are given. The EOS of the inner core is softened by the occurrence of hyperons. Because of their higher mass the Fermi momentum per particle is lower and hence the pressure as well.

3.2 Neutron star cooling processes

As stated in the introduction, neutron stars cool mainly by neutrino processes. Because the surface area of a neutron star is so small only a small fraction of the energy is radiated away in the form of photons. Neutrinos can escape the neutron star without too much trouble. In the following sections I will go into detail about the six most important processes for neutrino cooling. I will cover the application of these processes in detail in chapter 4.

3.2.1 Direct URCA

The direct URCA process, proposed by Gamow and Schoenberg in 1941, is simply beta decay of neutrons and inverse beta decay (electron capture) of protons and electrons. Summarized in the following reactions:

$$n \rightarrow p + e^- + \bar{\nu_e} \tag{3.43}$$

$$p + e^- \rightarrow n + \nu_e \tag{3.44}$$

As both reactions occur simultaneously the number of neutrons, protons and electrons[8] remains constant (which it must for the star to have a fixed composition), but a tremendous amount of neutrinos can fly away in this manner, carrying a lot of momentum and thus a lot of thermal energy.
However, due to simultaneous conservation of momentum and energy these reactions can only occur if the neutron coming from reaction 3.44 can occupy a free level in or atop the Fermi-sea of neutrons. In general this is not the case, as we have seen in subsection 3.1.5. This implies that the reaction rates for the direct URCA processes are suppressed in some cases and that other reactions may be important in these situations. In 1991 Lattimer et al. derived the emissivity per cm^3 of the direct URCA process using Fermi's Golden Rule:

$$L_{Durca} = \frac{457\pi}{10080} G_F^2 cos^2(\theta_c)(1+3g_A^2)\Theta \frac{m_n^* m_p^* m_e^*}{\hbar^{10} c^3}(k_B T)^6 = 4.00 \cdot 10^{27} \left(\frac{n_e}{n_0}\right)^{\frac{1}{3}} \frac{m_n^* m_p^*}{m_n^2} \Theta T_9^6 \text{ ergs/cm}^3\text{s} \tag{3.45}$$

Here T_9 is the temperature of the emitting material in units of 10^9 K, $G_F \approx 1.436 \cdot 10^{-49}$ erg cm^3 is the weak coupling constant, $\theta_c \approx 0.239$ is the Cabibbo angle, $g_A \approx -1.261$ is the axial vector coupling constant and $\Theta = \Theta(p_{F_p} + p_{F_e} - p_{F_n})$ is the stepfunction fulfilling the condition that the neutron Fermi momentum should be smaller than the proton Fermi momentum plus the electron Fermi momentum, as was explained in subsection 3.1.5. The starred masses are the effective masses of the nuclei in the medium of the stellar core. In the last expression, n_0 is the standard nuclear number density.
The electrons in the direct URCA process can easily be replaced with muons, leading to the same emissivity. Only the treshold density will differ.

3.2.2 Modified URCA

In order to conserve energy and momentum at the same time the direct URCA process has been adapted by Chiu and Salpeter in 1964. They introduced the 'modified URCA' reactions:

$$N + n \rightarrow N + p + e^- + \bar{\nu_e} \tag{3.46}$$

$$N + p + e^- \rightarrow N + n + \nu_e \tag{3.47}$$

The bystander nucleon N can absorb the momentum needed for the proton to fit in the Fermi sea of protons, or it can cough up the momentum needed for the neutron to be able to sail atop of the Fermi sea of neutrons. This slow process is the least constrained and can happen in every neutron star core.

[8]In principle muons can interact instead of electrons, but muons only occur above a certain treshold density.

Superfluidity can suppress its rate, but I will come to that when I discuss pair formation processes. The emissivity of the modified URCA process is a little more complicated. If the process operates in a core consisting of npe(μ)-matter the bystander nucleon would of course be a neutron or a proton. This gives rise to two branches of the modified URCA process: a proton branch and a neutron branch. The method of calculation is the same as for direct URCA, but it has to be done twice in view of the two channels. There are also uncertainties which arise from the fact that the interaction is strong. To approximate the strong interaction the one-pion-exchance (OPE) model has been used by Friman and Maxwell and they obtained the following result for the neutron branch in 1979:

$$L_{Murca(n)} = \frac{11541}{60480\pi} G_F^2 cos^2(\theta_c) g_A^2 m_n^{*3} m_p^* \left(\frac{f_\pi}{m_\pi}\right)^4 \frac{p_{F_p}(k_B T)^8}{\hbar^{10} c^8} = 6.1 \cdot 10^{21} \left(\frac{m_n^{*3} m_p^*}{m_n^3 m_p}\right) \left(\frac{n_p}{n_0}\right)^{\frac{1}{3}} T_9^8 \text{ ergs/cm}^3\text{s} \tag{3.48}$$

Yakovlev and Levenfish showed in 1995 that the magnitude of the proton branch emissivity is of equal order of magnitude:

$$L_{Murca(p)} \approx \left(\frac{m_p^*}{m_n^*}\right)^2 \frac{(p_{F_e} + 3p_{F_p} - p_{F_n})^2}{8 p_{F_e} p_{F_p}} \Theta' L_{Murca(n)} \tag{3.49}$$

The total emissivity for a typical core composition is given by the following expression:

$$L_{Murca(n+p)} = 9.2 \cdot 10^{21} \left(\frac{m_n^{*3} m_p^*}{m_n^3 m_p}\right) \left(\frac{n_p}{n_0}\right)^{\frac{1}{3}} T_9^8 \text{ ergs/cm}^3\text{s} \tag{3.50}$$

The constants in the above equations are the same as in the subsection 3.2.1, the constant $f_\pi \approx 1$ is an effective pion-nucleon coupling constant. The Θ-function Θ' in eqation 3.49 now guarantees an other condition, namely:

$$p_{F_n} < 3p_{F_p} + p_{F_e} \tag{3.51}$$

In other words, Θ' equals one if $p_{F_n} < 3p_{F_p} + p_{F_e}$ and zero otherwise. This is satisfied almost everywhere in the neutron star core, except maybe at its utter boundary in some models. Modified URCA reactions via the neutron branch will still be possible there.

3.2.3 Processes involving pion condensates

The pion condensate interacts strongly with the nucleons. This interaction mixes neutron and proton states. These states are now best described as quasinucleons that have comparable momenta. Because their (Fermi) momenta are almost the same, we can construct a direct URCA-like process with the quasinucleons with a reasonable high neutrino output:

$$\tilde{n} \rightarrow \tilde{p} + e^- + \bar{\nu}_e \tag{3.52}$$

$$\tilde{p} + e \rightarrow \tilde{n} + \nu_e \tag{3.53}$$

Where p̃ and ñ are the quasiprotons and -neutrons.
The emissivity was calculated by Maxwell et al. in 1977, the calculation is very analogous to that of the direct URCA emissivity. The emissivity can in fact be given in terms of the direct URCA emissivity:

$$L_\pi = L_{Durca} \left(\frac{m_n^*}{m_p^*}\right) \left(\frac{\mu_\pi^2}{k_\pi m_e^*}\right) \frac{\theta^2}{16} \left(1 + \left(\frac{g_A k_\pi}{\mu_\pi}\right)^2\right) \tag{3.54}$$

Here θ is the condensation angle that is used as an order parameter to describe the phase transition to condensed matter, i.e. θ=0 means no condensation. The mass parameters μ_π and k_π are approximately equal to the pion mass. Filling in all the known constants:

$$L_\pi \approx 1.0 \cdot 10^{26} \left(\frac{m_n^*}{m_n}\right)^2 T_9^6 \text{ ergs/cm}^3\text{s} \tag{3.55}$$

All this hinges on the occurrence of a pion condensate, which remains unsure. However, according to some realistic models of dense stellar matter, above a density value of several standard nuclear densities condensation is likely.
One can of course imagine the condensation of other bosons, such as kaons. The arguments for creating such a condensate are essentially the same, however the mass of kaons and other bosons is higher, which makes their occurrence less apparent.

3.2.4 Pair breaking and formation processes and superfluidity

Apart from these URCA-like processes there is another process, sometimes referred to as minimal cooling: Cooper pair breaking and formation (PBF), which is responsible for substantial cooling if certain conditions are met, (see below). The PBF process was proposed by Flowers et al. in 1976, it was generalized by Yakovlev et al. in 1998 and further investigated in 2004 by Page et al. and again by Yakovlev et al. To understand the PBF process, consider the following.
If there is some attractive interaction between nucleons[9], Cooper pairing may occur resulting in a gap Δ in the energy spectrum of the nucleons, which equals half the energy gained by forming a pair. Consider for example neutrons in a singlet orbital angular momentum state. The symmetric nature of the spatial part of the wavefunction can lead to an effective attraction.
Pairing can most easily be understood in the quasi-particle language. In the new ground state of the system, i.e. with the gap, all the particles form pairs. The system can be excited by breaking one or more pairs. These excitations are the quasi-particles and the ground state is sometimes referred to as the quasi-particle vacuum. So, in fact these quasi-particles are single unpaired neutrons or protons in our quasi-particle language. Formation of a pair out of two unpaired nucleons, i.e. the annihilation of two quasi-particles, gives a neutrino pair via a neutral current reaction. This is the case because two unpaired nucleons constitute a higher energy than a pair of them and a pair of neutrinos. These nucleon pairs can then be broken by thermal excitations, i.e. the creation of two quasi-particles, which consumes thermal energy. The occupation of paired and unpaired states assuming thermal equilibrium is thus governed by Fermi-Dirac statistiscs. As pairs are continuously broken and formed, neutrinos and anti-neutrinos of all types are produced continuously as well. The neutrino producing reaction can be summarized like this:

$$B_u + B_u \rightarrow B_p + B_p + \nu + \bar{\nu} \tag{3.56}$$

in which B_u is an unpaired baryon and B_p is a paired baryon. In our quasi-particle language we get:

$$\tilde{B} + \tilde{B} \rightarrow \nu + \bar{\nu} \tag{3.57}$$

in which $\tilde{B}$ is a quasi-baryon. The amount of energy carried away due to the PBF process can also be calculated using Fermi's Golden Rule. Flowers et al. (1976) did this for neutron pairing in the singlet state to lowest order in $\frac{T}{T_c}$, where $T_c \approx 10^9$K assuming superfluidity of the BCS type. However there is a wide range of values for T_c in the literature, see for instance Levenfish 2007. Yakovlev et al. (1998) generalized the previous results to proton pairing and neutron pairing in the triplet angular momentum state. Superfluidity in this state only occurs at high densities because the anti-symmetric nature of the spatial part of the wave function has to be overcome for the particles to be able to interact strongly.
In general the PBF process is suppressed exponentially at low temperatures because of the lack of excitations from the ground state. There are simply no quasi-particles available to annihilate. Taking all this into account we have for the total emissivity, depending on the pairing type:

$$L_{cp} = 3.51 \cdot 10^{21} a_N F(x) \left(\frac{m_N^*}{m_N}\right) \left(\frac{p_{F_N}}{m_N c}\right) T_9^7 \text{ ergs/cm}^3\text{s} \tag{3.58}$$

[9] or other particles, but the argument is similar

The function F of x=Δ/k_BT takes the temperature dependence of the superfluidity and the quasi-pair excitations into account. A plot is given below. The constant a_N is of order unity and depends on the pairing type and the nucleon under discussion. For example a_N=4.17 for triplet state neutron pairing. This Cooper pairing of nucleons results in superfluidity, which can have a large effect on all the other

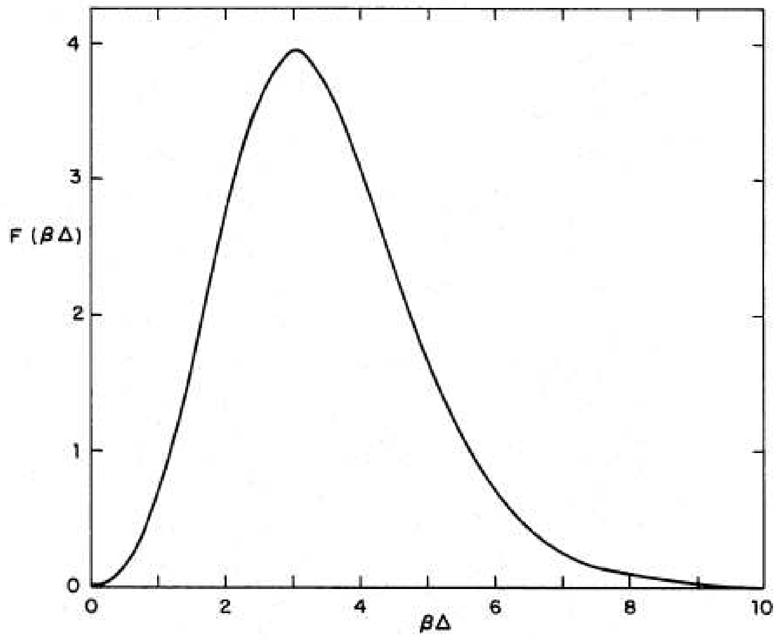

Figure 3.4: A plot of the function F, where Δ is the gap energy. The exponential reduction at low temperatures, i.e. high β=1/k_BT, is clearly visible. At low β, i.e. above the critical temperature T_c, the thermal excitation energy becomes of the same order as the gap energy and superfluidity is no longer present because no pairs can be formed. Taken from Flowers et al. 1976.

neutrino producing mechanisms. The reason for this is the formation of pairs, which can happen in the crust as well as in the core. All the neutrino processes work via single interacting particles except the PBF process. The fraction of particles that are free to interact when superfluidity is present is roughly given by $\exp(\frac{-\Delta}{k_B T})$. This would generally mean a tremendous suppression of the rate of all the possible processes, except the PBF process. One could of course postulate that no superfluidity is present in the core of a neutron star, but simple BCS theory predicts otherwise. It could be that the critical temperature is not what we expect at densities present in a neutron star. As I mentioned before, at present a wide range of critical temperatures is given, see again Levenfish et al. 2007. As is shown by several authors, see for example Hoffberg 1972, proton pairing in the $^1S_0$ (singlet) state may occur in the inner crust, as well as neutron pairing in the same state. In this case the state of the nucleon is labeled: $^{2S+1}L_j$. Here S is the spin multiplicity, L represents the angular momentum state: S for l=0, P for l=1 etc., and j is the total angular momentum quantum number. In the core, neutron pairing in the $^3P_2$ (triplet) state can occur, suppressing all neutrino processes that occur inside the core. Only above a density of about 3.5 ρ_0 a central non-superfluid kernel may exist if we assume neutron $^3P_2$ superfluidity.

3.2.5 Processes involving hyperons

At sufficiently high densities, Σ^- hyperons may form. It is possible to carry out the direct URCA process with them. Here are the equilibrium reactions:

$$\Sigma^- \to n + l + \bar{\nu}_l \tag{3.59}$$

$$n + l \to \Sigma^- + \nu_l \tag{3.60}$$

where the l stands for a lepton, since muons may occur as well in core matter at the densities considered. The contribution of this process to the total cooling can easiest be seen in the rescaling rule which is valid for any direct URCA process. The rescaling rule was derived by Prakash et al. in 1992 by considering the phase space decomposition of the emissivity. It is given by the following expression:

$$L_{Durca(12)} = L_{Durca(np)} \frac{m_1^* m_2^*}{m_p^* m_n^*} \Theta_{12l} \left(\frac{G_1^2 (f_{V1}^2 + 3g_{A1}^2)}{G_F^2 cos^2(\theta_C)(1 + 3g_A^2)} \right) \tag{3.61}$$

where Θ again represents the condition that the Fermi momenta of the interacting particles must satisfy. In our case that would be: $p_{F_\Sigma} < p_{F_n} + p_{F_l}$. The labels 1 and 2 are used to identify the reacting particles. In the above case particle 1 is the Σ^- and particle 2 is the neutron. Therefore G_1 is the weak coupling constant of particle 1, f_{V1} is the vector coupling constant of particle 1 and g_{A1} is the axial-vector coupling constant of particle 1. Filling in all the constants in the same way as before yields in our case:

$$L_H = 4.8 \cdot 10^{25} \left(\frac{n_l}{n_0} \right)^{\frac{1}{3}} \frac{m_{\Sigma^-}^* m_n^*}{m_n^2} \Theta_{\Sigma n l} T_9^6 \quad \text{ergs/s} \tag{3.62}$$

All this only works above a certain treshold density at which the hyperons form. This should be taken into account when one wants to calculate the total emissivity.

3.2.6 Pasta phase URCA

As is elaborated in section 3.1.3, the crust of a neutron star can have a layer, the mantle, in which the nuclei have strange shapes such as rods or plates. Since some of these shapes resemble Italian food they are sometimes called pasta phases.

In 2004 Gusakov et al. suggested the possibility of a fast neutrino process in this phase. Although the process can only occur in a thin layer in the crust it might be competitive with processes operating in the core.

The process requires protons to appear out of the nuclei. This only happens in the deepest layers of the mantle where the nuclei are formed by inverted cylinders or Swiss cheese. The periodic lattice structure of the nuclei introduces Bloch states resulting in a different relation between the momenta of the particles present. This opens the direct URCA process in the deepest layer of the neutron star mantle. A detailed calculation of the emissivity can be found in Gusakov et al. 2004. I will just quote the final result, which has exactly the same form as the direct URCA emissivity given in eq. 3.45 but is reduced by a factor of order 10^5 because of the lower density in the mantle. This seems a poor result since the layer in which the process occurs might be only 100 m thick. However, as I already remarked if the direct URCA process is forbidden in the core, which it very well may be, this process can make a difference. The total emissivity of mantle and core is approximated by:

$$L_{tot} = 4\pi R_{cc}^2 h L_{mantle} + \frac{4\pi}{3} R_{cc}^3 L_{core} \tag{3.63}$$

Adopting $R_{cc} = 10$ km and h = 100 m, the process in the mantle becomes important if $L_{mantle} > 30 L_{core}$, which is certainly the case if only the modified URCA process is allowed in the core.

3.2.7 Alternatives

Apart from these six processes other, less important, neutrino producing reactions have been proposed. I will name two.

Quark matter In the limit of extremely high densities (hadronic Fermi energies $\gg 1$ GeV), quarks are no longer confined to hadrons. If deconfined quarks can exist in the core of a neutron star they can take part in a direct URCA process involving up and down quarks instead of protons and neutrons respectively. However, since the densities in the core of a neutron star are thought to be less than those needed for deconfined quark matter, such processes seem highly unlikely.

Bremsstrahlung Nucleon-nucleon bremsstrahlung which produces neutrinos can occur in the neutron star crust, as well as in the core. The neutrinos resulting from bremsstrahlung cool a neutron star more efficient than thermal radiation at the initial stages. However, since the neutrino luminosity obtained by bremsstrahlung is even lower than that of the modified URCA process, because of the same temperature dependence and a lower intrinsic luminosity, I will not include bremsstrahlung in any of the following discussions.

The pictures on the next page nicely summarize a few of the possible processes, giving the surface temperature of a cooling neutron star as a function of time with the mass as an external parameter.

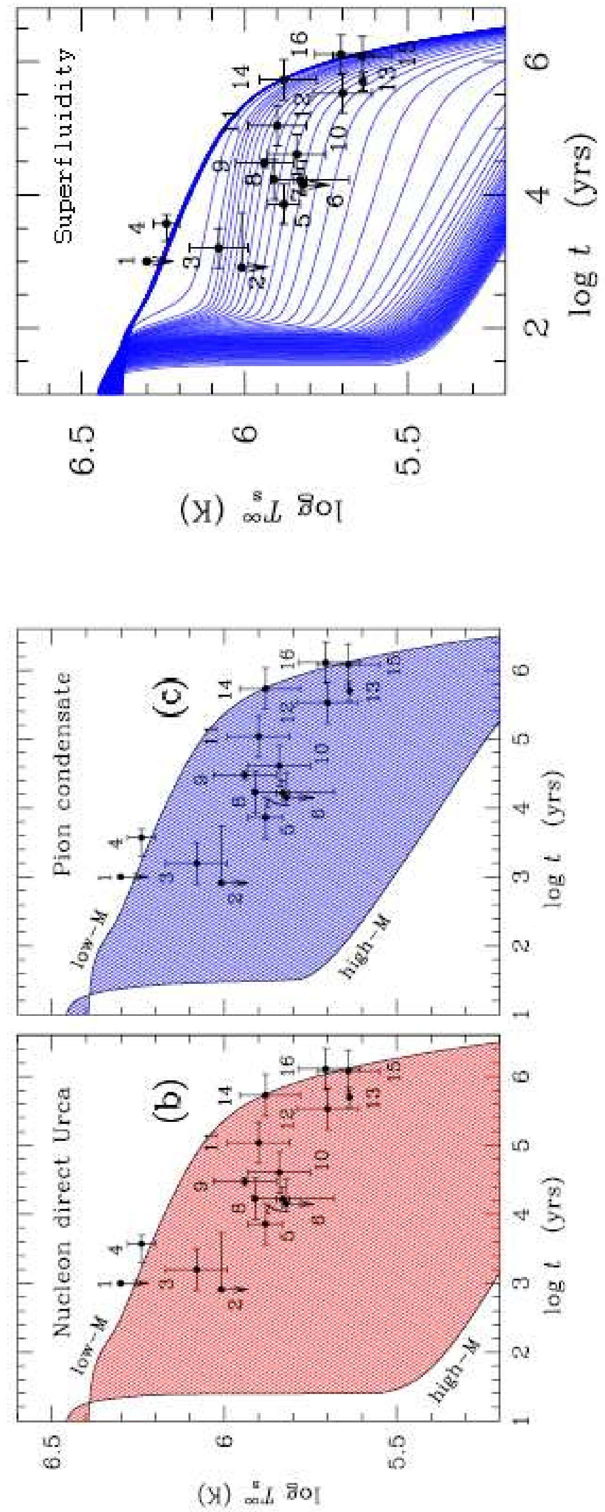

Figure 3.5: Surface temperatures as a function of time for cooling neutron stars with different cooling processes. Every line in the right figure corresponds to another mass, the uppermost line being the lightest neutron star. Observations of cooling neutron stars have been included in every figure. Taken from Yakovlev et al. 2007

Chapter 4

Application to Bob

Now that I have given a humble overview of all the exciting things happening in a neutron star I can try explaining why Bob could not be detected with a 300 ks Chandra survey. As stated in the introduction the inability to detect Bob implies a surface temperature measured at infinity of $3.5 \cdot 10^5$ K. In this chapter I will first compare a few scenarios using Frank Hemmes' results about how much energy could have been deposited into the star, where it might have ended up and how it might have affected different regions of the star. Then I will employ a crustal relaxation model to explain the behaviour of the crust. After that I will use a relation between the surface temperature and the temperature below the crust in equilibrium to finally derive how much energy can be radiated away using neutrinos. Having done that I will be able to tell which processes are at work inside Bob and thus explain its behaviour.

4.1 Heating of an accreting neutron star

In determining how much heat may be deposited in Bobs interior and on its surface, Frank employs the hypothesis of deep crustal heating developed by Brown et al. in 1998 together with two branches of periodic accretion models put forward by Lasota and Sunyaev et al. respectively. The essence of the deep crustal heating model is that every accreted nucleon deposits ~ 1.45 MeV of energy onto the neutron star. That would mean a total increase in thermal energy after accretion of

$$E_{th} \approx 1.1 \cdot 10^{43} ergs \tag{4.1}$$

Next, one postulates that an equilibrium is reached[1] between the energy deposited on the neutron star and the energy radiated away during quiescence. In other words, one assumes a very (c)old neutron star which starts to accrete matter at some point, this heats up the star in several accretion-quiescence episodes until equilibrium is reached. Assuming this accretion equilibrium, Brown et al. derived formulas 2.1 and 2.2 relating the quiescent luminosity and duration to the outburst luminosity and duration.

4.2 Crustal relaxation and its effect on Bob

In normal SXT's with outburst times of the order of days-months the crust and core stay thermally coupled. If the accretion period is much longer, for instance 11 years, the crust can become hotter than the core and stay thermally decoupled from it since the structures of crust and core are different. This possibility was put forward by Rutledge et al. in 2002 and it was tested by Shternin et al. in 2007 by modelling the specific neutron star KS 1731-260. The results of Shternin et al. resemble the measured

[1] Bob could of course have accreted matter one single time, but us witnessing that event seems unlikely

surface temperatures (Cackett et al. 2006) of KS 1731-260 remarkably well. The model gives a crust-cooling timescale of about two years. This is short enough to think that Bob had an outburst time of 11 years, Bob might have been accreting for 10 years after which relaxation set in. Comparing this model with Bobs surface temperature is hard since we have not seen Bob after it went into quiescence. When I employ Shternins model nonetheless I have to argue that Bobs core temperature was about an order of magnitude lower before accretion than that of KS 1731-260. I also have to account for the heat that went into the core during accretion as argued in the previous section. This can both be accomodated by using a generic model for the atmosphere, allowing certain neutrino processes and assuming a minimum age.

4.3 Core temperature and minimum age

Potekhin derived temperature profiles for neutron stars with accreted envelopes in 1996. A surface temperature[2] of $4.6{\cdot}10^5$ K implies a temperature beneath the crust of 10^7 K, assuming an envelope of accreted helium. Because of the degeneracy of the matter in the core one may assume the core to be isothermal. I have to remark that the temperature and therefore the luminosity should not be much lower than this since that would imply a recurrence time of more than 60 kyr. Such a long recurrence time is very hard to sustain accretion-wise, since that would mean a mass transfer rate from the secondary star of less than 10^{-13} $M_\odot$/yr, which is very unlikely. Measurement of a significantly lower temperature could imply that Bob has not yet reached accretion equilibrium so that it is still in the accretion driven heating phase. A core temperature of 10^7 K implies a minimum age before accretion of about 1 Myr, assuming a canonical mass, as can easily be seen from the figures at the end of the previous section.

4.4 Application of neutrino processes to Bob

In this section I will go into detail about the application of each of the six dominant neutrino processes. I will review the conditions for occurrence and discuss the relevance of the processes for the behaviour of Bob. The scope of this thesis is to be able to describe Bob satisfactory without resorting to too bold assumptions since, knowing so little about Bob, things should be kept as general as possible. I first give a table with all the integrated luminosities calculated for a canonical core radius of 10 km (for most processes) and a uniform core temperature of $1.0{\cdot}\ 10^7$ K. The emissivities, as given in section 3.2, are not very dependent on the density. Furthermore, the density of the core itself varies also just a little bit, see figure 4.1. Therefore, I took a mean density of ρ_0 for the core to calculate the total emissivities by multiplying the specific emissivities with the volume of the core that is relevant for each process. I will comment on these results later.

Process	Integrated luminosity
Thermal photon emission	$< 1.8{\cdot}\ 10^{30}$ ergs/s
Direct URCA	$1 \cdot 10^{32}$ ergs/s
Modified URCA	$3{\cdot}\ 10^{23}$ ergs/s
Pion condensate	$2.7 \cdot 10^{31}$ ergs/s
PBF process	$2.6 \cdot 10^{22}$ ergs/s
Hyperonic URCA process	$5 \cdot 10^{30}$ ergs/s
Pasta phase processes	$3{\cdot}\ 10^{25}$ ergs/s

As we can clearly see, modified URCA, PBF processes and pasta phase cooling are very unimportant in Bobs cooling behaviour as their luminosities are vastly exceeded by the thermal photon luminosity. The reason that the other luminosities are of the same order of magnitude as the photon luminosity, even

[2]measured at the star surface, considering gravitational redshift

though I stated in the introduction that neutrino processes are more important, is the low temperature of the core: most cooling neutron stars are hotter.

4.4.1 Direct urca

The most important thing incorporated in the direct URCA emissivity is a high enough proton fraction needed for this process. As pointed out in the previous chapter one could demand that Bob has a mass high enough to accomodate direct URCA or that the equation of state of Bob is softer than that of other neutron stars, such as RX J0822-43 or PSR 1055-52, which are known to be hotter but for which there is no reason as yet to assume a lower mass than Bob. I do not want to employ any of these two statements. Not the first statement because, although the statistics on neutron stars is rather poor, chances for a high mass neutron star are very limited following detailed mass studies of binary neutron stars. Therefore it would be preferable to explain Bob with a canonical mass of $\sim$1.4 $M_{\odot}$. Not the second statement because there is no known physical mechanism by which only one or a few neutron stars can have a soft EOS while others are known to have a more stiff one.

4.4.2 Modified urca

Due to the very heavy (T_9^8) dependence on the temperature, the modified URCA process does not contribute a significant part to the loss of heat obtained by accretion. It can still be a dominant process in the long term cooling if all the other processes are forbidden.

4.4.3 Pion condensate neutrino cooling

As soon as the electron Fermi energy exceeds the π^- rest energy a pion condensate field may form. This happens at a density of about $6.6{\cdot}10^{14}$gr/cm^3, if the EOS is not extremely stiff. In the cases of an FPS-model[3] (Friedman et al. 1989) or a SLy- model[4] (Douchin et al. 2001), which are the only models applicable to the entire neutron star, pions may occur. The exact point at which the condensate begins to form is stiffness dependent. The luminosity obtained might be an order of magnitude lower in the stiff case than the value given in the table on the previous page, but that is a pessimistic estimate. In the above estimate a radius of 5 km was used for the kernel in which pion condensate occurs, which is reasonable according to figure 4.1 on the next page.

4.4.4 Pair breaking and formation processes and superfluidity

Due to the exponential suppression of this process at low temperatures there will be no significant luminosity because of pair formation. The occurrence of superfluidity also affects the other processes. Especially neutron superfluidity in the triplet state suppresses the pion condensate process and hyperonic processes at a high level. This is a reason to assume a not too low mass ($M \gtrsim 1.1\ M_{\odot}$) for Bob. The mass-central density diagram in figure 4.2 nicely summarizes this. With a high mass, the density in the inner core may be high enough for $^3P_2$ neutron superfluidity to vanish, so that in a central kernel the aforementioned cooling processes can occur, although their integrated luminosity will diminish by maybe an order of magnitude.
Assuming a somewhat high mass is reasonable because to be able to accrete matter the mass should not be too low. The statement that an accreting neutron star has gained enough matter to cool significantly faster may sound attractive intuitively, but the timescales associated with that are way too long to

[3] somewhat soft
[4] somewhat stiff

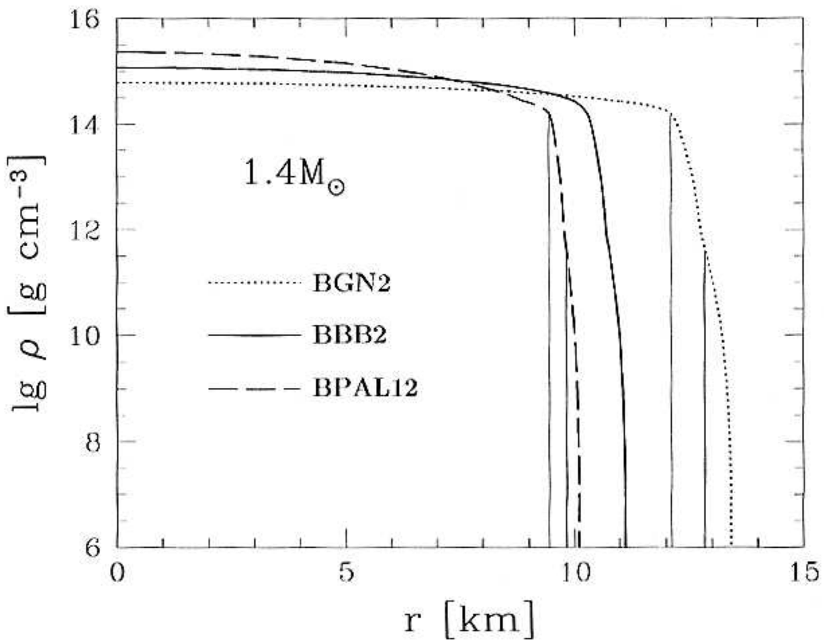

Figure 4.1: The density as a function of the radial coordinate of a neutron star with a canonical mass of 1.4 $M_\odot$, for different EOSs. The BGN2 EOS is the stiffest, prohibiting a pion condensate at this mass.

accomodate such a scenario in our case: $t_{acc} > 10^{11}$ yr, which is seven times the age of the universe. The statement is only sound for very fast accreting systems ($< \dot{M} > \sim 10^8$-10^9 $M_\odot$/yr).

4.4.5 Hyperonic processes

As can be seen in the above table, processes involving hyperons contribute significantly to the cooling. The condition for the formation of hyperons is almost the same as the one for the formation of a pion condensate, as pointed out in the previous chapter. The density at which a significant amount of hyperons is present is also about $6.6 \cdot 10^{14}$gr/cm^3. As explained, a radius of 5 km was used to estimate the total emissivity.

4.4.6 Pasta phase neutrino cooling

The luminosity of the pasta phase processes is higher than that of the modified URCA process, but still insignificant to the total amount of cooling. In a low mass star with no proton superfluidity in the inner core it may become important.

4.5 Summary

In order to shed a little more light on the results given above I will summarize the most important parts in a short way. We observed Bobs outburst and have to draw conclusions about the amount of heat deposited in the star. From the Chandra observations carried out by Jonker et al. we know that the thermal luminosity is lower than $1.8 \cdot 10^{30}$ ergs/s. I will compare this with the amount of heat radiated away in neutrino processes.

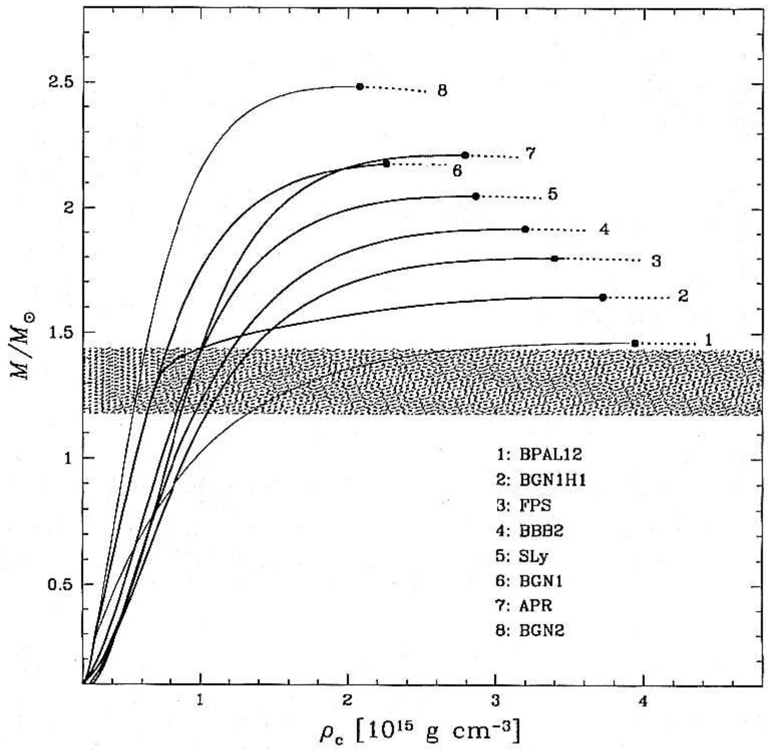

Figure 4.2: The central density as a function of the total mass of a neutron star for different EOSs. The lower the density at a specified mass, the stiffer the EOS. The shaded region corresponds to precisely measured masses of binaries. We also expect Bob to have such a mass.

Assuming accretion equilibrium, the amount of heat deposited in the neutron star during accretion and the amount of heat radiated away during quiescence should be equal, following Brown et al. 1998. One can of course postulate that equilibrium is not yet reached due to long quiescent episodes.

As soon as the accretion stops the crust starts to relax and becomes in thermal equilibrium with the core whose temperature has not changed much. The relaxation timescale is about two years, using the relaxation model of Shternin et al. 2007.

After relaxation, Bob is very faint and cannot be observed, putting a limit on the maximum thermal photon luminosity of $1.8 \cdot 10^{30}$ ergs/s. This infers an effective surface temperature of $4.6 \cdot 10^5$ K[5], which on its part implies a core temperature of 10^7 K following Potekhin et al. 1996. The total neutrino luminosity of all relevant processes summed together is summarized for different scenarios in the table below.

[5] again measured at the surface, considering gravitational redshift

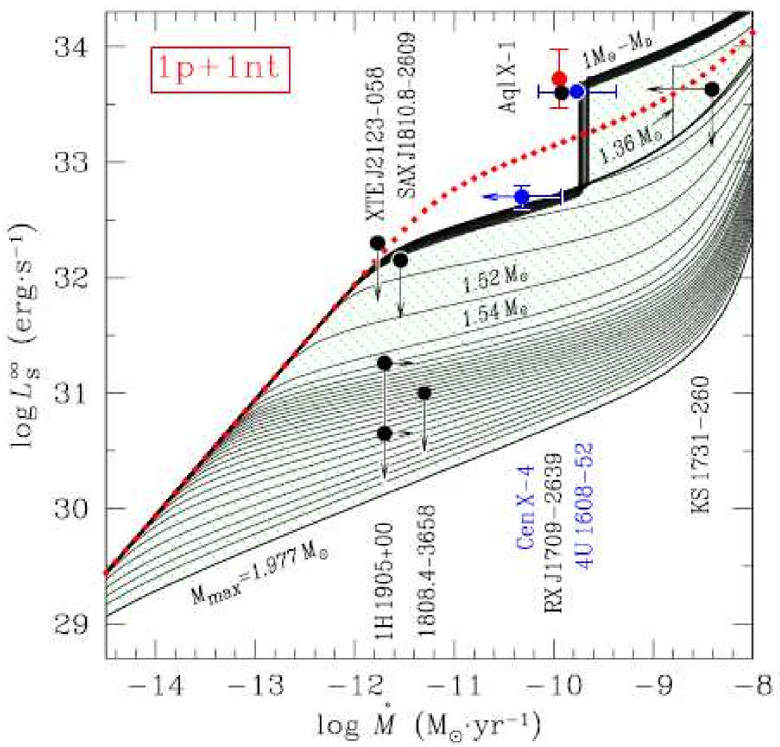

Figure 4.3: Quiescent luminosities as a function of the mean mass transfer rate for several SXTs, Bob should actually be placed more to the left, making it a lighter neutron star. Taken from Levenfish et al. 2007

Scenario	Total neutrino luminosity
No neutron superfluidity, rather soft EOS	$3.2\cdot 10^{31}$ ergs/s
No neutron superfluidity, rather stiff EOS	$4.9\cdot 10^{30}$ ergs/s
Neutron superfluidity, rather soft EOS	$2.5\cdot 10^{30}$ ergs/s
Neutron superfluidity, rather stiff EOS	$2.5\cdot 10^{30}$ ergs/s
Neutron superfluidity, no hyperons	$2\cdot 10^{30}$ ergs/s
Neutron superfluidity, no pions	$5\cdot 10^{29}$ ergs/s
Neutron superfluidity, no fast processes	photon cooling dominates
No neutron superfluidity, no fast processes	photon cooling dominates

As can be easily seen, in most cases the neutrino luminosity is comparable to the thermal photon luminosity. This is mostly attributable to the low temperature in the core. This significantly shortens the recurrence time for outbursts, which is easier to explain with the current accretion models. If the star is light, the central density drops below the treshold density for the occurrence of pions or hyperons, which deprives the star of moderately fast cooling mechanisms. The occurrence of neutron superfluidity everywhere in the core has the same effect. For a lighter star one thus has to employ longer recurrence times, which are hard to explain in the framework of accretion because the mass transfer rate is very low.

Chapter 5

Conclusions

Let us briefly recall what we set out to do. In the introduction, we said that "an explanation will be given why Bob is as faint as he is.". Now it is time to see whether we have succeeded in doing so.
First, we tried to make a reasonable estimate on the recurrence time of Bobs accretion by modelling the accretion processes and checking their stability. The result of this is an estimate of $t_r \approx 1 - 2 \cdot 10^4$ yr. Although these values were obtained using a crude model, we think they are nonetheless reasonable as no out of the ordinary physics had to be invoked to arrive at them. Also, one has to keep in mind that the processes involved are not yet fully understood. Combined with the lack of observational data on Bob, this makes more accurate estimates difficult.
All this assumes an accretion equilibrium is reached: the energy gained during accretion is radiated away during quiescence. This would mean Bob is an old cooling neutron star of, judging from its temperature, at least one million years old. At a certain point, after those at least a million years of cooling, Bob started to accrete matter, most likely reheating himself. After several phases of accretion an equilibrium was reached between the heat deposited during and the heat radiated away after accretion. Now, we return to equation 2.4. Acquiring the incandescent luminosity is simply dividing the amount of energy stored by the time available to reemit it. Or, as an equation[1]:

$$L_i = \frac{fE_{tot}}{t_r} \,\mathrm{ergs}^{-1} \tag{5.1}$$

This is now basically a relation between the storage fraction parameter f, mentioned in the summary of Frank Hemmes' work, and the quiescent luminosity. From here, we can discern two extreme situations. We can either make things difficult for ourselves, or fairly easy.
The difficult approach would be using the lowest value for t_r and setting $f = 0.9$, in other words: almost all heat generated in the star stays there. If this is done, it yields an quiescent luminosity of $L_i = 3.16 \cdot 10^{31}$ ergs/s, which is more than ten times the luminosity observed in the x-ray spectrum. We can account for this excess of energy by incorporating mildly enhanced neutrino emission in Bobs inner core, i.e. hyperonic processes and processes involving a pion condensate. A way to arrive at this result is to forbid neutron superfluidity and use a NN-interaction model that implies a medium to soft EOS. To be able to employ neutrino processes we have to assume there is at least some part of the star that is not superfluid. This would require a central density of at least 3.5 ρ_0, which would imply a mass greater than 1.1 $M_\odot$.
The most optimistic result we get by setting $f = 0.1$, which is the minimal value assumed by Jonker et al. (2006) and using a long recurrence time. If we do so, the luminosity becomes $L_i = 1.77 \cdot 10^{30}$ erg s^{-1}. This is still slightly below the observational limit on the luminosity, and so it would not require

[1] This equation yields a slightly different luminosity than equation 2.1. This is because in equation 2.1 a different amount of energy gained per nucleon is assumed in their approximation

additional cooling mechanisms. There is one thing we should remark in this case: a long recurrence time, which infers a low mass transfer rate from the secondary star to the accretion disc ($<10^{-13}M_{\odot}$/yr) is very hard to explain with the current binary evolution models.
However, should future measurements constrain the luminosity to below this value, neutrino processes may still be at work in the core. But there is a remark to make: Due to the low temperature in the core accompanying such a low thermal luminosity, neutrino processes are operating at a much slower pace because of their large temperature dependence. Within the 'deep crustal heating' model, the only possibility left is stating that accretion equilibrium is not yet reached and that Bobs temperature is rising because of periodic accretion. An extreme case of stating that equilibrium is not yet reached is supposing that Bob only accreted matter one single time, but us witnessing that event seems unlikely.
In spite of the severe constraints imposed, it is still possible to explain the behaviour of Bob within the 'deep crustal heating' model. However, if the critical value for the viscosity parameter α would for some physical argument be higher, or if it is highly implausible to assume a non-superfluid kernel in the star, it might be necessary to invoke an ADAF, jet-formation or the propellor effect. These last two effects cause that not all matter in the accretion disc reaches the star. We did not invoke these mechanisms as it was our aim to explain Bob while using the least amount of fancy physics possible. Also, a jet coming from the neutron star would likely be observed, but it is not in Bobs case. Summarizing, with current knowledge, we think that Bob still remains a strong argument in favour of the 'deep crustal heating' theory.

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