Dynamical Stability in the Acquisition and Performance of Rhythmic Ball Manipulation: Theoretical Insights With a Clinical Slant

Dagmar Sternad and Tjeerd M. H. Dijkstra†*

Abstract: Three experiments illustrate how a task-based approach and dynamical modeling of a perceptual-motor task can provide a useful framework for understanding functional and dysfunctional behavior. In the chosen task, subjects held a racket and bounced a ball rhythmically in the air with invariant ball amplitude. As such, the task could be cast into a mechanical model that encompassed the movements of the actor (racket) and the manipulandum (ball). In this form, the movement task is a dynamical system that displays dynamical stability, i.e., performance where perturbations die out by themselves. The hypothesis is that skilled actors seek to perform with this "passive" stability as it alleviates the control demands because perturbations do not require explicit corrections. In the experimental data, this strategy could be characterized by a single parameter, the acceleration of the racket at impact, which provided quantitative predictions. Experiment 1 established that subjects with normal sensorimotor functions indeed performed with racket acceleration values that were predicted to provide passive stability. Experiment 2 showed that subjects improved their skill over a course of 40 practice trials, as evidenced in decreased variability and accompanied by a change in racket acceleration toward values that provided optimal stability. In experiment 3, perturbations were applied and the subjects' adaptability was tested. When perturbations were large enough, subjects altered their racket timing to resume contact that provided stability. The results are discussed with their relevance to clinical contexts: How can such a task-based approach provide insights into the control of functional and dysfunctional movements? Can such behavioral results serve as a diagnostic tool? How can this approach to sensorimotor behavior stimulate physicians and therapists to progress therapeutic measures?

This work was made possible by the awards SBR #9710312 and Behavioral Neuroscience #0096543 of the National Science Foundation awarded to Dagmar Sternad and grant SBR #9809447 awarded to Tjeerd Dijkstra.

*Department of Kinesiology, Pennsylvania State University, University Park, Pennsylvania, U.S.A.; and †Department of Medical Physics and Biophysics, University of Nijmegen, Nijmegen, The Netherlands

Address correspondence and reprint requests to Dr. Dagmar Sternad, Department of Kinesiology, 266 Rec Hall, Pennsylvania State University, University Park, PA 16803, U.S.A.; e-mail: dxs48@psu.edu

Copyright © 2004 by Lippincott Williams & Wilkins

ISSN: 0736-0258/04/2103-0215

Key Words: Ball-racket bouncing, Attractors, Dynamic stability, Variability.

(*J Clin Neurophysiol* 2004;21: 215–227)

The acquisition of perceptual-motor skills and their control in the face of ever-changing task demands is fundamental to everyday life. We are continually engaged in perceptual-motor activities ranging from archetypal movement patterns, such as reaching and walking, to more advanced skills, such as tying shoelaces or hammering a nail into the wall. How important perceptual-motor skills are in our life is evidenced by the fact that we even intentionally create such challenges for ourselves in leisure time activities, exactly *because* they pose new problems for our perceptual-motor system. Yet, probably the most urgent reminder that a meaningful daily existence relies on coordinative skills is when such skills are impaired or lost, as is the case in many neurologic disorders. Although a detailed documentation of the specific impairments of a disease is extremely important for diagnosis, a deeper understanding of these impairments and their cure can only be obtained when control and coordination in healthy humans is understood. However, knowledge about motor control, i.e., how our central nervous system controls and coordinates movements, is still at a surprisingly rudimentary level. Therefore, a first needed step is to differentiate our understanding of coordination in healthy persons and to develop analysis methods to capture both functional and dysfunctional behavior (e.g., Beuter et al., this issue).

The field of motor control comprises a number of different levels of analyses, ranging from microscopic physiological descriptions of synaptic transmission and single-nerve functions to macroscopic descriptions of complex "normal" behavior where measures are as coarse-grained as performance errors. Although the detailed insights of neural structure and function often fail to translate into understanding of more real-life functions of the system, the more macroscopic approaches often lack in theoretically grounded results that generalize across tasks. For example, many results

Copyright © by the American Clinical Neurophysiology Society. Unauthorized reproduction of this article is prohibited.

have been accumulated about the neurotransmitter dysfunctions in Parkinson disease, yet leaving unanswered the question of how basal ganglia dysfunction creates a problem for movement initiation. On the other side, describing how different people use their fingers and hands to tie shoelaces without a theoretical framework does not provide insight *per se* into the complex mechanisms of what control signals to the multiple finger muscles are issued and coordinated (Fadiga and Craighero, this issue). An intermediate level of analysis is needed where perceptual-motor performance is examined in a way that addresses theoretically stringent questions and that yield generalizable answers about how our neuromechanical system works.

In the relatively short history of motor control, different theoretical perspectives have provided the background for formulating the questions and paradigms, ranging from information theory that viewed the central nervous system as an information-processing system (Scarborough and Sternberg, 1998), to control theory that interpreted the central nervous system as the controller of the biomechanical periphery or "plant" (Jagacinski and Flach, 2003). Although many insights have been gained about limb movements, what has been missing in many of these paradigms is to take the relation of the movement with the task and the environment into consideration. Most actions occur within a task context that is defined over the executing limbs and the objects in the environment. For example, the action of hammering a nail into a surface can only be understood if the properties of the hammer and the nail, the orientation to gravity, and the tolerance for accuracy is taken into account. Data of joint angles or muscle activation patterns alone without knowing the task constraints are insufficient to reveal the underlying principles of control. Environmental conditions pose important constraints and are an integral part of the task performance, and should therefore be included in the system of analysis.

In the approach we present here, we define the system of analysis as the task that is composed of the actor and the environment. This system is viewed as a complex system, such that methods from nonlinear dynamics provide an appropriate tool to model and analyze behavior. This dynamical systems approach to perceptual-motor control has gained an increasing appeal over the past two decades (Kelso, 1995; Kugler and Turvey, 1987; Sternad, 2000a). Although most studies have been directed to reveal principles of interactions among limbs that characterize stable action patterns and transitions as typical for locomotor actions, some lines of work have addressed the interactions between an individual agent and the environment. These few examples comprise navigation in a cluttered environment (Fajen and Warren, 2003; Warren, 1998), postural balance in a moving visual environment (Cabrera and Milton, 2002; Dijkstra et al., 1994a, 1994b; Schöner, 1991), and the perceptual-motor tasks of balancing a pole (Cabrera and Milton, 2002, 2004).

To investigate the coordination of a perceptual-motor coordination, research in our laboratory investigated a simple task as a model system: bouncing a ball rhythmically on a racket (de Rugy et al., 2003; Katsumata et al., 2003; Schaal et al., 1996; Sternad, 2000b; Sternad et al., 2000, 2001). The task is to hit the ball with a racket, such that the ball bounces to an approximately constant height with an invariant period between ball–racket contacts. This deceptively simple task poses all the archetypal questions of motor control: How does the hand's movement synchronize with the ball's trajectory to intercept the ball at the right time with the right velocity to achieve an invariant ball amplitude? How are the racket's or hand's movements controlled such that the right muscle forces are produced to achieve the desired contact that, in turn, will lead to the next ball trajectory? How do actors improve their performance and become more steady with practice? The task has the advantage that it is sufficiently unfamiliar that there is room for improvement to study motor learning. Yet, it is also relatively easy and can be learnt within a practice session so that skilled performance can be studied in healthy subjects. Another not negligible issue is that it is interesting enough to serve as a task to study developmental questions in children and clinical populations.

The major theoretical motivation for choosing this task is that the skill resembles a ball-bouncing model that has been analyzed in the literature on nonlinear dynamics (Guckenheimer and Holmes, 1983; Tufillaro et al., 1992). The dynamical system of a periodically bouncing ball, or particle on a planar surface has been studied in the applied mathematics literature as it demonstrates the hallmark features of nonlinear dynamical systems: The system displays multiple stable periodic solutions and period-doubling bifurcations that lead to chaos (for review see Milton et al., this issue). The bridge that Sternad and colleagues made was to interpret the periodically moving surface as a racket that is manipulated by an actor in a rhythmic fashion and the bouncing particle as a ball. With this analogy, one can ask whether humans establish stable attractor regimes when performing rhythmic ball-bouncing. To perform with dynamical stability has the advantage that perturbations relax back to its stable state that relieves the actor of excessive corrective actions. Such a solution would be less computationally expensive for the central nervous system because it obviates continuous perceptual tracking of the ball and appropriate compensatory actions by the motor system. The question is whether humans are sensitive to such stable regimes.

THEORETICAL PREDICTIONS

The ball-bouncing model, as developed in mathematics, consists of a periodically moving planar surface and a ball moving in the vertical direction obeying ballistic flight

© 2004 Lippincott Williams & Wilkins

Copyright © by the American Clinical Neurophysiology Society. Unauthorized reproduction of this article is prohibited.

and inelastic impact (Guckenheimer and Holmes, 1983; Tufillaro et al., 1992). By using this model, we explicitly focus our analysis on the task dynamics, ignoring physiological details of the movement execution. This reflects our belief that the primary constraints are task-related and physiological constraints, though important, are secondary in their influence. The movement task is illustrated in Fig. 1A, and Fig. 1B shows an exemplary time series of racket and ball displacements. Both racket and ball motions are confined to the vertical dimension to keep the model tractable. Between impacts, the ball trajectories follow ballistic flight with the gravitational constant $g = 9.81$ m/s^2. At impact, the ball bounces up in the air with a coefficient of restitution α, expressing the energy lost at each impact. This coefficient captures that a ball impacting a stationary racket with velocity $v = -1.0$ m/s bounces up with velocity $v' = -\alpha\ v = -\alpha^*(-1.0 \text{ m/s}) = \alpha$ m/s (negative velocities denote downward movement and positive velocities upward movement). The primary issue is how the actor controls the racket to maintain a stable rhythmic pattern with invariant ball amplitudes between bounces. We hypothesize that actors contact the ball such that they obtain "passive" stability. This term defines a type of performance that is resistant to perturbations, without adjusting the racket on a bounce-to-bounce basis. In case of disturbances of the ball trajectory, the actor does not need to actively adjust the racket movements because the deviations die out by themselves.

For modeling passively stable performance, we assume that the racket trajectory is periodic with a fixed period and amplitude. More specifically, the period between two bounces is constant for the duration of a trial. It can then be shown that the condition for stable ball movements is that the racket movement should impact the ball at times of negative racket acceleration, i.e., the racket trajectory is decelerating in the upward direction before and when contacting the ball. A detailed derivation and analysis can be found in (Dijkstra et al., 2004). Thus, for the system to be stable in a stationary state, the critical parameter is the racket acceleration at impact. We denote this parameter by *AC*.

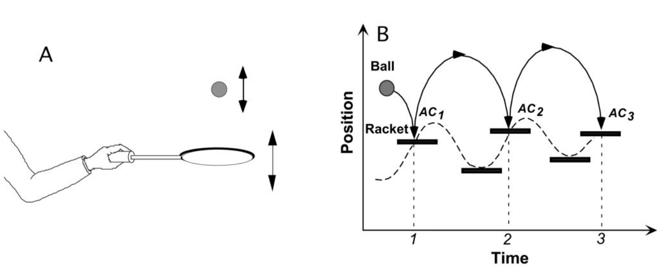

FIGURE 1. (**A**) Sketch of the task. (**B**) An exemplary trajectory of the ball (*solid line*) and the racket indicated by the black bars (*dashed line*). The dashed vertical lines highlight that most analyses focus on the discrete impacts, in particular the parameter racket acceleration at impact *AC*.

In such a stable solution, deviations correct themselves. To appreciate this solution, the left panels in Fig. 2 illustrate a simulation where two ball trajectories are perturbed during the third cycle, one to a higher- and one to a lower-than-previous amplitude. It can be seen that after a few cycles, both ball trajectories converge back to the preperturbation amplitude. Importantly, the racket does not change its periodic trajectory. The bottom left panel of Fig. 2 shows that the ball–racket contacts occur at a moment where the racket is decelerating in the upward direction, i.e., *AC* is negative, -9.5 m/s^2, in the stationary state as shown by the solid line. The two dashed lines illustrate the *AC* values corresponding to the two perturbed trajectories. It can be seen that the original *AC* value has been regained within 3 or 4 cycles.

These predictions were obtained from local linear stability analysis of the model that revealed that one asymptotically stable state exists if *AC* is between zero and a negative value determined by g and α:

$$-2g\frac{(1+\alpha^2)}{(1+\alpha)^2} < AC < 0 \qquad (1)$$

When inserting the values for normal gravity ($g = 9.81$ m/s^2) and $\alpha = 0.50$, stability is obtained if *AC* is between -10.90 m/s^2 and 0 m/s^2. Thus, once the actor has chosen the amplitude and frequency of racket movement such that *AC* is in this range, the resulting performance is "passively" stable. It should be pointed out that in Fig. 2B there is one *AC* value occurring at time $t = 2.8$ seconds, which is smaller than -10.90 m/s^2 that falls outside of the stable regime for *AC*.

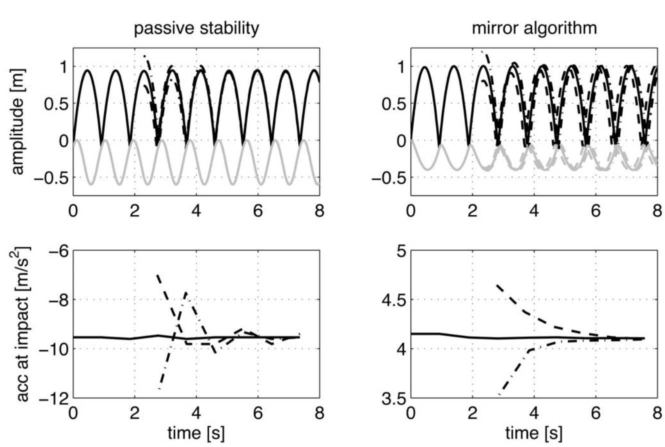

FIGURE 2. Simulation of the trajectories of ball and racket for the regime with passive stability (**left panels**) and with the mirror algorithm (**right panels**). In both simulations, perturbations were applied in cycle 3. The two top panels show the simulated trajectories. The *dashed lines* represent the two perturbed trials, whereas the *solid line* corresponds to the unperturbed trial. The two bottom panels show the corresponding accelerations of the racket at impact *AC*.

© 2004 Lippincott Williams & Wilkins

Copyright © by the American Clinical Neurophysiology Society. Unauthorized reproduction of this article is prohibited.

However, we defined *AC* as the *stationary* value of the acceleration at impact, not the instantaneous value at each impact (more details are found in the Appendix).

To emphasize that this passively stable regime is not trivial and other solutions are possible, a different strategy is illustrated for comparison in Fig. 2. This so-called mirror algorithm was developed and implemented on a robot system that juggled a ball in two and three dimensions (Bühler and Koditschek, 1990; Bühler et al., 1994). The two right panels in Fig. 2 show a simulation of their strategy. The mirror algorithm generates a racket trajectory that similarly maintains an invariant periodic pattern. However, when applying the same perturbations to the ball trajectory in cycle 3, it becomes evident that the racket continuously adapts its trajectory with respect to the ball. The racket motions indeed mirror the motions of the ball due to a continuous visual coupling between ball and racket. This implies that when the ball trajectory is perturbed, the racket trajectory is changed as well. The trajectory of the racket is a scaled copy of the ball trajectory where the value of the scaling factor depends on the coefficient of restitution α (for details see Appendix). Interestingly, we observe in the bottom right panel that *AC* is positive, a necessary consequence of the mirror algorithm, because the ball acceleration is negative before the impact (and equal to $g = -9.81$ m/s^2). However, it should be pointed out that the fact that *AC* is positive in no way means that this strategy is unstable. It is simply a different strategy that involves continuous perceptual control and leads to the same goal of bouncing a ball to a target height.

To summarize, both strategies generate stable rhythmic bouncing movements, albeit with different means. This is evidenced in different values of the impact parameter *AC*. The mirror algorithm uses continuous visual feedback from the ball trajectory into the generation of racket movements. The passive stability strategy involves no such feedback, but relies on the actor choosing a period and amplitude such that *AC* is negative and within the range specified by Eq. 1. Both strategies are feasible. However, we hypothesize that trained subjects will favor the passively stable regime and exploit the stability properties of the task. This solution necessitates less control and is computationally less expensive, leaving attentional resources free for other demands. To test this hypothesis, we will use *AC* as the primary parameter. As a secondary hypothesis, we expect that stable performance is associated with less variability. To test this hypothesis we use a measure of variability in the task variable ball amplitude. Thirdly, we hypothesize that with practice subjects change their strategy from initially relying on visuomotor coupling, i.e., a performance similar to the mirror algorithm, to one where self-stabilizing properties are exploited. Specifically, we expect that *AC* changes from more positive to negative values during practice. To test these hypotheses, we conducted a series of experiments. We will review three experiments where subjects perform at a stationary state (experiment 1), where they practice a sequence of trials (experiment 2), and where they are confronted with perturbations (experiment 3).

METHODS

Subjects performed the experimental task with a custom-made apparatus where they held a tennis racket in their hand and bounced a ball. The ball was attached to a 1-m-long boom rotating on a hinge joint. Because of this fixture, the ball trajectory was confined to a one-dimensional curvilinear path and the ball could not be lost in the performance. Within the observed ball amplitudes the ball trajectory could be assumed to be linear. Further, in this restricted version, the task closely resembled the model's assumptions of one-dimensional racket and ball motions. An accelerometer attached to the racket close to the "sweet spot" measured the racket's acceleration. The coefficient of restitution α was experimentally determined and has the value $\alpha = 0.52$. Because of the attachment, the ball's flight was no longer in normal gravitational conditions, and experimental determination of g yielded a value of 5.6 m/s^2. The range of stable solutions for these parameter values was between -6.16 and 0 m/s^2. Subjects were instructed to bounce the ball rhythmically with a steady ball amplitude throughout the duration of a trial. This typically comprised 50 to 70 ball–racket impacts.

The raw data were time series of ball and racket displacement as shown in Fig. 1B. The primary dependent variable was the acceleration of the racket at impact with the ball *AC*. The average values across the impacts during one trial were used to test the model's predictions about stability of performance. In addition, the amplitudes of the ball trajectories were determined as the distance between the ball–racket impact and its peak. Variability of task performance was captured by the standard deviations of the amplitudes across one trial in *SDA*.

RESULTS

Experiment 1: Dynamical Stability in Steady Ball Bouncing

In a first series of experiments Sternad and colleagues tested the prediction that human actors indeed exploit the stability properties of the system. Six subjects were instructed to bounce the ball with constant ball amplitude throughout each trial. In a given trial, subjects were instructed to bounce the ball at either a low, intermediate, or high amplitude. No explicit targets were given because this may have introduced error correction processes. The intermediate amplitude was defined to be the height at which each subject preferred to bounce the ball. High amplitude should be in a high, but still comfortable and approximately linear range. The experimental session consisted of two blocks. In the first block subjects performed the task with their eyes open, whereas in the

© 2004 Lippincott Williams & Wilkins

Copyright © by the American Clinical Neurophysiology Society. Unauthorized reproduction of this article is prohibited.

second block they performed with their eyes closed. In the latter condition subjects began the bouncing actions with their eyes open and only closed their eyes after having established a stable movement pattern. At this moment, the data collection was started. Three trials at each amplitude condition were presented in random order. All trials were performed with the racket held in the dominant hand. Each of the 18 trials lasted 30 seconds.

The major results are shown in Fig. 3, which displays the trial means of the six subjects in all experimental trials and conditions. Each symbol represents one trial mean of *AC* and *SDA* plotted against each other. First, it can be seen that all *AC* values were scattered across a range between −6.80 and +1.55 m/s^2, with the overall mean at −3.16 m/s^2. The mean is well within the hypothesized range of −6.16 and 0 m/s^2. The inserted histogram of all *AC* data highlights the distribution of the *AC* values around that mean. Importantly, six data points were positive and five data points were more negative and outside the predicted range. This shows that different performance was possible and the observed negative values were, therefore, not trivial.

The variability associated with these *AC* values was measured in the standard deviations of the task criterion ball amplitude *SDA*. The graph shows that the *SDA* associated with each *AC* was lowest for the intermediate range of *AC* approximately between −5 and −2 m/s^2. For values closer to zero or more negative, the variability increased in a U-shaped pattern. This means that performance at nonoptimal *AC* values had a "cost" of higher variability. The superimposed

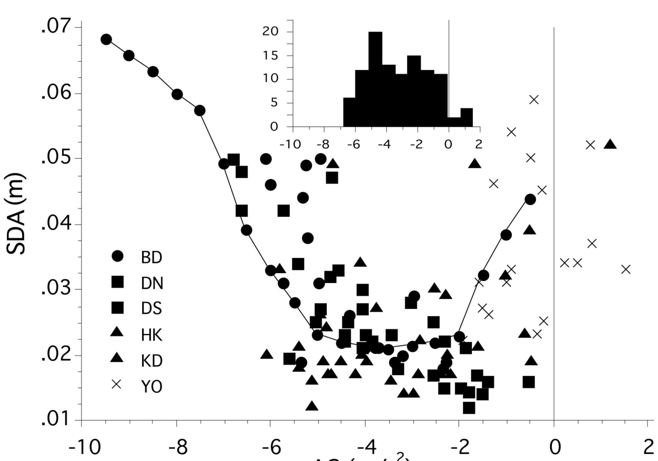

FIGURE 3. Standard deviations of ball amplitude (*SDA*) plotted against acceleration at impact (*AC*) from all subjects and all conditions in experiment 1. Each data point is a trial mean. The different symbols denote the six subjects. The *solid line* represents the predicted function of variability calculated from a Lyapunov stability analysis. The inserted figure is a histogram of the *AC* values from all subjects and conditions.

curve represents a mathematical foundation for this observation. The solid line is the result of a nonlocal Lyapunov stability analysis that determined the range of optimal stability for *AC* (Sternad et al., 2001). The units of the Lyapunov prediction have been scaled to fit to the scale of the variability estimates and are therefore of a qualitative nature. As can be seen, the Lyapunov predictions form an approximate U-shape and the data cluster around this prediction quite closely.

This pattern of data confirmed the first hypothesis that actors indeed performed the task with negative *AC*, and, following from the theoretical analysis, performed the task with passive stability. The second hypothesis was also supported in that variability was lowest for performance solutions with optimal *AC* values. These results were replicated several times in different experimental setups, including one where the ball was not confined to one dimension and both racket and ball could move in three dimensions (Katsumata et al., 2003; Sternad et al., 2001).

From these results, a straightforward expectation is that actors with more developed coordinative skills or special practice should show higher sensitivity to this passive solution. Indeed, another look at Fig. 3, which differentiates performance for different subjects by different symbols, reveals that the subject HK had the best performance with least variability. This subject had practiced the task a lot and also was an expert tennis player. However, subject YO, who showed a number of trials with positive *AC* values, had never played a racket sport before. Taken together, these first observations lead to the expectation that the dependent measures *AC* and *SDA* provide sensitive measures for skill development or also skill deficits. As such, it can be expected that these measures may also serve a useful clinical assessment of motor function. Yet, before such application becomes viable, a controlled experiment with healthy subjects is required that tests whether practice leads to changes in the exploitation of passive stability.

Experiment 2: Skill Acquisition as Attunement to Stability

To examine whether the acceleration at impact *AC* can serve as an indicator for changing performance as a function of practice, we conducted an experiment in which 12 subjects performed the experimental task across a series of 40 trials. The same apparatus with the same data collection and processing methods were used as in the previous experiment. To ensure that our measures were as sensitive as possible, we selected subjects that reported relatively little to no practice in racket sports. All subjects performed the task for 40 trials in a sequence without prior practice. They were split evenly into two groups performing two different conditions. For the first group, a difficult condition was created to allow maximal room for improvement. First, subjects were instructed to perform the task with their nonpreferred hand, because pilot

Copyright © by the American Clinical Neurophysiology Society. Unauthorized reproduction of this article is prohibited.

studies had shown much poorer performance in this situation. Second, a previous experiment had revealed that when a metronome paced the ball-bouncing actions, the pattern became more difficult for subjects and more variable. Hence, more change could be expected when a metronome drove performance. To contrast this difficult condition, the second group of six subjects performed the same number of trials with their preferred hand and without the explicit presence of a metronome. To ensure that both groups performed with a similar period and amplitude, the subjects in group 2 were initially paced at the same period as the metronome trials (714 milliseconds). Only after having established this period did subjects continue the trial without explicit external pacing. For each trial, data were collected for 30 seconds. To capture the change in performance with practice, the sequence of 40 trials was divided into 4 blocks consisting of 10 consecutive trials. Means of *AC* and the task performance measure *SDA* across 10 trials were calculated. For each performance group, the estimates of all six subjects were averaged.

In virtually all motor-learning studies, the typical signature of improvement in task performance across practice has been a decrease in variability. Hence, a first analysis tested whether such a decrease was observed. The standard deviations of ball amplitude *SDA* are shown in Fig. 4A. The group means of *SDA* performed with the nonpreferred hand significantly decreased over the four blocks. In contrast, no such change in performance was seen for the subjects performing with their preferred hand (and without explicit pacing). Concomitantly, the *AC* results in Fig. 4B displayed a similar pattern across the four blocks. *AC* decreased significantly for performance with the nonpreferred hand and less so for the preferred hand. However, contrary to the variability results for group 2, which showed no change across blocks, a small decrease in *AC* was observed even for the preferred hand. This trend was revealed in statistical analyses by pairwise differences between the first and the last block. This result is interesting because it indicates that *AC* is even more sensitive to changes in performance than traditional variability measures.

To visualize this change in the parameter *AC*, Fig. 5 plots the trial averages of two selected subjects across the sequence of 40 trials. Interestingly, for the nonpreferred hand (and metronome pacing), the average impact *AC* data start with positive values that only become negative after the third trial and then decrease relatively consistently. In the theoretical section, we suggested that positive *AC* values may be an indicator for a mirror-algorithm strategy, a regime with continuous visuomotor coupling. Following this logic, the result suggests that the actor started with a performance strategy where he tightly coupled the racket to the ball motion. Only after a few trials he became sensitive to the passive solution and incrementally changed the racket–ball contacts to exploit these properties. The same gradual but smaller decrease is seen for the subject performing with his preferred hand.

Taken together, these results confirm the individual observations from experiment 1 and support hypothesis 3: Practice makes subjects better in exploiting the stability properties of the task. On the assumption that positive *AC* values indicate continuous coupling between the visually perceived ball trajectory and the racket movement, and if more negative *AC* values are equivalent to attunement to passive stability of the task, then indeed, learning is a "road

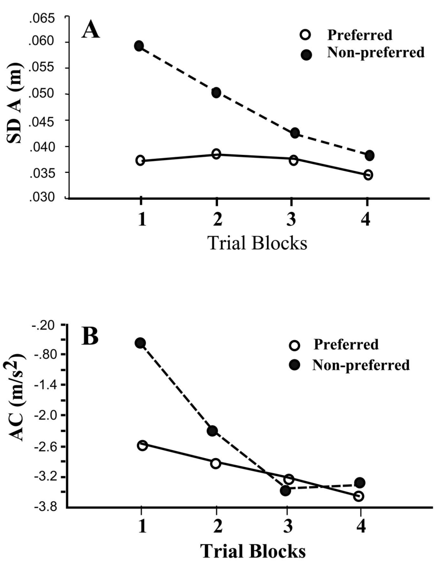

FIGURE 4. (**A**) Standard deviations of ball amplitude as a function of trial blocks in experiment 2. The *open circles* are the averages of six subjects who performed the task with their preferred hand without metronome pacing; the *filled circles* are the average of six subjects performing with their nonpreferred hand with metronome pacing. In each block, 10 trials are averaged. (**B**) Acceleration of the racket at impact (*AC*). The grouping of the data is identical to that in panel **A**.

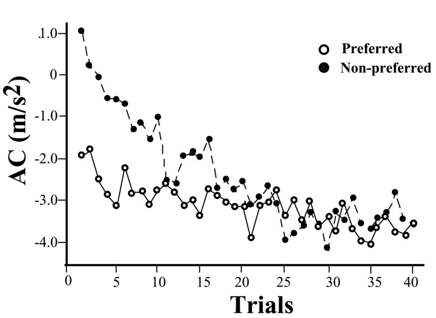

FIGURE 5. Individual means of acceleration at impact (*AC*) in the 40 trials of two selected subjects, performing with their preferred (*open circles*) and their nonpreferred (*filled circles*) hand.

© 2004 Lippincott Williams & Wilkins

Copyright © by the American Clinical Neurophysiology Society. Unauthorized reproduction of this article is prohibited.

to automatic" as Milton and colleagues argue in their overview article in this issue. These results permit the conclusion that sensitivity to task stability is an ability that characterizes degrees of sensorimotor functions and dysfunctions. Indeed, this ability lies at the heart of many types of movements and has been argued to be a hallmark of functional performance. Yet, the present experimental results give unambiguous quantitative support to this hypothesis. Using these results as a benchmark, it will be of interest to test clinical populations in their ability to exploit task stability.

Experiment 3: Actively Tracking Passive Stability

Adaptations to unforeseen changes and perturbations in the environment are omnipresent in our everyday life. Although healthy people successfully deal with constantly changing situations, the lack of such adaptability is one of the first manifestations of a motor disorder. For instance, patients with Parkinson disease often are able to maintain postural stability but have problems to compensate for small perturbations and regain their balance. Also in dual tasks additional cognitive or motor requirements interfere with their postural control leading to significantly higher postural sway (Marchese et al., 2003; Visser et al., 2003). Similarly, elderly people have a significantly higher propensity to fall when walking or climbing stairs, incurring the risk of fractures and other injuries (Startzell et al., 2000). This is due to their diminished ability to deal with small, unforeseen disturbances. Although these behavioral symptoms are recognized, little detail is known about exactly *how* the intact nervous system counteracts quickly and continuously such small and large perturbations. The following experiment investigated this ability in the ball-bouncing task by confronting subjects with unforeseen perturbations (de Rugy et al., 2003).

Figure 2 illustrates how, in task performance with passive stability, perturbations of the ball trajectories converge back to the stable regime *without* requiring racket adjustments. However, this behavior was demonstrated for perturbations that were small enough so that the ball–racket impact following the perturbation was at a phase that still had negative *AC* values. What happens if perturbations are larger and lead to ball–racket impacts that are outside the stable range, no longer providing for these self-stabilizing processes? How do subjects deal with perturbations that do require active adjustments to reestablish the rhythmic pattern?

Six subjects performed rhythmic ball-bouncing in a virtual reality setup with and without perturbations. In the virtual environment, the subject manipulated a real tennis racket in front of a large screen onto which the visual display was projected. The subject stood upright at a distance of 1.5 m from the screen and held the racket horizontally at a comfortable height. A rigid rod with two hinge joints and one swivel joint was attached to the racket surface and ran through a noose that rotated a wheel by its vertical motion. The movements of the racket were measured by an optical encoder and on-line projected onto the screen. To simulate the mechanical contact between the racket and the ball, a solenoid was triggered for each computed virtual impact, which activated a mechanical brake for a duration of 30 milliseconds. The force developed by this brake onto the rod was adjusted to that produced by a tennis ball falling on the racket.

Custom-written software computed online the ball trajectories based on the measured racket movements and the contact parameters of ball and racket. The simulated ball trajectories were projected onto the screen so that the subject only interacted virtually with the ball. The ball's trajectory was calculated using the equations of ballistic flight and inelastic impact. In the perturbation trials the coefficient of restitution of the ball-racket impact was changed at every fifth contact leading to unexpected ball amplitudes.

The task for the subject was to rhythmically bounce the virtual ball for the duration of each trial as in the physical setups before. The only difference was that there was a target height displayed on the screen. The task of the subject was to bounce the ball so that its peak amplitude reached the target line after every bounce. The experiment comprised two conditions: unperturbed and perturbed. In the unperturbed condition, the coefficient of restitution remained constant at $\alpha = 0.50$ during the entire trial. In the perturbed trials, perturbations were introduced by changing α at every fifth impact. That is, α was set to 0.50 over four consecutive impacts, but was randomly changed at every fifth impact of the trial. Although the occurrence of a perturbation was in principle predictable, subjects could not preplan their responses as the values and direction of the perturbed α_Ps were randomly specified. α_P was determined for each perturbed impact to be any value within the ranges 0.30 and 0.40, or within 0.60 and 0.70. One trial gave approximately 50 cycles and contacts and 10 perturbations for each perturbed trial. The experiment consisted of 15 trials per condition (40 seconds in each trial).

To evaluate whether performance was consistent with passive stability, the values of *AC* were examined in both the unperturbed and perturbed trials. In the unperturbed conditions overall trial averages were calculated. The overall mean across all subjects was -2.16 m/s^2 replicating the previous results in the virtual setup. In the perturbed trials, the impacts were sorted by impact number, where impact 1 denoted the first impact following the perturbed impact, with the constant $\alpha = 0.50$. Impacts 2, 3, and 4 were the impacts before the next perturbed impact. For each subject, all *AC* values for a given impact number were averaged and are shown in Fig. 6A. The six lines represent the average pattern of each of the six subjects across impacts. What can be seen is that for each

© 2004 Lippincott Williams & Wilkins

Copyright © by the American Clinical Neurophysiology Society. Unauthorized reproduction of this article is prohibited.

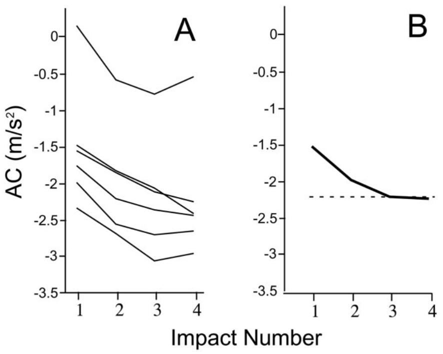

FIGURE 6. Acceleration of the racket at impact as a function of the impact number from experiment 3. Impact 1 is the impact following the perturbation. Impacts 2, 3, and 4 denote the following impacts. (**A**) Each line corresponds to one subject. (**B**) The *solid line* represents the average across all six subjects. The *dashed line* is the overall trial average from the unperturbed trials.

individual, the *AC* values were more positive at impact 1 immediately after the perturbation, and decreased to more negative values across the four unperturbed impacts. Although one subject displayed significantly higher values of *AC*, his pattern had the same trend and no other abnormalities were observed.

The overall averages of *AC* are shown in Fig. 6B together with the trial average measured in the unperturbed condition, shown by the dashed constant line. It is evident that after the perturbation, subjects quickly stabilized their movements and reestablished ball–racket contacts close to the unperturbed performance. On average, it took two cycles to stabilize performance. Statistical analyses confirmed this decrease and identified that impacts 1 and 2 are significantly different from impacts 3 and 4. At impacts 3 and 4, the *AC* values were no longer different from the baseline level obtained in the unperturbed trials. This approximately exponential approach to constant negative *AC* values is similar to the learning curves reported in experiment 2, although it happened here at a much shorter time scale.

A comparison of this return pattern with the simulated relaxation pattern of the passively stable model in Fig. 2 reveals interesting differences. In the simulated return, the perturbed *AC* values were either more positive or more negative, depending on the direction of the perturbation. That is, if the ball amplitude was higher than before, then it would contact the racket trajectory later, given an unchanged racket trajectory, leading to more negative *AC* values. Conversely, smaller-than-normal ball amplitudes hit the racket earlier, leading to more positive *AC* values. This separation was not seen in the experimental performance. The uniformly positive *AC* values are evidence that subjects manipulated the racket trajectory to prepare for the next ball–racket contact. This adaptation in the first cycle after the perturbation will be scrutinized next.

To this end, the continuous racket trajectories were parsed into individual cycles separated at the ball–racket impact. Figure 7 shows these cycles of one subject from all 15 perturbed trials. The cycle trajectories were sorted by cycle number and superimposed on each other. C-1 refers to the cycle immediately following the perturbation, C-2 to C-5 the subsequent cycles. The racket trajectories illustrate how the movements were modulated due to the perturbation. During C-1, when the ball amplitude was higher or lower than expected, the trajectories were lengthened or shortened. This can be seen in the fanning out of the trajectories at the end of the cycle. Two groups of trajectories separate: a shortening was observed for α_P lower than $\alpha = 0.50$, leading to a smaller ball amplitude, which required a shortened cycle period if the ball was to be hit at the same *AC* value. Conversely, lengthening of the cycle was observed for α_P higher than $\alpha = 0.50$, as the ball amplitudes were higher and the periods between bounces longer. The period modulation was such that the next impact occurred at a phase closer to the stationary *AC* value. This separation in the trajectories was no longer as clear in C-2 and had disappeared by C-4. This means that a regular contact was obtained and the period was reset to normal. Interestingly, the amplitude did not show similar adjustments.

These qualitative impressions were corroborated in an analysis that calculated racket periods and amplitudes and averaged them by cycle number. Figure 8 summarizes the modulation in period and amplitude of the racket movement after a perturbation. The dependent measures are the differences in period and amplitude at each postperturbation impact compared with the average period and amplitude computed over all cycles. This facilitated a comparison across subjects who had different mean values of period and amplitude. Figure 8A shows that the racket period was increased or decreased by 0.20 seconds during C-1 in response to the perturbation. In contrast, the amplitude modulations in Fig.

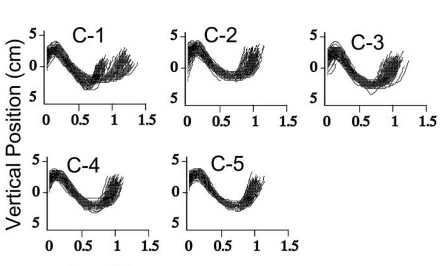

FIGURE 7. Racket trajectories of one selected subject parsed and sorted by cycle number. C-1 is the cycle immediately following the perturbation, whereas C-2, C-3, C-4, and C-5 are the following cycles until the next perturbation. The trajectories were parsed at the moment of contact.

 © 2004 Lippincott Williams & Wilkins

Copyright © by the American Clinical Neurophysiology Society. Unauthorized reproduction of this article is prohibited.

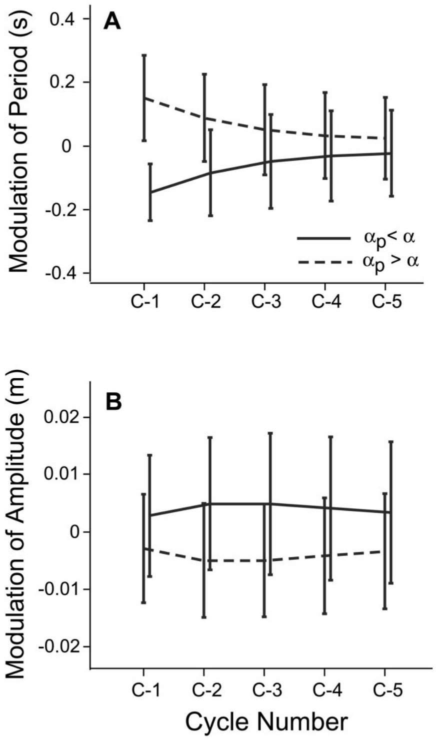

FIGURE 8. Modulation of racket period (**A**) and amplitude (**B**) as a function of cycle number in the perturbed trials. The data of all subjects are averaged. The *vertical bars* denote the standard deviations. The *dashed lines* correspond to those trials where the perturbed coefficient of restitution α_P was greater than $\alpha = 0.50$. The solid lines correspond to those trials where the perturbed coefficient of restitution α_P was smaller than $\alpha = 0.50$.

8B show no such difference across the cycles for the two α_P conditions. The small constant offset between the two α_P conditions is not statistically significant.

These results unequivocally show that, when necessary, subjects modulate their racket trajectory to regain a regular bouncing pattern. Interestingly, the adjustments were made in the period only. Further, these modulations establish a contact that is always more positive than the preperturbation *AC*. With a view to the mirror algorithm illustrated above, this finding suggests that immediately with the perturbation the control of the racket trajectory switched to a strategy that mirrored the ball velocity, hence producing *AC* values that were more positive. However, a strategy like the mirror algorithm would produce racket trajectories with also a larger or smaller amplitude, and this modulation in the amplitude was not observed in the data. Hence, de Rugy et al. (2003) presented a model in which these results were replicated with a discrete coupling between the racket movements and the ball trajectories at the moment of contact. Rhythmic racket trajectories were generated by an oscillator whose period was parameterized as a function of the velocity of the ball immediately after the contact. Adjustments were achieved by a resetting of the period directly following the impact as a function of the ball velocity after contact. The simulation replicated most of the features in the data.

DISCUSSION

The first objective of the reviewed research was to show how a dynamical model of a perceptual-motor task can provide a useful framework for understanding functional and dysfunctional behavior. Such a task-based approach encompasses the actor and his/her environment as its unit of analysis. Bouncing a ball with invariant amplitude was the example we chose. With its analysis of racket and ball trajectories that represent the interface between the actor's movements within the environment, it adopts a relatively coarse-grained level of analysis. Instead of recording neurophysiological variables, such as electromyographic profiles or single-neuron recordings of muscle and brain cells, it focuses on behavioral or mechanical variables, such as the displacements and accelerations of the racket movements. We propose that it is at this interface where successful solutions and their constraints are defined. Drinking from a water glass requires grasping and transporting the glass with adequate muscle force, which depends on the weight, content, and orientation of the glass in a continuously changing way. Recordings of muscle activation and their evaluation are only meaningful when analyzed with respect to these environmental contingencies.

We would like to suggest that this approach holds interesting implications for the clinical context and for rehabilitation. Although physiological measures such as spike trains of cell activity provide important insights into the damage of the neurophysiological structures, such information often remains unconnected to the pertinent question of how to reconstruct functional behavior. Rehabilitation aims to reestablish functional behavior, even when the damage of physiological structures is irreversible. A task analysis may offer new perspectives for developing rehabilitation strategies. For instance, understanding the task and its constraints may reveal new solutions or the task constraints can be altered so as to create options for a damaged nervous system to develop new successful solutions. In this very general way, we hope that this article stimulates physicians and therapists to think on this more coarse-grained level.

A second and more specific objective of this review was to illustrate how the concept of stability can provide useful insights into motor behavior. Stability, and its close associate variability, have played a central role in the inquiry of the acquisition and control of movements, albeit with many different definitions and levels of rigor (see also Milton et al., this issue). Most commonly, successful performance has been accompanied by low variability in important task

© 2004 Lippincott Williams & Wilkins

Copyright © by the American Clinical Neurophysiology Society. Unauthorized reproduction of this article is prohibited.

parameters, which in turn was equated with high stability. For instance, performance of patients with neurological disorders is in almost all cases more variable, which has commonly been equated with a lack of stability and controllability. This, however, is not completely correct. Stereotypy, a pathology that refers to abnormal constancy and rigidity, clearly highlights that lack of variability not necessarily indicates more stable behavior (Newell et al., 1993). Similarly, it was reported that Down syndrome patients who are usually hypotonic show a higher level of co-contraction in voluntary movements (Aruin et al., 1996). Such excessive co-contractions and hypertonic behavior are most likely a system response to compensate for the perceived lack of control. Such preemptive control decreases variability at the expense of decreased adaptability. However, such a strategy should not be confounded with a "smart" exploitation of task stability. Hence, whereas intuitive in common language, the equation of stability and variability blurs over some finer-grained distinctions that have been highlighted in dynamical systems theory.

Stability in a dynamical system—and all movements constitute dynamical systems—is a well-defined concept and quantifies how fast the system recovers from a small perturbation. A pictorial interpretation of stability can be gleaned from Figs. 3 and 4 in the overview article by Milton et al. in this issue. With the image of a ball in a potential well, stability is high when the well is deep and steep. Given a displacement of the ball, the steepness of the well determines the time until the ball settles back to the well's minimum. Further, the system itself need not apply corrective control: the return to the stable state is inherent in the system dynamics. Variability, however, can be conceptualized as the response of a dynamical system to a continuous barrage of small perturbations or noise. Taking recourse again to the same Fig. 2, variability depends both on the steepness of the potential well *and* the magnitude of the noise. Higher noise levels push the ball further away from its lowest point, and thus increase variability. Similarly, the same noise has a larger effect when the well is shallow. Hence, although the concepts of stability and variability are certainly related, they are not equal.

This pictorial comparison highlights that the two concepts give two somewhat independent pieces of information. Variability holds information of both stability *and* noise and, hence, does not allow a direct inference about stability. Stability offers a "smart" solution for behavior that is resistant to perturbations. It is therefore an anchor point for successful behavior. Hence, what is needed for a rigorous analysis of variability and stability in movement control is a model of the task and behavior. This was exactly the objective of this line of research.

To permit a quantitative analysis for the chosen task of bouncing a ball, the task was cast into a mechanical model that encompassed the movements of the actor and manipulanda. In this form, the movement task is a dynamical system that displays stability, i.e., perturbations die out by themselves. The hypothesis is that the actor, when engaged in a task, seeks to perform with stability. We termed this strategy "passive" because this strategy alleviates the control demands, i.e., (sufficiently small) perturbations do not require corrections. This passive stability was characterized by a single parameter, the acceleration of the racket at impact, which provided quantitative predictions about stability. Variability was measured separately in standard deviations of the task parameter ball amplitude. Armed with these variables, a series of experiments examined performance at steady state, the process of acquisition, and the response to perturbations.

In experiment 1, we established that indeed subjects with normal sensorimotor functions tended to perform with racket-acceleration values that were predicted to provide passive stability. This result has been corroborated in several other variants of the ball bouncing task, for instance when performed in three dimensions, or with different coefficients of restitutions, or with different experimental setups (de Rugy et al., 2003; Katsumata et al., 2003; Schaal et al., 1996; Sternad et al., 2000, 2001). A closer look at the individual data of experiment 1 showed that racket–ball contacts with negative accelerations were by no means necessary and, therefore, not trivial. Positive accelerations and negative values outside the predicted range were also observed. Interestingly, such "deviant" behavior was associated with higher variability. Because these solutions are no longer passively stable, this variability may be the expression of corrective processes. One expectation from this distinction into different regimens is that the structure of variability in the passively stable and not passively stable performance should be different. More fine-grained time-series analyses, as developed in nonlinear dynamics, may offer another route to explore the corrective, i.e., deterministic sources of the observed fluctuations and those due to noise (Müller and Sternad, 2004; Riley and Turvey, 2002).

Experiment 2 examined the process of acquisition and improvement in the framework of stability and variability. Proponents of a dynamical systems approach and the closely associated ecological approach to perception and action, as for instance pioneered by Fowler and Turvey (1978) and Gel'fand and Tsetlin (1971), have emphasized that learning is a search process that explores the perceptual-motor workspace defined by the task (Müller and Sternad, 2004; Newell, 1986, 1991; Newell et al., 1989). Such search can be traced when the workspace with its stable solutions is known. Dynamical principles, such as attunement to attractors, intermittency, relaxation, as found in nonlinear systems, have given a new language in revealing principles of learning (Kelso and Zanone, 2002; Schöner, 1989; Shockley et al., 2001; Wagman et al., 2002). The results of experiment 2

© 2004 Lippincott Williams & Wilkins

Copyright © by the American Clinical Neurophysiology Society. Unauthorized reproduction of this article is prohibited.

showed that improvement in performance is paralleled by an increased attunement to passive stability. The parameter *AC* changed in an approximately exponential fashion over the course of 40 practice trials (Sternad et al., 2000). In fact, this parameter was even more sensitive than the standard variability measure *SDA* to capture changes. The comparison of the results of variability in ball amplitude and *AC* in Figs. 6A and 6B revealed that *AC* indicated change in performance with the preferred hand. In comparison, *SDA* did not indicate any improvement. This discrepancy highlights again that variability and stability should not simply be regarded as inverse concepts.

Having supported the major predictions, experiment 3 went one step further: It tested how actors use active correction mechanisms when unforeseen events occur that no longer fall into the passive stability range. The results revealed that subjects are sensitive to the dynamics of the ball-bouncing task and modulate their movements immediately following the perturbation. We showed that a one-time adaptation of the period of the racket oscillation sufficed to put the system back into the range of the passive stability. Again, armed with the quantitative benchmark of stability, we were able to characterize the subjects' responses. Such adaptation to uncertain or changing environments is a hallmark of healthy behavior and has been subject to a number of investigations (Davidson and Wolpert, 2003).

Sensitivity and continuous adaptation to movement dynamics has also been pivotal in research on multijoint movements. For example, in a reaching action that involves the upper and lower arm segments, intersegmental torques arise that transfer torques from one limb segment to the other. From a mechanical analysis, it can be quantified how the forearm is moved by movements of the upper arm without any active muscular torque generation. Hence, to produce a given elbow flexion, different muscle torques are required, depending on the concurrent shoulder movements and its orientation to gravity and to the upper arm. In short, humans have to continuously be sensitive to these changing dynamics of their own limbs. In fact, not only do actors have to take these additional torques into account, it has been argued that humans make use of these interaction torques (Bernstein, 1967; Schneider et al., 1987, 1989; Zernicke et al., 1991). That smooth movements require such attunement has been made evident in studies on discrete and rhythmic movements in patients with large-fiber neuropathy. Ghez, Sainburg, and colleagues showed that these patients no longer exploited these interaction torques and therefore produced very jerky and unusually curved movements trajectories (Ghez and Saiburg, 1995; Sainburg et al., 1993). It will be of interest to what degree patients with neurologic deficits deal with perturbations in the ball-bouncing task. However, such studies await to be done in the future.

In conclusion, the results verified that bouncing a ball provided a useful model system to gain insight into organizational principles of performing a rhythmic perceptual-motor task. Note that by adopting this theoretical approach, we refrained from searching for cognitive representations or physiological control signals. Rather, we aimed to identify conditions and constraints where such control is minimized and tested whether humans exploited these conditions.

APPENDIX

In this appendix, we provide the mathematical details of the simulations depicted in Figs. 2 and 3. We denote ball position and velocity by x_b and v_b, respectively. The dynamics (time evolution) of x_b and v_b between bounces is given by the differential equations of free flight:

$$\dot{x}_b = v_b$$

$$\dot{v}_b = -g \tag{A1}$$

where the over dot denotes differentiation with respect to time and g denotes the acceleration of gravity. At impact, we assume an inelastic bounce as given by:

$$\alpha(v_b^- - v_r^-) = -(v_b^+ - v_r^+) \tag{A2}$$

with α as the coefficient of restitution, v_r racket velocity, and v^- denoting velocities before impact and v^+ after impact.

The racket dynamics in the mirror algorithm is given by:

$$E = gB - gx_b - v_b^2/2$$

$$\dot{x}_r = -\frac{1}{\tau}\left(x_r + \left(\frac{1-\alpha}{1+\alpha} + k_{11}E\right)x_b\right) \tag{A3}$$

with B denoting target amplitude and E the difference between required and actual mechanical and potential energy of the ball. Further, τ denotes a time constant and k_{11} a gain factor. From Eq. A3, it is evident where the name "mirror algorithm" comes from, because in the stationary state the right-hand side equals zero, which implies:

$$x_r = -\left(\frac{1-\alpha}{1+\alpha} + k_{11}E\right)x_b \tag{A4}$$

which shows that the racket trajectory x_r is a mirrored copy (caused by the minus sign) of the ball trajectory x_b. In the original mirror algorithm, Bühler and colleagues used Eq. A4 instead of A3, and thus the racket trajectory was an exact mirror of the ball trajectory including the bounce. Because it is physically not realistic for the racket trajectory to change that suddenly, we imbued the racket with the first-order

Copyright © by the American Clinical Neurophysiology Society. Unauthorized reproduction of this article is prohibited.

dynamics of Eq. A3. The time constant τ is a rough estimate of the reaction time. A second-order dynamics might be more realistic, however, at the expense of an extra dimension for the state space and an extra parameter.

For the simulations in Figs. 2 and 3, the following parameters were used: acceleration of gravity $g = 9.81$ m/s^2, coefficient of restitution $\alpha = 0.50$, target height of ball $B = 1$ m, time constant of the mirror map dynamics $\tau = 0.1$ second, and gain constant $k_{11} = 0.03$ s^2/m^2. The perturbations consisted of shifting the ball up or down by 0.1 m. The dynamics were numerically integrated with Euler integration with a time constant of 0.1 millisecond and implemented in Matlab 6.5.

To simulate the strategy that displays passive stability, we took a sinusoidal motion with a fixed racket amplitude a_r and frequency f_r:

$$x_r = a_r \sin(2\pi f_r) \tag{A5}$$

The name "passive stability" derives from the fact that the amplitude and frequency have to be chosen such that the resulting dynamical system (consisting of A1, A2, and A5) is stable. However, once chosen, the bouncing is fixed, hence the qualifier "passive." In the simulations we took racket amplitude $a_r = 0.3$ m and frequency $f_r = 1$ Hz. These were chosen such that the ball amplitude was 1 m and the racket amplitude was comparable to the mirror algorithm, with the constraint that racket acceleration at impact in the stationary state (*AC*) had to be within the limits as specified by Eq. 1. The parameters of the numerical integration were identical to those of the mirror algorithm.

A final comment pertains to the use of the symbol *AC* to denote the racket acceleration at impact in the stationary state. In the main text of this article, we use *AC* in a mathematically sloppy way to denote all racket accelerations at impact, irrespective whether the system is in the stationary state. This simplifies the discussion at the expense of mathematical rigor. Strictly speaking, *AC* is a parameter that has a fixed value given a fixed coefficient of restitution, acceleration of gravity, and amplitude and frequency of the racket motion. One can extract this parameter directly from the data by averaging the individual accelerations at impact from each bounce of a trial, assuming that the system is at the stationary state. However, when the system is not in the stationary state, the accelerations at impact are a time series that have values that depend on the state variables of the system. Strictly speaking, a different symbol should be used. Because it does not affect the main message in the present review, we have confined the presentation to use one symbol, *AC*.

REFERENCES

Aruin AS, Almeida GL, Latash ML. (1996) Optimization of a simple two-joint synergy in individuals with Down syndrome. *Am J Ment Retard* 101:256–268.

Bernstein N. (1967) *The coordination and regulation of movement*. London: Pergamon Press.

Bühler M, Koditschek DE. (1990) From stable to chaotic juggling: theory, simulation, and experiments. IEEE International Conference on Robotics and Automation, Cincinnati, Ohio.

Bühler M, Koditschek DE, Kindlmann PJ. (1994) Planning and control of robotic juggling and catching tasks. *Int J Robot Res* 13:101–118.

Cabrera JL, Milton JG. (2002) On-off intermittency in a human balancing task. Phys Rev Lett 89:158702-1–4.

Cabrera JL, Milton JG. (2004) Human stick balancing: tuning Levy flights to improve balance control. *Chaos* (In press).

Davidson PR, Wolpert DM. (2003) Motor learning and prediction in a variable environment. *Curr Opin Neurobiol* 13:232–237.

de Rugy A, Wei K, Müller H, Sternad D. (2003) Actively tracking "passive" stability. *Brain Res* 982:64–78.

Dijkstra TMH, Katsumata H, de Rugy A, Sternad D. (2004) The dialogue between data and model: passive stability and relaxation behavior in a ball bouncing task. *J Nonlinear Sci*. (In press)

Dijkstra TMH, Schöner G, Gielen CCAM. (1994a) Temporal stability of the action-perception cycle for postural control in a moving visual environment. *Exp Brain Res* 97:477–486.

Dijkstra TMH, Schöner G, Giese MA, Gielen CCAM. (1994b) Frequency dependence of the action-perception cycle for postural control in a moving visual environment: Relative phase dynamics. *Biol Cybern* 71:489–501.

Fajen BR, Warren WH. (2003) Behavioral dynamics of steering, obstacle avoidance, and route selection. *J Exp Psychol Human Percep Perform* 29:343–362.

Fowler CA, Turvey MT. (1978) Skill acquisition: An event approach with special reference to searching for the optimum of a function of several variables. In: Stelmach GE, ed. *Information processing in motor control and learning*. New York: Elsevier.

Gel'fand IM, Tsetlin ML. (1971) Mathematical modeling of mechanisms of the central nervous system. In: Gel'fand IM, ed. *Models of the structural-functional organization of certain biological systems*. Cambridge. MA: MIT Press.

Ghez C, Sainburg R. (1995) Proprioceptive control of interjoint coordination. *Can J Physiol Pharmacol* 73:273–284.

Guckenheimer J, Holmes , P. (1983) *Nonlinear oscillations, dynamical systems, and bifurcations of vector fields*. New York: Springer.

Jagacinski RJ, Flach JM. (2003) *Control theory for humans: Quantitative approaches to modeling performance*. Mahwah, NJ: Erlbaum.

Katsumata H, Zatsiorsky V, Sternad D. (2003) Control of ball-racket interactions in the rhythmic propulsion of elastic and non-elastic balls. *Exp Brain Res* 149:17–29.

Kelso JAS. (1995) *Dynamic patterns: the self-organization of brain and behavior*. Cambridge, MA: MIT Press.

Kelso JAS, Zanone PG. (2002) Coordination dynamics of learning and transfer across different effector systems. *J Exp Psychol Human Percep Perform* 28:776–797.

Kugler PN, Turvey MT. (1987) *Information, natural law, and the self-assembly of rhythmic movement*. Hillsdale, NJ: Erlbaum.

Marchese R, Bove M, Abbruzzese G. (2003) Effect of cognitive and motor tasks on postural stability in Parkinson's disease: a posturographic study. *Mov Disord* 18:652–658.

Müller H, Sternad D. (2004) Decomposition of variability in the execution of goal-oriented tasks: three components of skill improvement. *J Exp Psychol Human Percep Perform* 30.

Newell KM. (1986) Constraints on the development of coordination. In: Whiting HTA, ed. *Motor development on children: Aspects of coordination and control*. The Hague: Nijhoff.

Newell KM. (1991) Motor skill acquisition. *Ann Rev Psychol* 42:213–237.

Newell KM, van Emmerik REA, Sprague RL. (1993) Stereotypy and variability. In: Corcos DM, ed. *Variability and motor control*. Champaign, IL: Human Kinetics.

Newell KM, Kugler PN, van Emmerik REA, McDonald PV. (1989) Search strategies and the acquisition of coordination. In: Wallace SA, ed. *Perspectives on the coordination of movement*. Amsterdam, North Holland: Elsevier Science Publishers, 650–664.

Riley MA, Turvey MT. (2002) Variability and determinism in motor behavior. *J Mot Behav* 34:99–125.

Sainburg RL, Poizner H, Ghez C. (1993) Loss of proprioception produces

 © 2004 Lippincott Williams & Wilkins

Copyright © by the American Clinical Neurophysiology Society. Unauthorized reproduction of this article is prohibited.

deficits in interjoint coordination. *J Neurophysiol* 70:2136–2147.

Scarborough D, Sternberg S. (1998) *Invitation to cognitive science, Volume 4: Methods, models, and conceptual issues*. Cambridge, MA: MIT Press.

Schaal S, Sternad D, Atkeson CG. (1996) One-handed juggling: a dynamical approach to a rhythmic movement task. *J Mot Behav* 28:165–183.

Schneider K, Zernicke RF, Schmidt RA, Hart TJ. (1987) Intersegmental dynamics during the learning of a rapid arm movement. *J Biomech* 20:816.

Schneider K, Zernicke RF, Schmidt RA, Hart TJ. (1989) Changes in limb dynamics during the practice of rapid arm movements. *J Biomech* 22:805–817.

Schöner G. (1989) Learning and recall in a dynamic theory of coordination patterns. *Biol Cybern* 62:39–54.

Schöner G. (1991) Dynamic theory of action-perception patterns: the "moving room" paradigm. *Biol Cybern* 64:455–462.

Shockley K, Grocki M, Carello C, Turvey MT. (2001) Somatosensory attunement to the rigid body laws. *Exp Brain Res* 136:133–137.

Startzell JK, Owens DA, Mulfinger LM, Cavanagh PR. (2000) Stair negotiation in older people: a review. *J Am Geriatr Soc* 48:567–580.

Sternad D. (2000a) Debates in dynamics: a dynamic systems' perspective to action and perception. *Hum Mov Sci* 19:407–423.

Sternad D. (2000b) Juggling and bouncing balls: parallels and differences in dynamic concepts and tools. *Int J Sports Psychol* 30:462–489.

Sternad D, Duarte M, Katsumata H, Schaal S. (2000) Dynamics of a bouncing ball in human performance. *Phys Rev E* 63:011902-1–011902-8.

Sternad D, Duarte M, Katsumata H, Schaal S. (2001) Bouncing a ball: tuning into dynamic stability. *J Exp Psychol Human Percep Perform* 27:1163–1184.

Tufillaro NB, Abbott T, Reilly J. (1992) *An experimental approach to nonlinear dynamics and chaos*. Redwood City, CA: Addison-Wesley.

Visser M, Marinus J, Bloem BR, Kisjes H, van den Berg BM, van Hilten JJ. (2003) Clinical tests for the evaluation of postural instability in patients with Parkinson's disease. *Arch Phys Med Rehabil* 84:1669–1674.

Wagman JB, Shockley K, Riley MA, Turvey MT. (2002) Attunement, calibration, and exploration in fast haptic perceptual learning. *J Mot Behav* 31:481–510.

Warren WH. (1998) Visually controlled locomotion: 40 years later. *Ecol Psychol* 10:177–221.

Zernicke RF, Schneider K, Buford JA. (1991) Intersegmental dynamics during gait: implications for control. In: Patla AE, ed. *Adaptability of human gait.* Elsevier Science Publishers, 187–202.

© 2004 Lippincott Williams & Wilkins

Copyright © by the American Clinical Neurophysiology Society. Unauthorized reproduction of this article is prohibited.