Ural Federal University

Patterns of quantum state bitstrings

Vladimir Mazurenko

- Quantum computing
- Dissimilarity of bitstrings
- Phase transitions

Ekaterinburg 2021

arXiv.org > quant-ph > arXiv:2107.09894

Quantum Physics

[Submitted on 21 Jul 2021]

Certification of quantum states with hidden structure of their bitstrings

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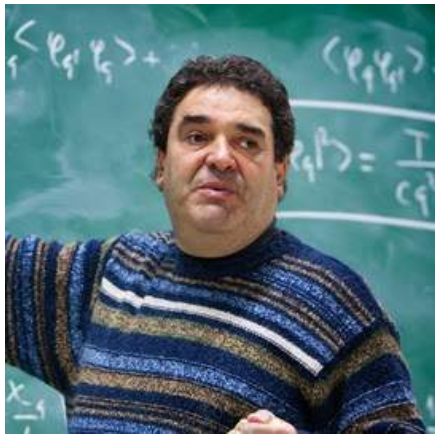

Askar Iliasov

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Quantum computing: motivation

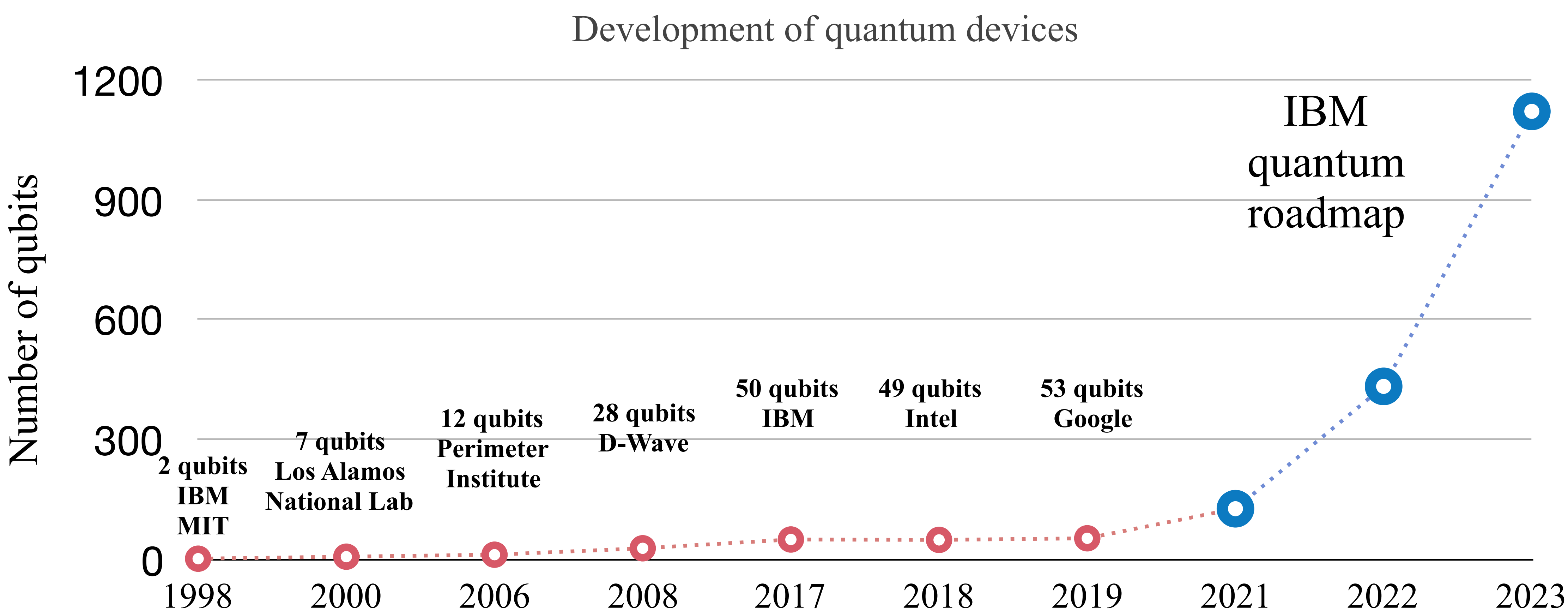
Classical bits	Quantum bits
0 1	$\|\Psi\rangle = a_0\|0\rangle + a_1\|1\rangle$
00 01 10 11	$\|\Psi\rangle = a_0\|00\rangle + a_1\|01\rangle + a_2\|10\rangle + a_3\|11\rangle$
000 001 010 011 100 101 111	$\|\Psi\rangle = a_0\|000\rangle + a_1\|001\rangle + a_2\|010\rangle + a_3\|011\rangle$ $+a_4\|100\rangle + a_5\|101\rangle + a_6\|110\rangle + a_7\|111\rangle$
a single combination of bits	superposition of all possible combinations (bitstrings or basis functions)

The ultimate goal is to solve computational problems beyond the capabilities of classical supercomputers

- Material science
- Search
- Quantum machine learning

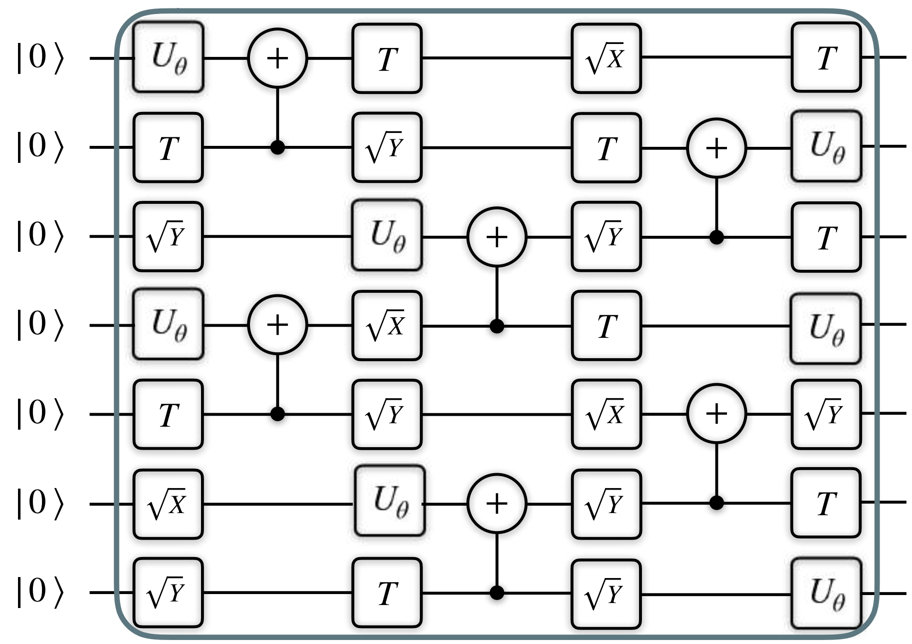
Quantum computing

Problem 1: storing $|\Psi\rangle$



Quantum computing

Problem 2 (Manipulation): to generate an entangled states from the trivial $|000\ldots0\rangle$



$$|\Psi\rangle$$

Variational approach [Nature 549, 242 (2017)]

$$|\Psi(\boldsymbol{\theta})\rangle = U(ent) \times U(\boldsymbol{\theta})|0000\rangle$$

entanglement gates

rotation gates

Idea

- quantum computer is used to calculate observables $\langle\Psi(\boldsymbol{\theta})|AB|\Psi(\boldsymbol{\theta})\rangle$
- classical computer is used to prepare new set of parameters
- fixed sequences of gates

Quantum computing

Problem 3 (Characterization):

- to design and debug quantum circuits
- to distinguish a given state from others
- to control the evolution of the state

Approaches:

- Quantum tomography
- Classical shadow
- Dissimilarity of bitstrings

Quantum tomography

characterization via reconstruction of the density matrix

V.V. Dodonov, V.I. Man'ko [Physics Letters A 229, 335 (1997)]

- Reconstruction of the density matrix on a classical computer

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}$$

- Average value of the spin projection to the direction $\mathbf{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\phi)$

$$f(\theta,\phi) = \mathrm{Tr}(\rho\sigma_n) = (\rho_{11} - \rho_{22})\cos\theta + \sin\theta(\rho_{12}e^{i\phi} + \rho_{21}e^{-i\phi})$$

We need to choose three directions (three bases) to define the density matrix elements

- Calculation of the standard correlation functions and fidelities

- System size limit: ~ 10 qubits

Classical shadow

characterization without reconstruction of the density matrix

Hsin-Yuan Huang, Richard Kueng, John Preskill,
Nature Physics 16, 1050 (2020)

- Classical snapshot of unknown state

$$\hat{\rho} = \underbrace{\mathcal{M}^{-1}}_{\text{inverted channel}} \left(\underbrace{U^\dagger}_{} \left|\underbrace{\hat{b}}_{\text{bitstring}}\right\rangle\!\left\langle \hat{b} \right| \underbrace{U}_{\text{basis}} \right)$$

- Construction of the classical shadow

$$S(\rho; N) = \{\hat{\rho}_1, \dots, \hat{\rho}_N\}$$

- Calculation of the standard correlation functions and fidelities

$$\hat{o}_i = \mathrm{tr}\left(O_i \hat{\rho}\right)$$

- Up to date the only highly-structured quantum states

Patterns in bitstrings

example of a highly-structured state

Cat states

$$|\Psi_\theta\rangle = \cos(\frac{\theta}{2})|0\rangle^{\otimes N} + \sin(\frac{\theta}{2})|1\rangle^{\otimes N}$$

measurements

0000000011111111000000000000000
111111110000000011111111111111111
11111111000000001111111100000000
00000000111111111111111100000000
11111111000000000………..

examples of bit-string arrays

$\theta = \frac{\pi}{8}$

$\theta = \frac{\pi}{2}$

Patterns in bitstrings

example of a highly-structured state

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examples of bit-string arrays

$\theta = \frac{\pi}{8}$

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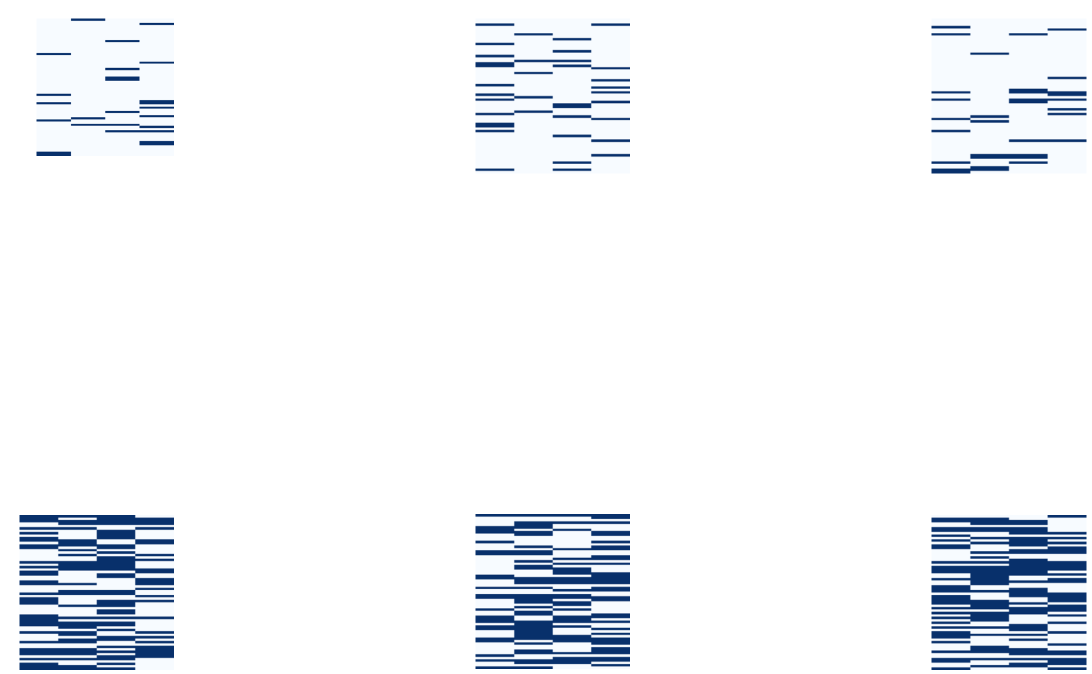


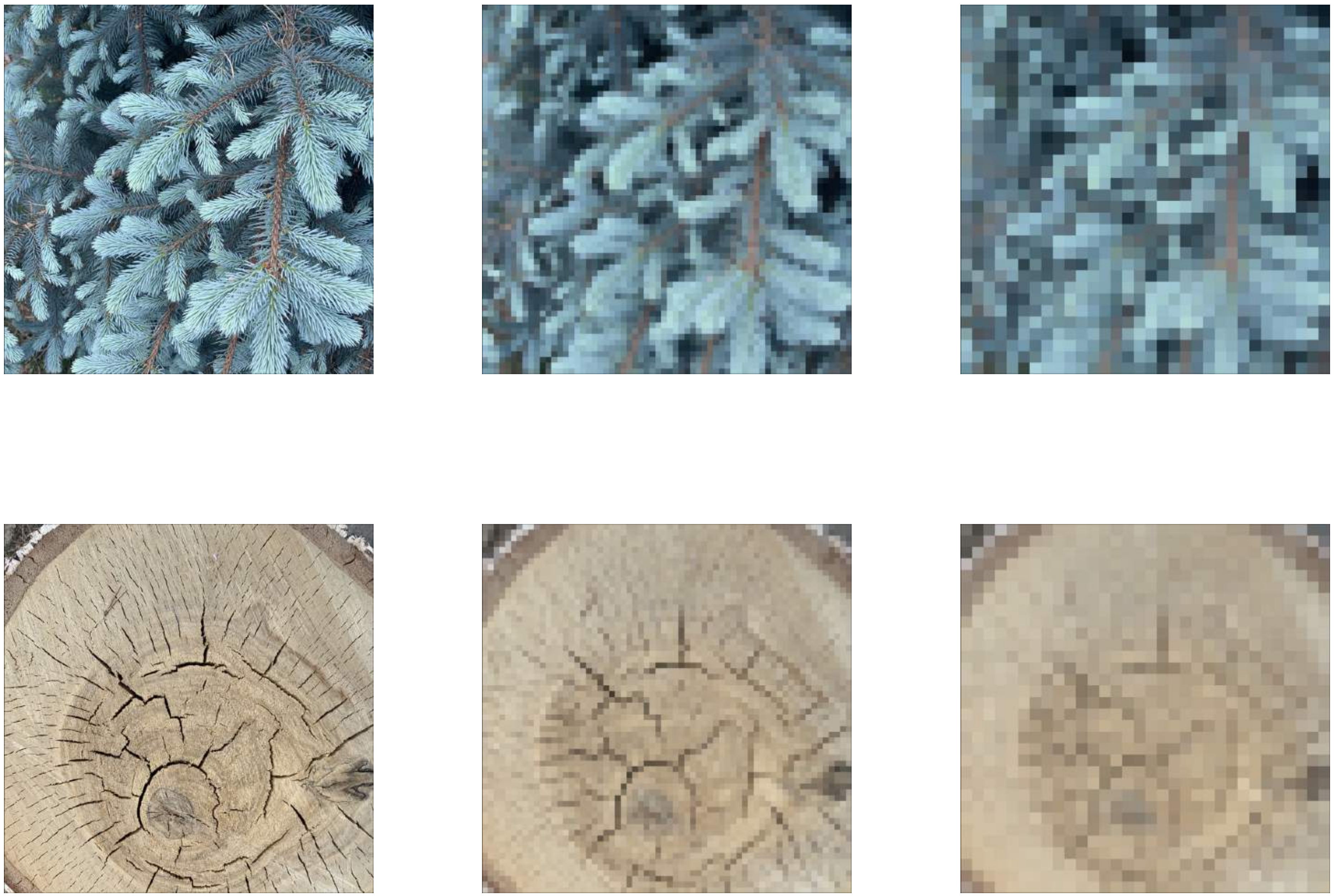
We need an algorithm to reproduce our perception of these images on the quantitative level

Structural complexity

To quantify the patterns, blur them!

A.A. Bagrov et al., PNAS (2020)

Initial	32x32 filter	64x64 filter



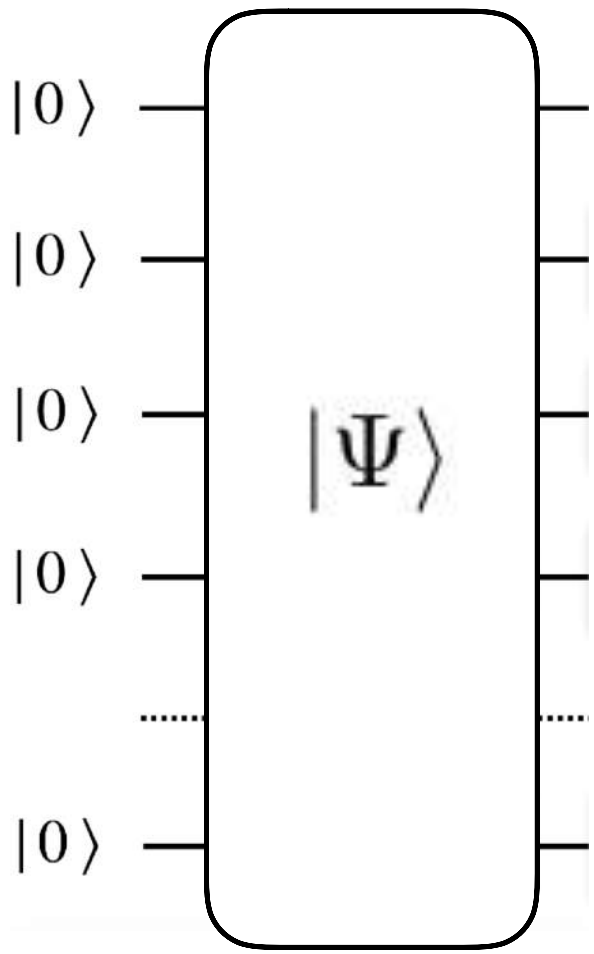
→ 0.32

→ 0.15

Structural complexity (classic) → Dissimilarity (quantum)

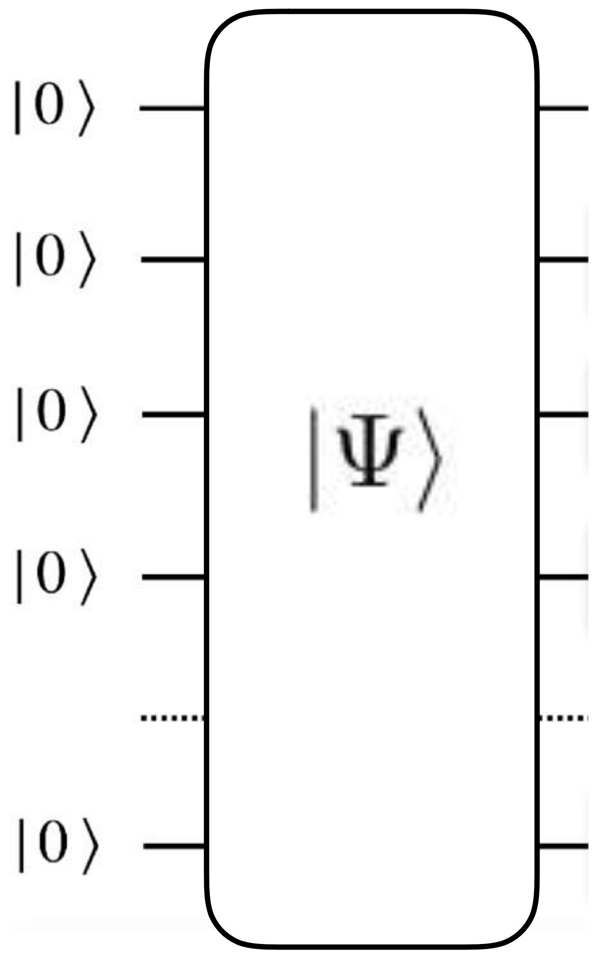
Dissimilarity of quantum states patterns

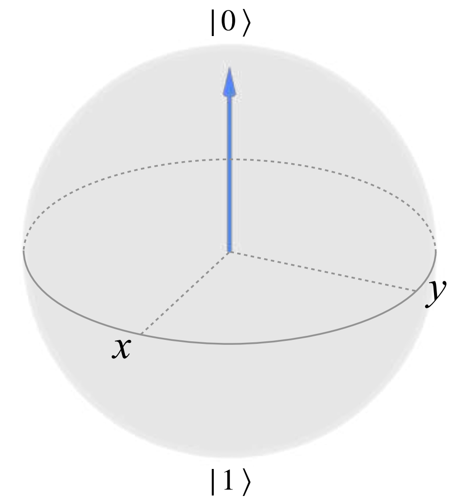
Initialization of the quantum state



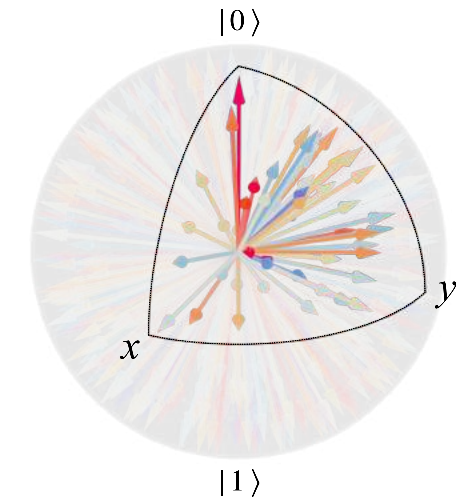
Basis matters

Basis choice





z basis



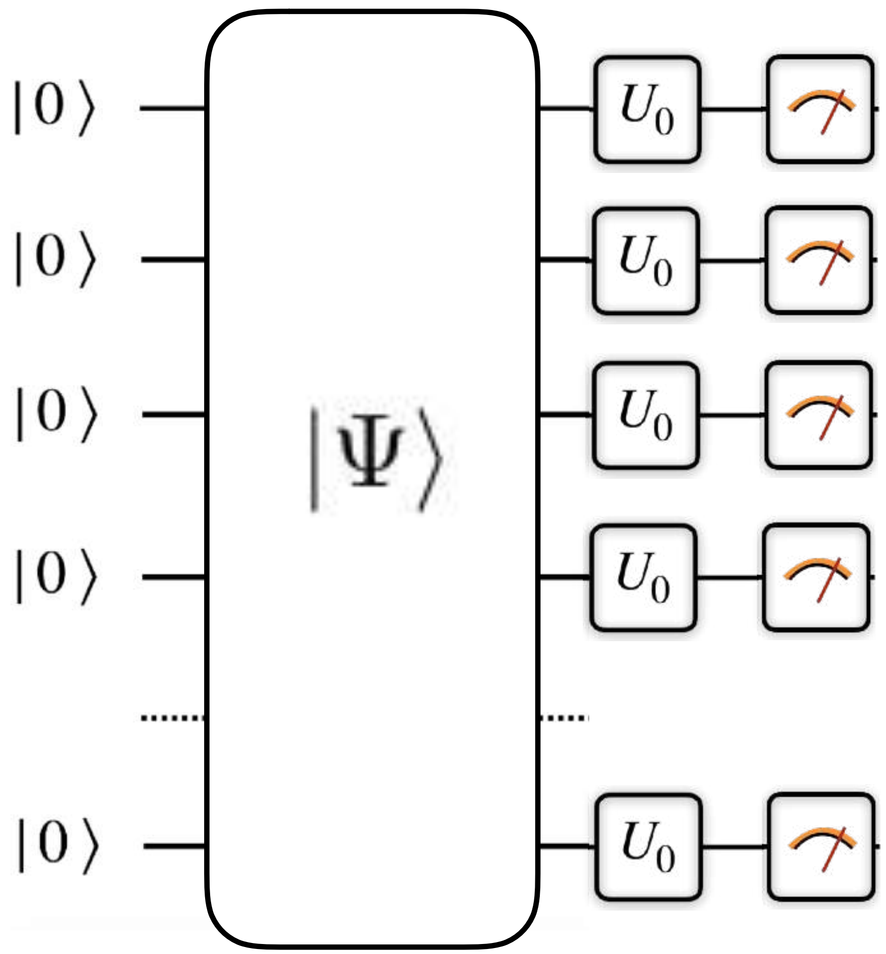
random basis

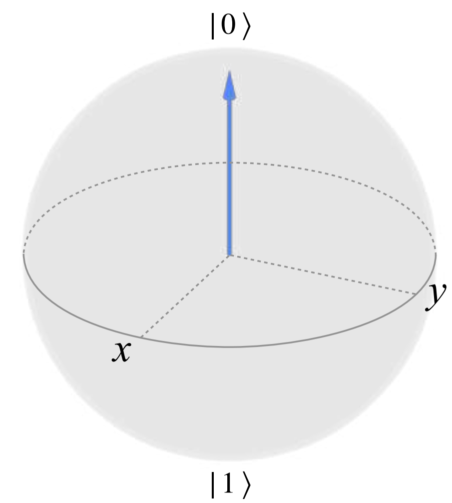
Example

$|X\rangle = (\mathrm{H}|0\rangle)^{\otimes N}$ is trivial in the x basis and uniform in z basis

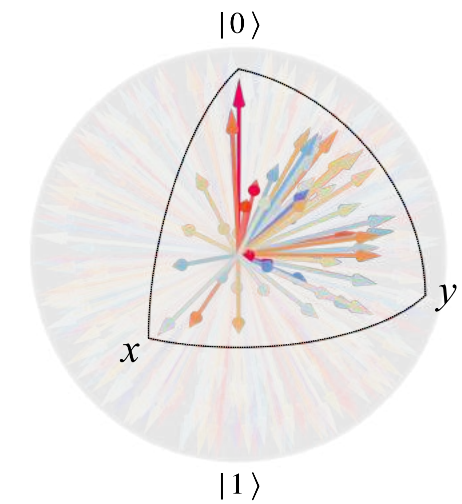
Dissimilarity of quantum states patterns

Measurements





z basis



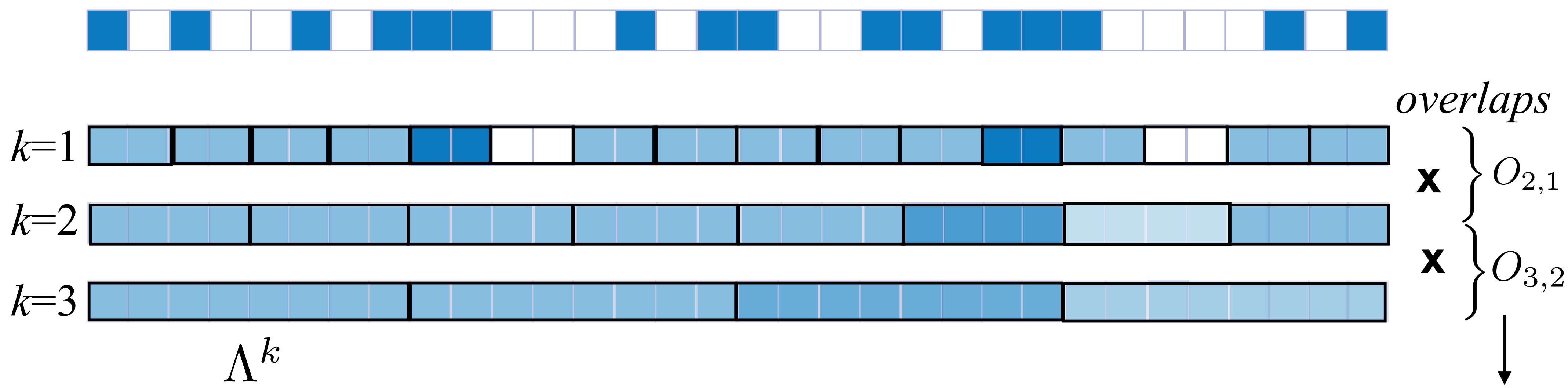
random basis

Bit-string array

$$\underbrace{01011010}_{\text{shot 1}}\underbrace{00111010}_{\text{shot 2}}\underbrace{01100100}_{\text{shot 3}}\underbrace{01111010}_{\text{shot 4}}$$

Dissimilarity of quantum states patterns

Renormalization



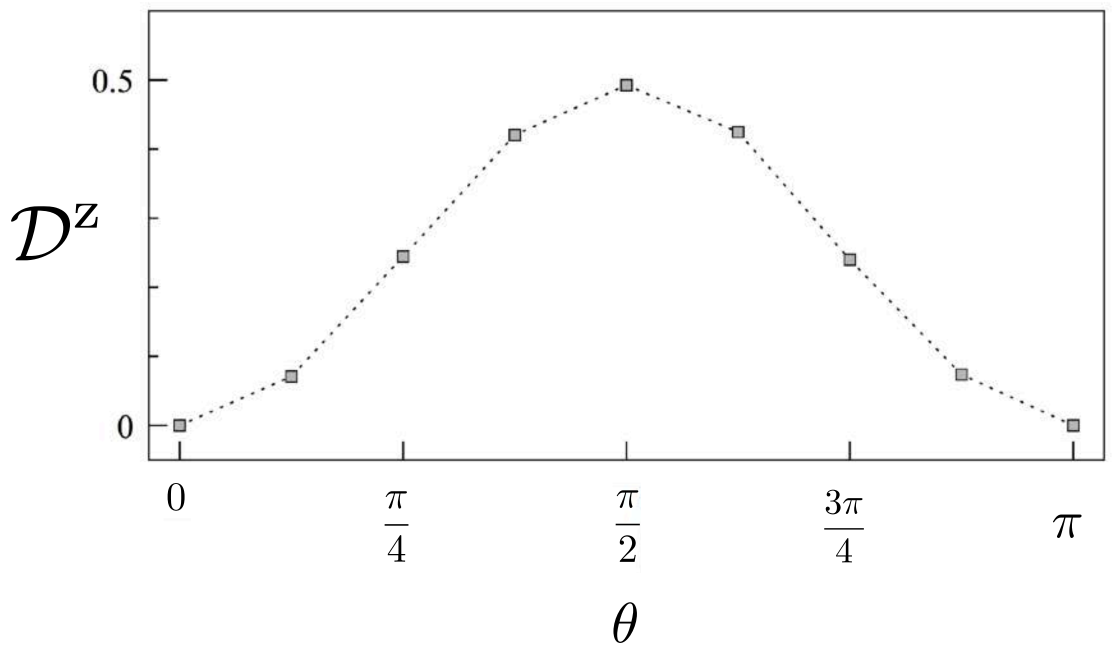
k-resolved dissimilarity $\mathcal{D}_k = \left|O_{k+1,k} - \frac{1}{2}\left(O_{k,k} + O_{k+1,k+1}\right)\right|$

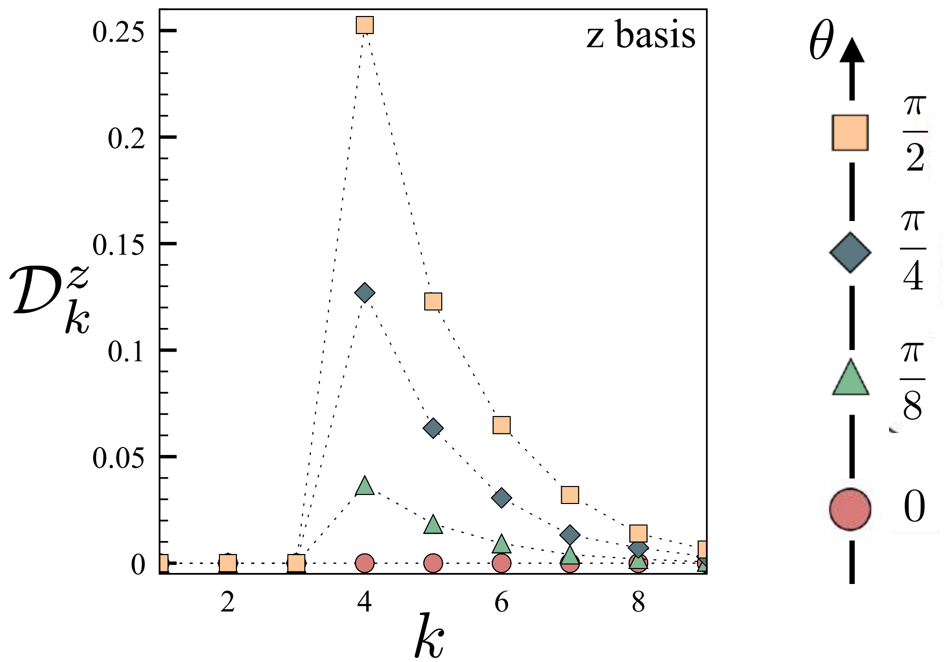
dissimilarity $\mathcal{D} = \sum_k \mathcal{D}_k$

Cat states

$$|\Psi_\theta\rangle = \cos(\frac{\theta}{2})|0\rangle^{\otimes N} + \sin(\frac{\theta}{2})|1\rangle^{\otimes N}$$

Dissimilarity of the 16-qubit system





All the states are distinguishable with the dissimilarity

Dicke states

$$|\Psi_D\rangle = \frac{1}{\sqrt{C_D^N}} \sum_j P_j(|0\rangle^{\otimes N-D} \otimes |1\rangle^{\otimes D})$$

sum over all possible permutations

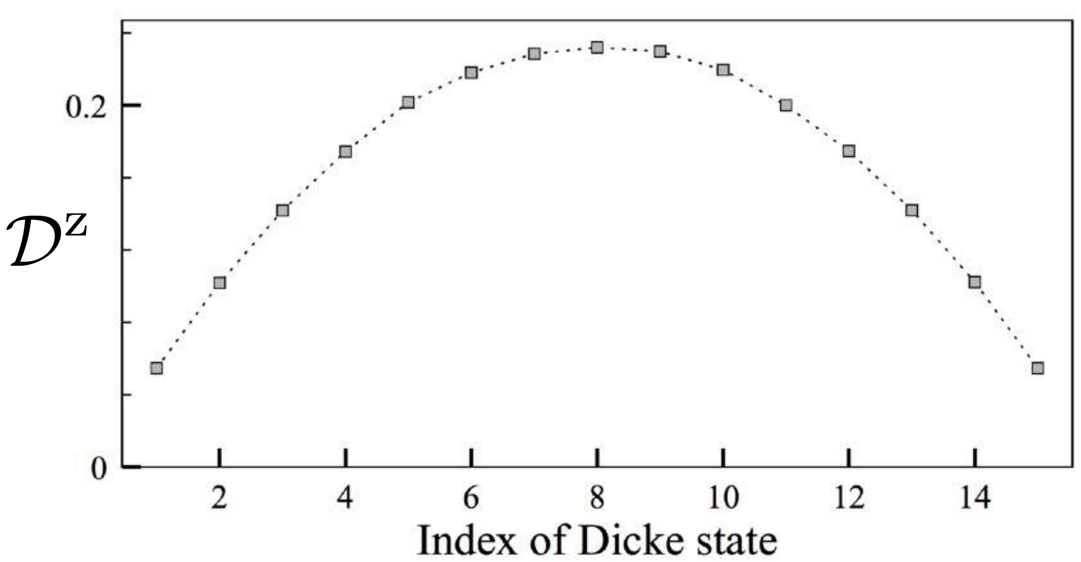
R.H. Dicke, Phys. Rev. (1954)

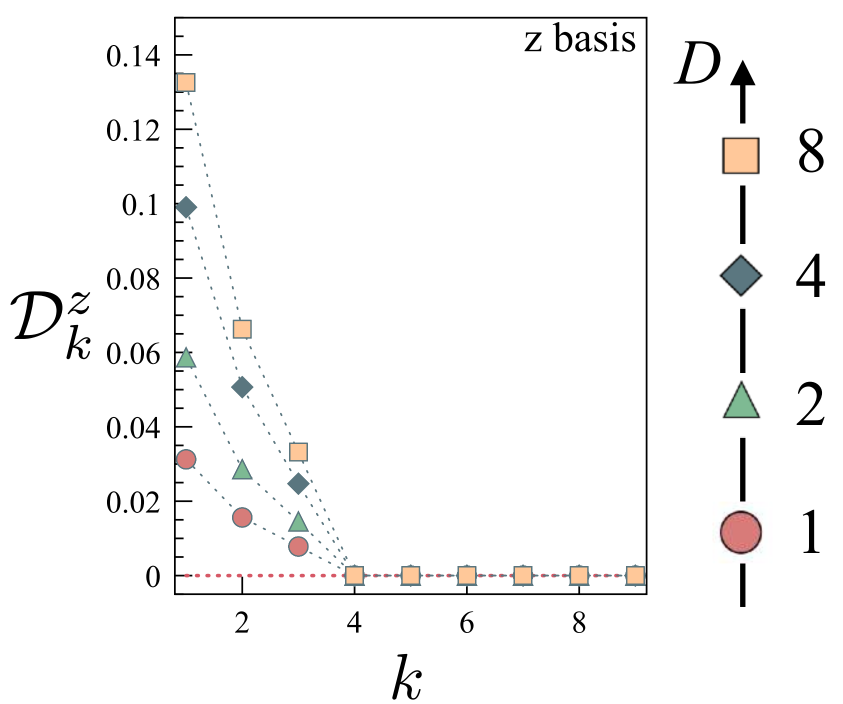
- Dicke states were experimentally realized in a system of 10000 atoms [PNAS 115, 6381 (2018)]
- Verification of the Dicke states and determination of their depth are actual problems [PRApplied 12, 044020 (2019)]

4-qubit example

$$|\Psi_1\rangle = \frac{1}{2}(|0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle)$$

Dissimilarity of 16-qubit system





Entanglement entropy

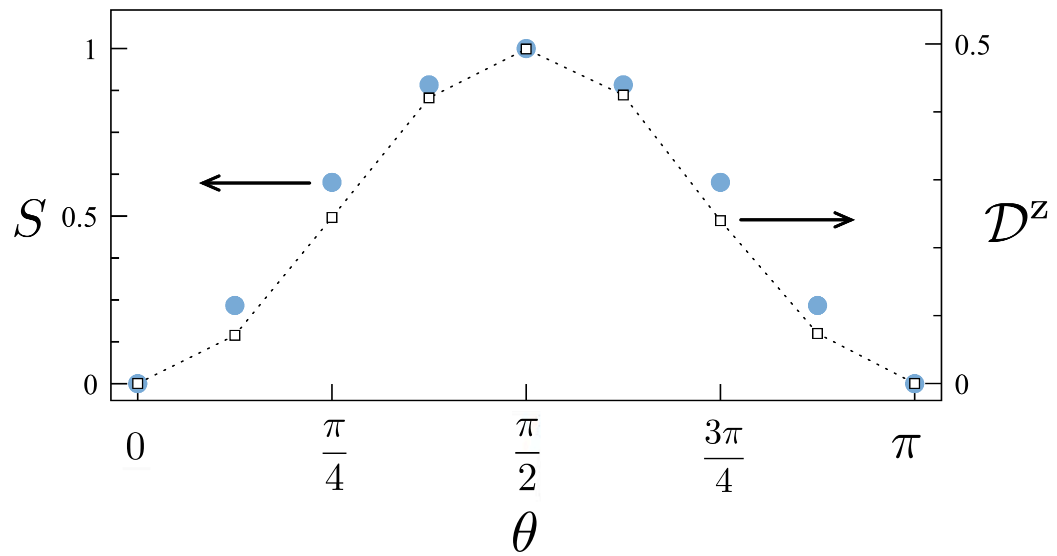
von Neumann entropy

$$S(\rho_{\mathrm{A}}) = -\mathrm{Tr}_{\mathrm{A}}\rho_{\mathrm{A}}\log_2(\rho_{\mathrm{A}})$$

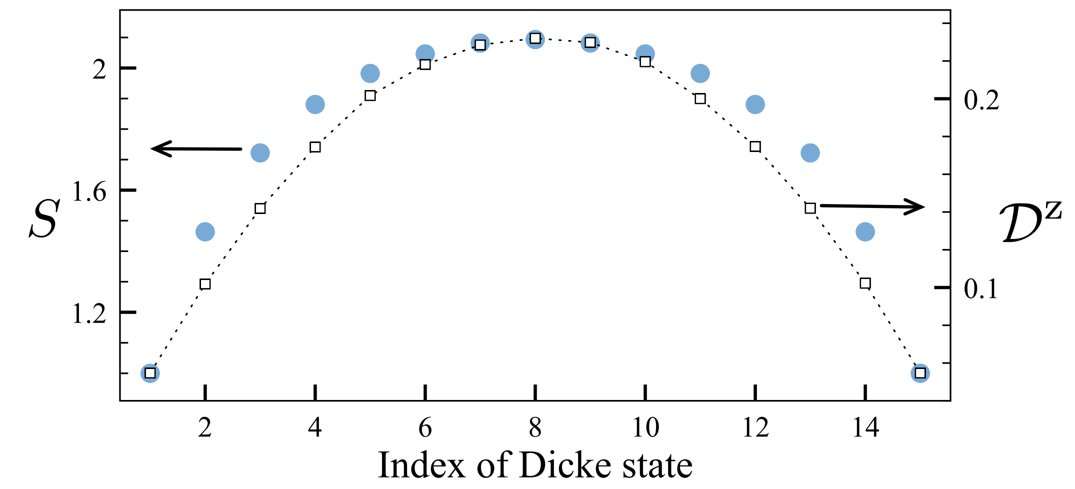
reduced density matrix

$$\rho_{\mathrm{A}} = \mathrm{Tr}_{\mathrm{B}}\rho_{\mathrm{AB}}$$

Cat states



Dicke states



Possibility to estimate the entropy changes for large-scale quantum states

Quantum states can be different

Localized examples

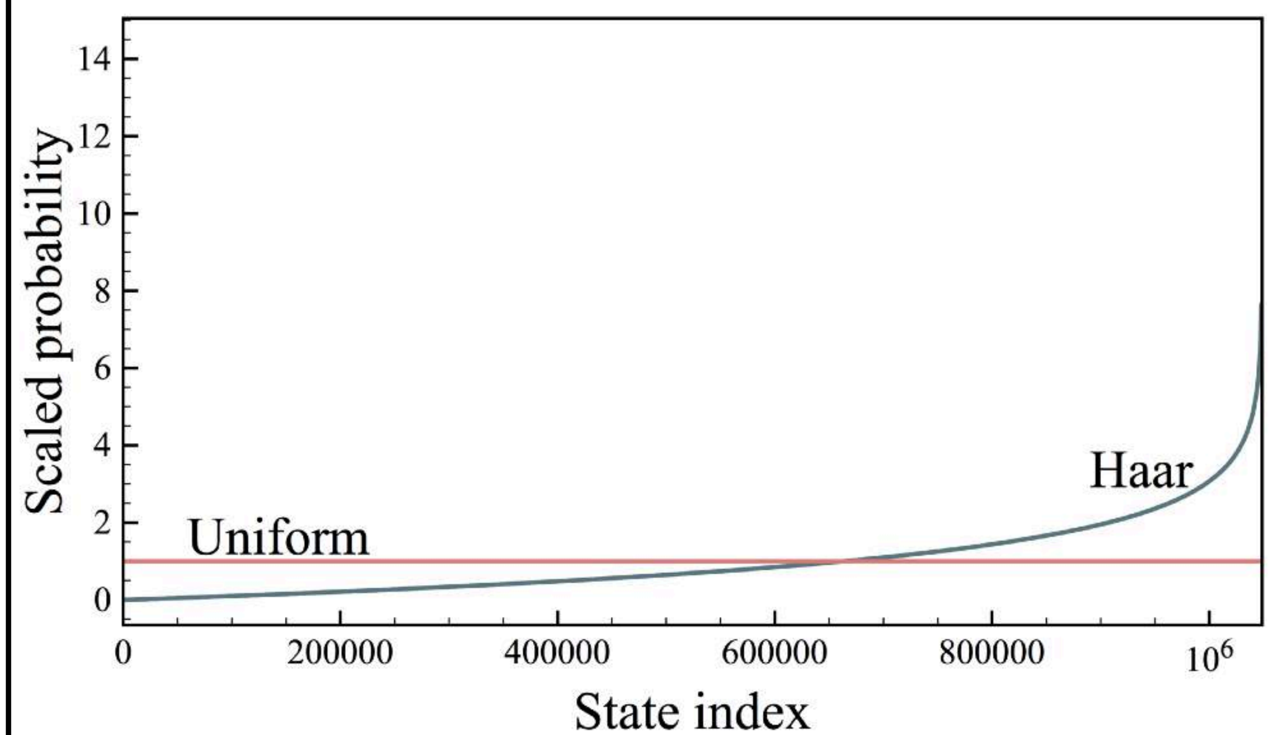
- $|\Psi_\theta\rangle = \cos(\frac{\theta}{2})|0\rangle^{\otimes N} + \sin(\frac{\theta}{2})|1\rangle^{\otimes N}$

two basis functions
giving nonzero contributions

- $|\Psi_D\rangle = \frac{1}{\sqrt{C_D^N}} \sum_j P_j(|0\rangle^{\otimes N-D} \otimes |1\rangle^{\otimes D})$

Dicke state has C_D^N nonzero weights

Delocalized examples



Distribution of the probabilities of
Haar-random state bitstrings
obeys the Porter-Thomas law

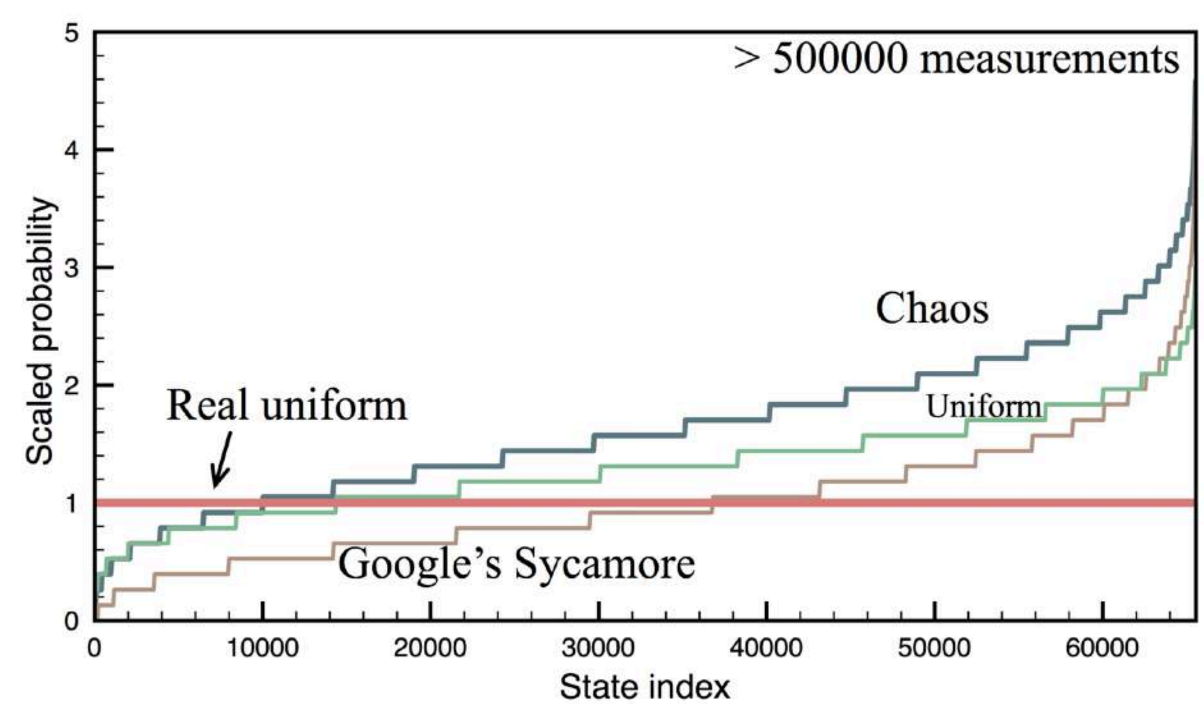
$$Pr(p) = 2^N e^{-2^N p}$$

Chaotic quantum states

Quantum chaos is useful

- Demonstration of quantum supremacy [Nature 574, 505 (2019)]
- Superdense coding of quantum states [PRL 92, 187901 (2004)]
- Transport phenomena [PRL **126**, 230501 (2021)]

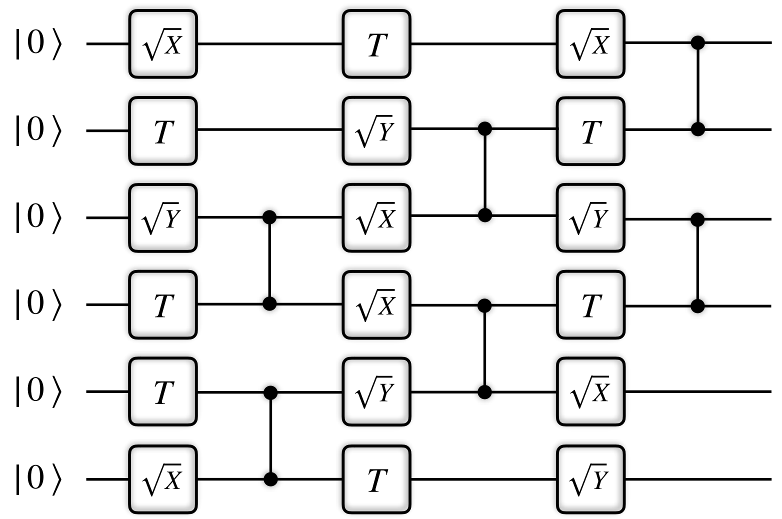
Detection of quantum chaos (16 qubits example)



It is not possible by calculating probabilities with a limited set of measurements

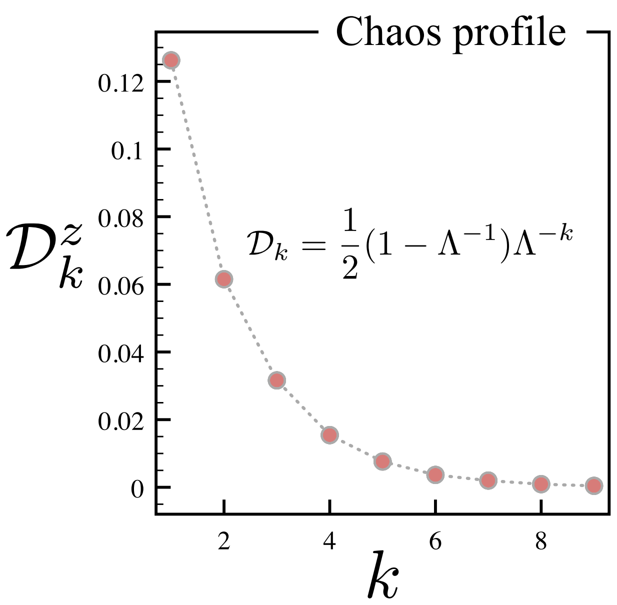
Dissimilarity of quantum chaos

quantum circuit

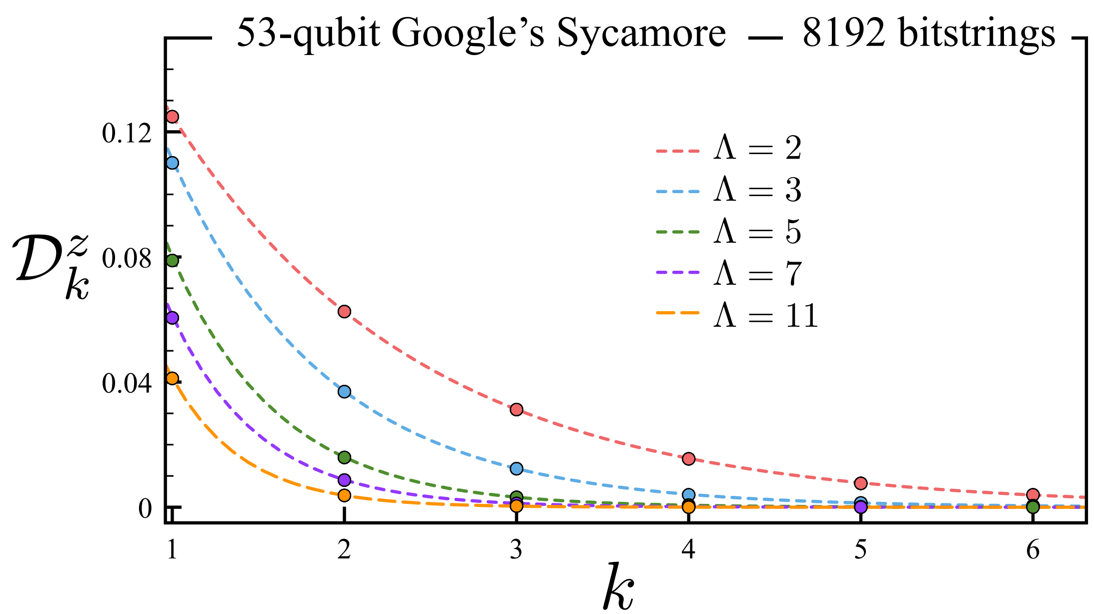


J. Richter and A. Pal, PRL (2021)

universal signature



experimental test



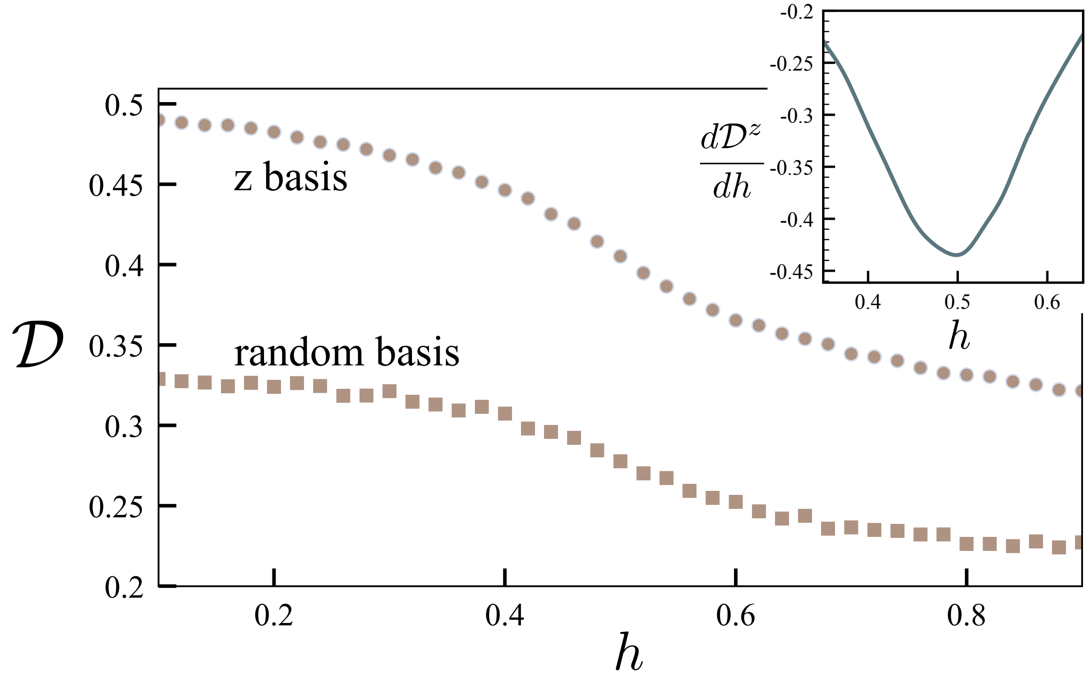
Phase transitions: Ising model

Spin Hamiltonian

$$H = J \sum_{ij} \hat{S}_i^z \hat{S}_j^z + h \sum_i \hat{S}_i^x$$

- classical test in the field of the quantum phase transitions
- ferromagnetic-paramagnetic transition at $h = J/2$

Dissimilarity (16 qubits)

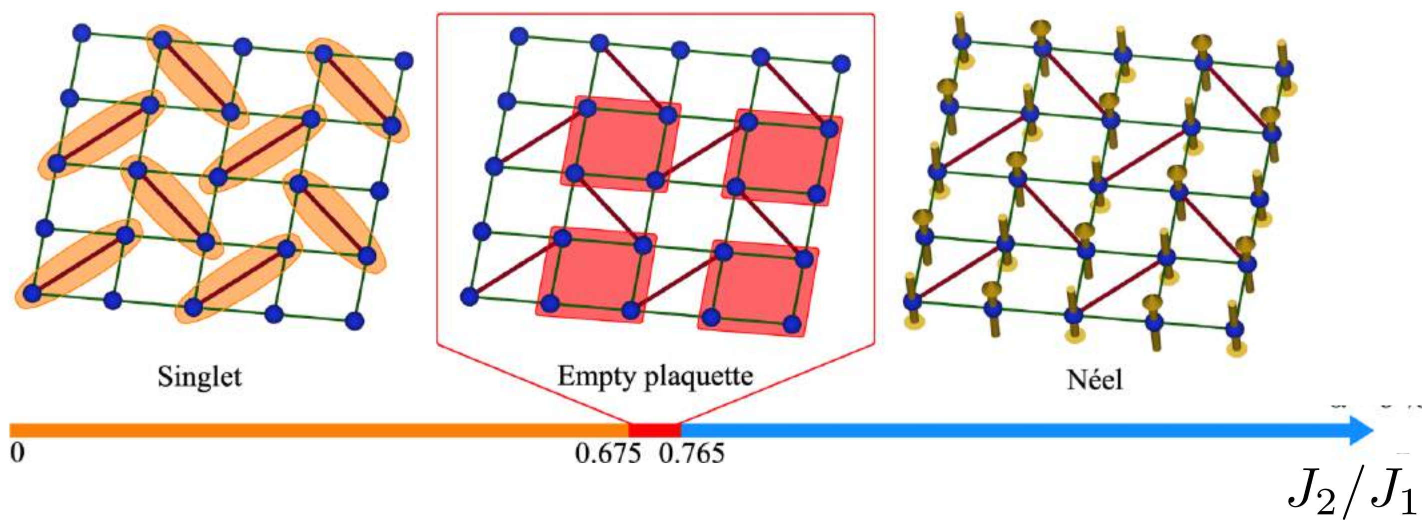


Shastry-Sutherland model

Spin Hamiltonian

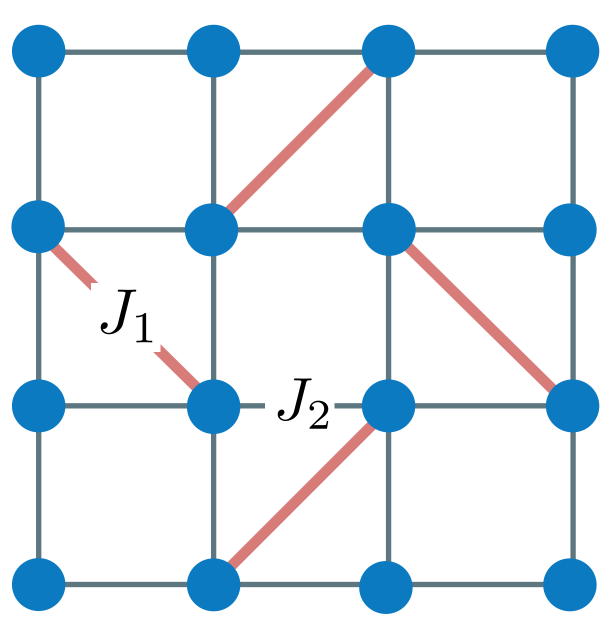
$$H = \sum_{\text{dimer}} J_1 \hat{\mathbf{S}}_i \hat{\mathbf{S}}_j + \sum_{\text{inter-dimer}} J_2 \hat{\mathbf{S}}_i \hat{\mathbf{S}}_j$$

Phase diagram

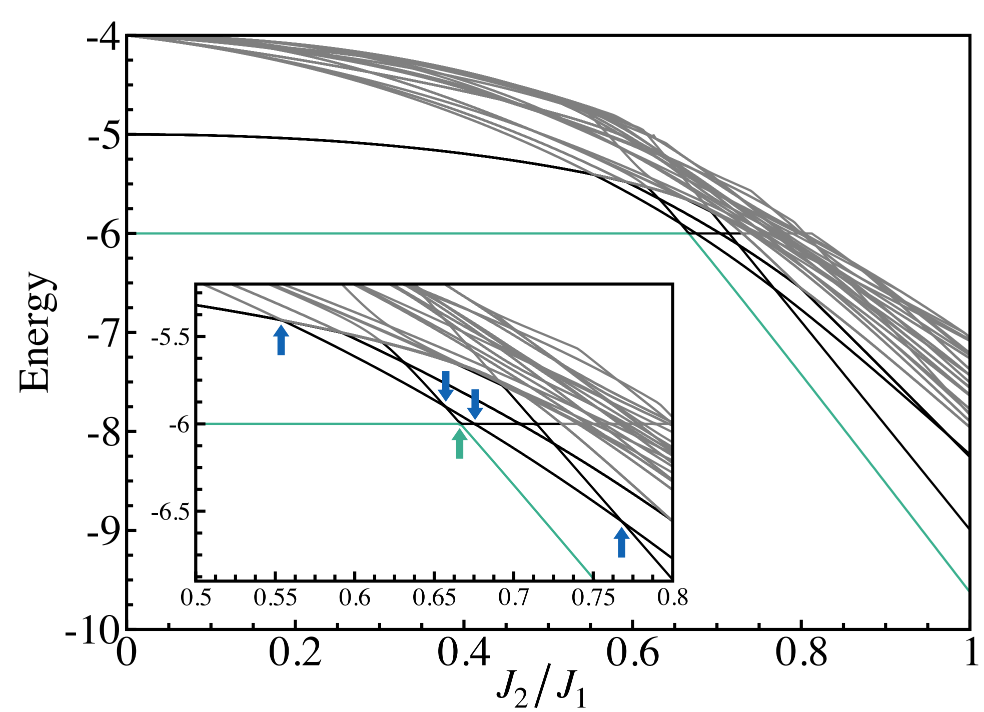


M.E. Zayed et al., Nat. Phys. (2017) D.I. Badrtdinov et al., PRB (2020)

Lattice



Exact diagonalization spectrum (16 qubits)



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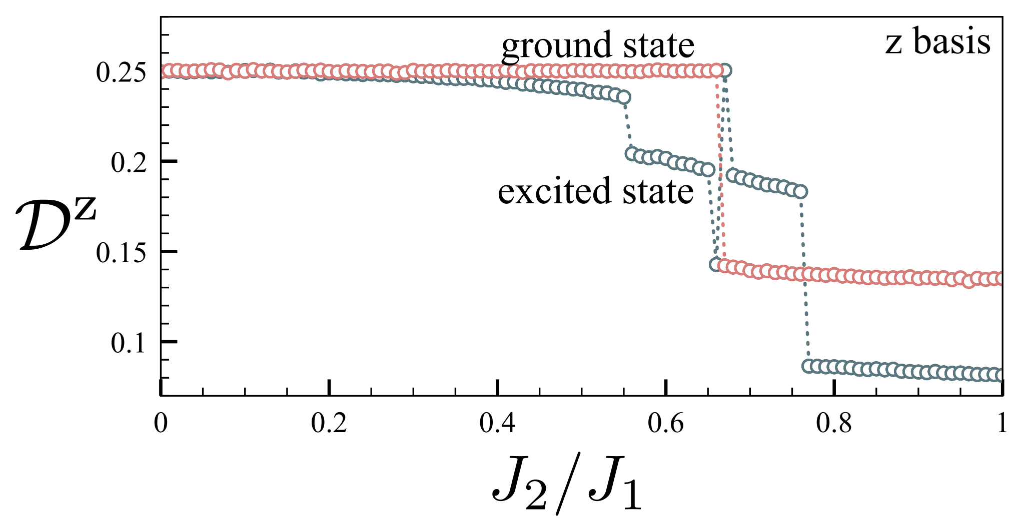
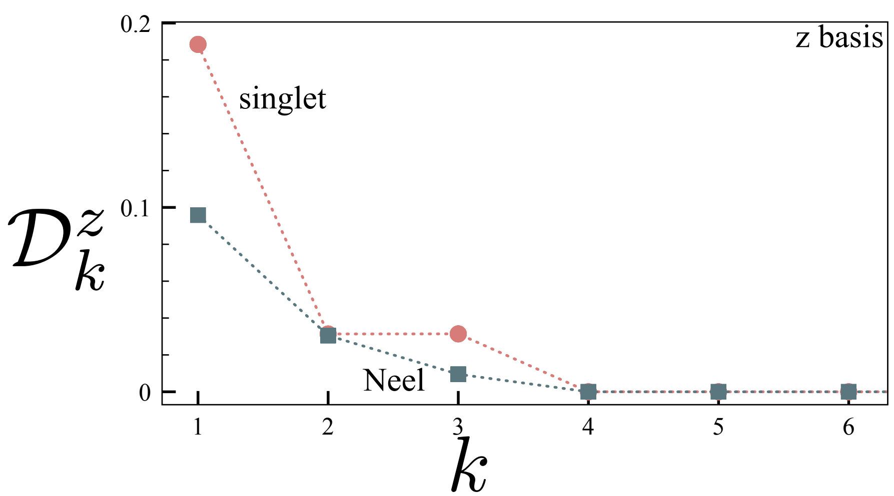
Dissimilarity (16 qubits)

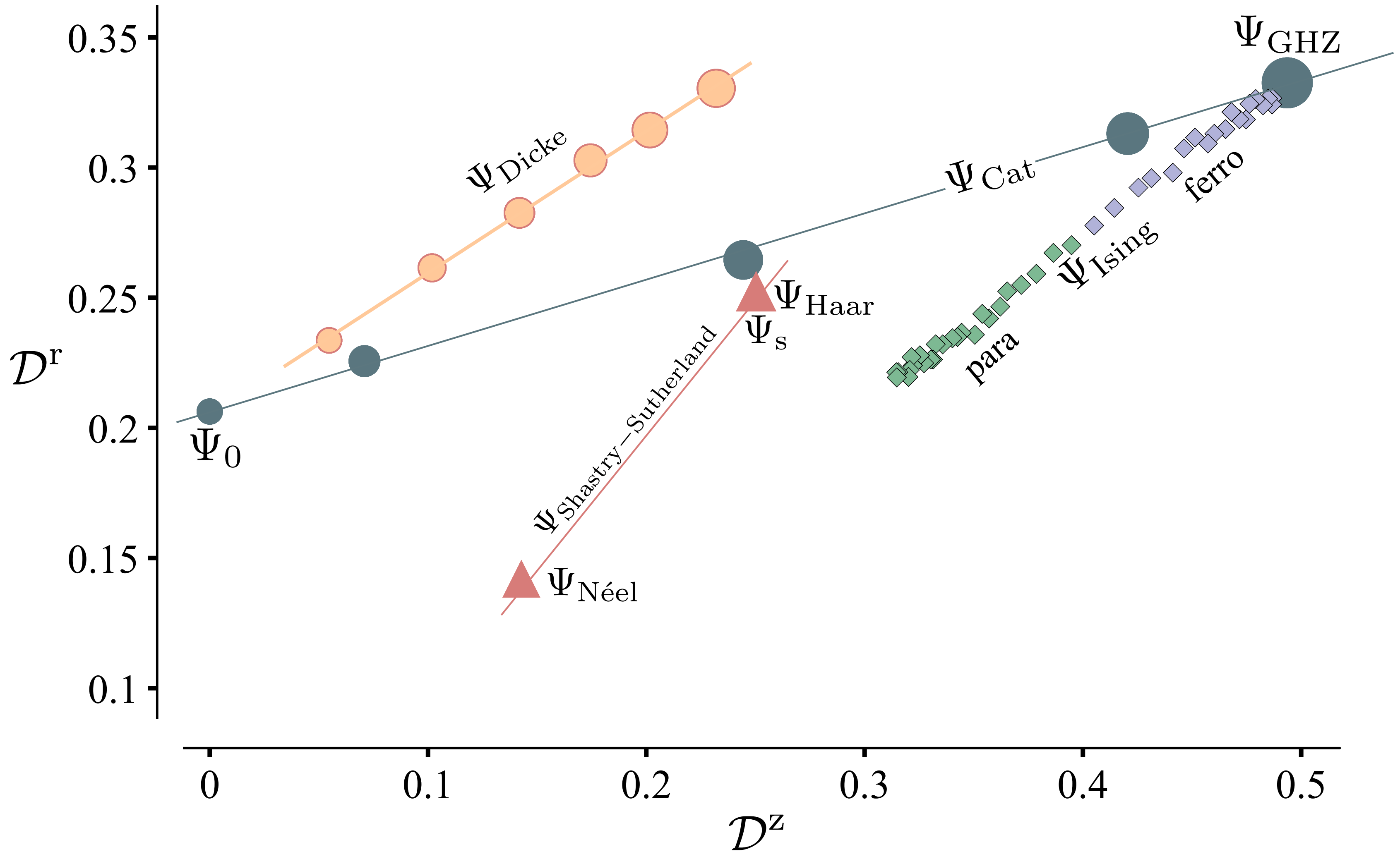


Partial dissimilarity



An accurate detection of the phase boundaries without calculations of order parameters

Dissimilarity map



- low-dimensional visualisation of quantum states
- possibility to trace the quantum state modifications

Summary

- $\mathcal{D} = \sum_k \mathcal{D}_k$ and $\{\mathcal{D}_k\}$ can be used for certification of the quantum states

- Dissimilarity procedure is cheap (from 1000 to 1000000 qubits right now)

- Accurate detection of quantum phase transitions