Exercise class 4. Magneto-optical phenomena.

1. Linearly polarized light is reflected from an isotropic medium in a magnetic field. The incidence of light is normal. The magnetic field is applied along the normal to the sample. Starting from the expression of the dielectric permittivity tensor in the presence of the magnetic field, calculate eigenvalues and eigenvectors of the tensor. Do these eigenvalues and eigenvectors have any physical meaning? Derive the relation between the amplitudes of the reflected and incident wave on a surface. How does the polarization of light changes upon reflection? (HINT: the off-diagonal components of the dielectric permittivity tensor are orders of magnitude smaller than the diagonal ones!)

2. An electromagnetic wave interacts with a ferromagnetic medium, which has the magnetization $\mathbf{M}_0$, and the magnetic component of the wave $\mathbf{h}$ induces in the medium a smaller magnetization $\mathbf{m}$. Interaction of the electric component of light with the medium is neglected ($\varepsilon = 1$). This is so-called magnetic dipole approximation. The medium was isotropic, but anisotropy was induced by a magnetic field $\mathbf{H}_0$ along the $z$-axis ($h << H_0$). Starting from the equation of motion, which is basically the law of conservation of the angular momentum, find the expressions for frequency dependencies of the tensor components for the magnetic susceptibility $\chi''(m_i = \sum_j \chi_{ij} h_j)$. If dissipation is neglected, the equation of motion, which is also called the Landau-Lifshitz equation, states:

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{H},$$

where $\mathbf{H} = \mathbf{H}_0 + \mathbf{h}$, $\mathbf{M} = \mathbf{M}_0 + \mathbf{m}$ and $\gamma$ is the gyromagnetic ratio. At which frequency does the diagonal component $\chi_{xx}$ have the resonance? What is the physical origin of this resonance? Show that $\chi_{ij} = \chi_{ji}^*$. How would linearly polarized wave propagate through such a medium?

Tips: a) transform the equation of motion from time to frequency domain b) neglect the terms of the second order of smallness (i.e. neglect terms such as $m_i h_j$)